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An Analysis of the Effect of Topography on the Martian Hadley Cells

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ABSTRACT

Previous work with Mars general circulation models (MGCMs) has shown that the north–south slope in Martian topography causes asymmetries in the Hadley cells at equinox and in the annual average. To quantitatively solve for the latitude of the dividing streamline and poleward boundaries of the cells, the Hadley cell model of Lindzen and Hou was modified to include topography. The model was thermally forced by Newtonian relaxation to an equilibrium temperature profile calculated with daily averaged solar forcing at constant season. Two sets of equilibrium temperatures were considered that either contained the effects of convection or did not. When convective effects were allowed, the presence of the slope component shifted the dividing streamline upslope, qualitatively similar to a change in season in Lindzen and Hou’s original (flat) model. The modified model also confirmed that the geometrical effects of the slope are much smaller than the thermal effects of the slope on the radiative–convective equilibrium temperature aloft. The results are compared to a simple MGCM forced by Newtonian relaxation to the same equilibrium temperature profiles, and the two models agree except at the winter pole near solstice. The simple MGCM results for radiative–convective forcing also show an asymmetry between the strengths of the Hadley cells at the northern summer and northern winter solstices. The Hadley cell weakens with increasing slope steepness at northern summer solstice but has little effect on the strength at northern winter solstice.

1. Introduction

The Martian atmosphere is affected by several asymmetries not present in the terrestrial one. One such asymmetry, referred to as the topographic dichotomy, is that the zonally averaged surface topography is at higher elevation in the Southern Hemisphere than the Northern Hemisphere (Fig. 1). Another asymmetry exists because of Mars’ relatively eccentric orbit (the Martian orbital eccentricity is 0.0934 whereas the terrestrial is 0.0167), which causes the solar insolation to differ by 44% between perihelion and aphelion. Since Southern Hemisphere summer is near perihelion and thus Northern Hemisphere summer is near aphelion, the solar insolation varies greatly between the solstices. Atmospheric dust, important radiatively, also shows a strong seasonal variation, with Southern Hemisphere spring and summer as the favored seasons for large-scale dust storms (Kahn et al. 1992).

Haberle et al. (1993) noted a factor of 2 difference in Hadley cell intensity between the two solstices using the National Aeronautics and Space Administration (NASA) Ames Mars general circulation model (MGCM), which they attributed to the variation in the solar constant between these two seasons. However, Joshi et al. (1995) found that when the solar forcing was held constant between the two seasons, a factor of 1.5 difference in Hadley cell intensity was still present in the Oxford “intermediate” MGCM simulations. Wilson and Hamilton (1996) observed with the Geophysical Fluid Dynamics Laboratory (GFDL) MGCM that the zonal mean component of topography inhibited Hadley cell intensity during Northern Hemisphere summer. Haberle et al. (1993) and Basu et al. (2006) remarked on the sensitivity of the Martian Hadley circulation to off-equatorial heating. Basu et al. (2006) also found that the relative strength of the circulation at the opposite solstices depends on dust loading.
Webster (1977) realized that elevated regions on Mars may act as heat sources for the adjacent atmosphere because the (nondusty) Martian atmosphere is effectively transparent to solar radiation. Two recent studies have focused on the effects of the north–south slope in the zonally averaged topography on the Hadley circulation. Richardson and Wilson (2002) noted in the GFDL MGCM results that the annually averaged zonal mean circulation contained a stronger Northern Hemisphere Hadley cell, which also extended southward across the equator. They performed two further experiments in which the argument of perihelion was shifted by 180° (to test the effect of seasonal differences in the strength of the solar forcing) and in which the zonal mean component of topography was removed (leaving only the mountain or “wave” component). The removal of the zonal mean component of topography created two cells of nearly equal strength and shape, while the shift in the argument of perihelion produced little change from the full MGCM run. These results suggest that the north–south slope in topography is important, but the strength of the solar forcing is secondary.

Similarly, Takahashi et al. (2003) found that in their own MGCM results at equinox the northern cell was stronger than the southern and extended across the equator into the Southern Hemisphere. They conducted three runs at perpetual equinox in which variations in only one of the following were included: topography, surface thermal inertia, and surface albedo. The runs with either surface thermal inertia only or surface albedo only did not match the control run with all three parameters, but the run with topography did. Two subsequent experiments in which either only the zonal mean component of topography or only the zonal wave component were included showed that—as in the study by Richardson and Wilson (2002)—the zonal mean component of topography is the dominant factor in causing an asymmetric Hadley circulation.

Both Richardson and Wilson (2002) and Takahashi et al. (2003) concluded that the cause of the asymmetry was an upslope (i.e., southward) shift in the peak heating. Takahashi et al. (2003) went on to state that the convective heating term was the main influence in this shift. In this paper, we expand the analysis of the effects of convection and the north–south topographic slope to include other seasons, most notably the solstices. We also not only use a simple MGCM, but also apply a modified version of the Hadley cell model of Lindzen and Hou (1988) that includes the effects of topography. To drive both of these models, we use a simple radiation scheme that assumes a gray atmosphere and no dust. While some aspects of the Martian atmosphere may be poorly represented by this assumption, we show that it captures the important aspects of Hadley cell dynamics in section 4d by comparing our scheme with the GFDL MGCM, which contains a nongray radiation scheme. The gray radiation scheme has the advantage of being analytical [such that it can be used in the modified Lindzen and Hou (1988) model] and it allows us to formulate a conceptually simple description of Hadley cell dynamics in the presence of a north–south slope.

In section 2, we develop the simple radiative transfer model that we use to calculate equilibrium temperature. The effects of convection are included in our radiative–convective model but are not included in our “pure” radiative model. The equilibrium temperatures are used in section 3, where we derive a model based on Lindzen and Hou (1988) that predicts the latitude of the dividing streamline and the poleward extent of the Hadley cells in the presence of nonzero topography. We use this model to solve for the boundaries of the Hadley cells with and without the zonal mean component of topography and also explicitly as a function of season [recall that Richardson and Wilson (2002) concentrated their study on the annual average, and Takahashi et al. (2003) focused on the equinox]. In section 4 we present a simple MGCM and compare it to the modified Lindzen and Hou (1988) model in section 5.

2. Calculation of equilibrium temperature

In both the modified Lindzen and Hou (1988) model (section 3) and the simple MGCM (section 4), the external thermal forcing is applied through Newtonian relaxation to a radiative equilibrium state, represented by the equilibrium temperature $T_{eq}$. It should be stressed that $T_{eq}$ is a proxy for the diabatic heat source and does not correspond to a physical parameter that can be measured. We have deliberately chosen a conceptually simple radiation scheme in order to more clearly understand its
effect on the results. Our experiments use two types of equilibrium configurations. The first is referred to as pure radiative equilibrium. Assuming the Eddington approximation, no dust, no scattering, no solar absorption by the atmosphere, and a gray atmosphere in the long wave, the solution to the radiative transfer equation is

\[
\sigma T_{eq,R}(\phi, L_s, \tau) = \begin{cases} 
Q_o(\phi, L_s)[0.5 + 0.75\tau(p)] & \tau \neq \tau_o, \\
Q_o(\phi, L_s)[1 + 0.75\tau_o(p_o(\phi))] & \tau = \tau_o,
\end{cases}
\]

where \(T_{eq,R}\) is the pure equilibrium temperature, \(\sigma\) is the Stefan–Boltzmann constant, \(\phi\) is latitude, \(L_s\) is the ecliptic longitude of the sun (in Mars-centered coordinates), \(p\) is pressure, \(Q_o\) is the daily averaged net solar flux, \(\tau\) is the optical depth, and \(\tau_o\) is the optical depth at the surface. Under the assumption of constant opacity, \(\tau(p) = p\tau_o/p_o\), where \(p_o\) is a reference pressure set to the mean surface pressure of 6 hPa and \(\tau_o\) (here taken to be 0.2) is the optical depth at \(p_o\). Likewise \(\tau_o[p_o(\phi)] = p_o(\phi)\tau_o/p_o\), where \(p_o\) is the height of the surface in pressure coordinates. Note that \(\tau\) increases downward.

From Peixoto and Oort (1992, 99–100),

\[
Q_o(\phi, L_s) = \frac{S_o}{\pi}(1 - A)\left[\frac{1 + e \cos(L_s - L_{sp})}{1 - e^2}\right]^2 \times (\gamma \sin \phi \sin \delta + \sin \gamma \cos \phi \cos \delta),
\]

where \(S_o = 600 \text{ W m}^{-2}\) is the mean solar constant, \(A\) is the albedo (set to a constant value of 0.15), \(e = 0.0934\) is the eccentricity, \(L_{sp} = 252^\circ\) is the \(L_s\) of perihelion (located near northern winter solstice), \(\delta\) is the declination of the sun (in Mars-centered coordinates), and \(\gamma\) is the hour angle of sunrise and sunset. Furthermore, \(\cos \gamma = -\tan \phi \tan \delta\) and \(\sin \delta = \sin \mu \sin L_s\), where \(\mu = 25^\circ\) is the obliquity. The square of the term in brackets in (2) represents the correction to \(S_o\) due to the changing distance from the sun in an elliptical orbit. For Earth, this quantity varies from 0.968 to 1.069 between perihelion and aphelion, respectively, whereas for Mars it varies from 0.837 to 1.217.

The second prescribed equilibrium state is radiative–convective equilibrium, in which the temperature above the convective layer is the same as in the pure radiative equilibrium state, and the temperature within the convective layer follows an adiabat. Thus,

\[
\sigma T_{eq,RC}(\phi, L_s, \tau) = \begin{cases} 
Q_o(\phi, L_s)[0.5 + 0.75\tau(p)] & \tau < \tau_i, \\
Q_o(\phi, L_s)[0.5 + 0.75\tau_i(\phi)]\left[\tau(p)/\tau_i(\phi)\right]^{4R_c/p} & \tau \geq \tau_i,
\end{cases}
\]

\[\text{at constant pressure. The height of the convective layer is calculated by requiring continuity of temperature and radiative flux (surface plus atmosphere) at } \tau_i. \]

Explicitly,

\[
\sigma T_{eq,RC}(\tau_o) = \int_{\tau_i}^{\tau_o} \sigma T_{eq,RC}(\tau) \exp \left[\frac{-(\tau - \tau_i)}{\mu}\right] d\tau + \int_{\tau_i}^{\tau_o} \sigma T_{eq,RC}(\tau) \exp \left[\frac{-(\tau - \tau_i)}{\mu}\right] d\tau,
\]

where \(\mu = 2/3\) is the average cosine of the emission angle and \(\tau_i\) is a variable of integration. Substitution of (1) and (3) for \(T_{eq,R}\) and \(T_{eq,RC}\), respectively, in (4) shows that \(\tau_i\) is a function of only \(R\), \(c_p\), and \(\tau_o\) (i.e., the height of the surface). The key difference between \(T_{eq,R}\) and \(T_{eq,RC}\) is that for the latter the equilibrium temperature aloft (within the convective layer) depends on the height of the surface, whereas the former does not.

The Martian atmosphere follows a seasonal cycle in which atmospheric CO₂ freezes out during the winter,
causing the surface pressure to vary about its annually averaged value by over 1 hPa (e.g., Hess et al. 1980).

Haberle et al. (1993) found that the mass change does not significantly affect the zonal mean circulation except at the lowest levels over the polar caps; therefore, we have chosen to keep atmospheric mass constant. When \( T_{eq} \) falls below the CO\(_2\) condensation temperature \( T_{co2} \), it is instantaneously reset to \( T_{co2} \). From the Clausius–Clapeyron equation,

\[
T_{co2} = \left[ \frac{1}{T_1} - \frac{R}{L} \ln\left(\frac{p}{p_1}\right) \right]^{-1},
\]

where \( T_1 = 136.6 \) K is the reference temperature at the reference pressure \( p_1 = 1 \) hPa (James et al. 1992), \( p \) is in hPa, \( L = 5.9 \times 10^5 \) J kg\(^{-1}\) is the specific latent heat of sublimation, and \( T_{co2} \) is given in K.

Figures 2 and 3 show zonally averaged \( T_{eq,R} \) and \( T_{eq,RC} \) given by (1) and (3), respectively, for \( L_s = 0^\circ \), \( L_s = 90^\circ \), and \( L_s = 270^\circ \). The following three topographies are considered: flat topography at zero elevation (“flat”), Mars Orbiter Laser Altimeter (MOLA) topography (“full”; Fig. 4a), and zonal mean MOLA topography (“mean”; Fig. 4b). For the pure radiative case, \( T_{eq} \) does not vary between the different topographies—it is as if the surface has been pasted onto the temperature profile. Conversely, the surface modifies the equilibrium temperature aloft in the radiative–convective case. The equilibrium temperature contours appear to be pushed up by the higher surface, changing the meridional temperature gradient so that the equilibrium temperature is higher above an elevated surface.

Our values for \( T_{eq,R} \) (and hence \( T_{eq,RC} \)) at high altitudes (not shown) are warmer and more isothermal than the corresponding values for \( T_{eq,RC} \) plotted in Joshi et al. (1995) and Haberle et al. (1997). While these authors use different radiative transfer models, the discrepancy is likely a result of the choice of \( \tau_{oo} \) and the gray atmosphere assumption, which is not fully accurate for Mars. Hinson et al. (2008) observed in radio occultation
data that the depth of the mixed layer varies with elevation, an effect that is not captured in our model. Caballero et al. (2008) derive a radiative–convective model for a semigray atmosphere, which they apply to Mars. Their predicted height of the convective layer in coordinates \( p/p_{oo} \) is 0.44 (cf. our value of 0.51 for flat topography).

Figure 5 shows \( T_{eq,R} \) and \( T_{eq,RC} \) at a constant pressure level of 3.8 hPa or 5-km log pressure height, which is within the convective layer. It is evident that topography

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**Fig. 3.** As in Fig. 2, but for zonally averaged radiative-convective equilibrium temperature (K).

**Fig. 4.** The topography of Mars: (a) the full topography measured by MOLA (“full”) and (b) the zonal mean MOLA topography (“mean”). Elevations are in km; contour interval is 0.5 km.
changes the meridional gradient of $T_{\text{eq,RC}}$ (cf. the heavy solid and heavy dashed lines). For flat topography, $T_{\text{eq,RC}}$ is offset to a higher temperature from $T_{\text{eq,R}}$ because in the former convection acts to redistribute heat from the surface into the convective layer. In pure radiative equilibrium, a temperature discontinuity exists between the surface and the atmosphere directly above the surface. The radiative–convective equilibrium temperature profile follows a dry adiabat that is continuous with the surface temperature.

Figure 6 shows a diagram of the two different types of $T_{\text{eq}}$. The gray and black curve is the pure radiative equilibrium solution for $T_{\text{eq}}$ as a function of pressure and is the same regardless of surface height. If the surface is located at some elevation $p_o$, located at the level of the solid black horizontal line, then the equilibrium surface temperature is given by $T_g$ (black text). If the surface is then located at some higher elevation (lower pressure), represented by the solid gray horizontal line, the equilibrium surface temperature ($T_g$ in gray text) will decrease because of the reduced greenhouse effect. In an optically thin atmosphere such as Mars, however, this effect is extremely small, and the surface temperature can be considered independent of surface height.

The dashed lines in Fig. 6, drawn for both surface elevations, represent adiabats calculated such that the convective layer remains in energy balance with the rest of the atmosphere. Comparing the radiative–convective equilibrium temperatures at some pressure level $p_a$ within the convecting layer shows that they are higher above the elevated surface than over the lower surface. The radiative–convective equilibrium temperature profiles are being evaluated at different relative heights above the local surface (the primary heating element for the atmosphere), which dominates over the reduced greenhouse effect at the higher altitude. In this way, the height of the surface modifies the heating profile of the lower atmosphere. Moreover, the change in surface temperature due to the reduced greenhouse effect at higher elevations is again negligible, varying by less than 3 K for a difference in elevation of 5 km.

3. Hadley cell model with topography

Held and Hou (1980) derived a model of the steady, nearly inviscid, Boussinesq, axisymmetric, Hadley circulation in the absence of eddies. Diabatic heating was represented by Newtonian relaxation of temperature at a uniform rate $\alpha$ toward a prescribed distribution of equilibrium temperature $\theta_e$, which in their case was symmetric about the equator. Through consideration of the constraint of angular momentum conservation, they showed that the Hadley cell is confined to a finite region about the equator; poleward of the edge of the cell, there is no meridional circulation, and temperatures are in radiative equilibrium. Using simple arguments, they were able to solve for the location of the cell edge and other characteristics of the circulation.

Lindzen and Hou (1988) solved the problem in the nonsymmetric case, where the imposed equilibrium temperature maximum is located on the summer side of the equator. They found strong sensitivity—only a modest shift of the peak of $\theta_e$ off the equator, the rising branch of the circulation shifted much further, and the resulting
cross-equatorial cell is much stronger than the opposite cell in the summer hemisphere. Recently, Caballero et al. (2008) revised the Lindzen and Hou (1988) result using a more sophisticated, semigray specification of equilibrium temperature. Their radiation scheme as- sumed a CO2-like absorber, where absorption is zero everywhere outside of a narrow band at a single wavelength. Given the appropriate radiation parameters (i.e., total broadband optical depth, pressure broadening, and equivalent bandwidth), they derive an analytical solution for the Hadley cell width, depth, energy transport, and mass flux.

Here we follow the analysis of Lindzen and Hou (1988), modifying the problem to permit varying height of the surface and the equilibrium temperatures discussed in section 2. A schematic is shown in Fig. 7. The Hadley cells are confined between latitudes \( \phi_+ \) and \( \phi_- \), with the dividing streamline at \( \phi_1 \). The height of the surface as a function of latitude is given by \( z_b(\phi) \). Nonaxisymmetric effects are ignored. The governing principles are as follows: conservation of absolute angular momentum,

\[
M(\phi) = \Omega a^2 \cos^2 \phi + u(\phi, z)a \cos \phi, \tag{6}
\]

at upper levels, where \( z \) is height, \( \Omega \) is the planetary rotation rate, \( u \) is the zonal wind, and \( a \) is the planetary radius; gradient wind balance,

\[
2\Omega \sin \phi a \theta + \frac{1}{a} \tan \phi a^2 = -\frac{1}{a} \frac{\partial \Phi}{\partial \phi}, \tag{7}
\]

where \( \Phi \) is geopotential; hydrostatic balance,

\[
\frac{\partial \Phi}{\partial z} = \frac{g}{\theta_0} \frac{\partial \theta}{\partial \phi} \tag{8}
\]

where \( g \) is the acceleration of gravity at the surface, \( \theta \) is temperature, and \( \theta_0 \) a constant reference temperature (note that in the Boussinesq approximation, temperature and potential temperature are the same); and the steady thermodynamic equation,

\[
\frac{v}{a} \frac{\partial \theta}{\partial \phi} + \frac{w}{\partial z} = -\alpha(\theta - \theta_e), \tag{9}
\]

where \( v \) and \( w \) are the meridional and vertical velocities, respectively.

Now, consider the dividing streamline at \( \phi = \phi_1 \). Assuming that the zonal flow near the surface is weak, the air rises out of the boundary layer with angular momentum \( M_0 = \Omega a^2 \cos^2 \phi_1 \), from (6). Conservation of angular momentum then dictates that everywhere along this streamline (outside the boundary layer) \( M(\phi, z) = M_0 \), where \( M_0 \) is a constant; hence, within each cell and along the top of the model at \( z = h \), (6) requires that

\[
u(\phi, h) = \Omega a \frac{(\cos^2 \phi_1 - \cos^2 \phi)}{\cos \phi}. \tag{10}\]

At \( z = z_b(\phi) \), assume (again following Held and Hou) that \( u \) is much weaker than at the top, such that subtracting (7) applied at \( z = z_b \) from that applied at \( z = h \) and substituting from (10) gives

\[
\Omega^2 a^2 \frac{\sin^2 \phi}{\cos^3 \phi}(\cos^4 \phi_1 - \cos^4 \phi)
- \frac{\partial \Phi}{\partial \phi} \bigg|_{z=h} + \frac{\partial \Phi}{\partial \phi} \bigg|_{z=z_b}
- \frac{\partial}{\partial \phi} \Phi(\phi, z_b - \Phi(\phi, z_b)) \bigg|_{z=z_b} - \frac{dz_b \, \partial \Phi}{\partial \phi} \bigg|_{z=z_b}. \tag{11}
\]

Using (8) and its vertical integral, and integrating in \( \phi \), we obtain

\[
\frac{g}{\theta_0} \int_{z_b}^{h} \theta \, dz + \frac{g}{\theta_0} \int_{\phi_1}^{\phi} \theta(\phi', z_b) \frac{dz_b}{d\phi'} \, d\phi'
= \Phi - \Omega^2 a^2 \frac{1}{2} \left( \cos^2 \phi + \frac{\cos^4 \phi_1}{\cos^2 \phi} \right). \tag{12}
\]

where the second integral is along the boundary starting at an arbitrary latitude \( \phi_1 \), \( \phi' \) is a variable of integration, and \( \Phi \) is a constant of integration to be determined.

Next we apply the conditions invoked by Held and Hou (1980). Since \( \Phi \) must be continuous at the edges of
the cells, both at $z = h$ and $z_b$ (assuming that $z_b$ is continuous), it follows from (8) that the vertical integral of $\theta$ must be continuous there. Outside the edges of the cell, there is no overturning circulation, and $\theta = \theta_e$; hence,

$$\int_{z_b(\phi)}^{h} \theta(\phi_+, z) \, dz = \int_{z_b(\phi)}^{h} \theta(\phi_+, z) \, dz,$$  

with a similar expression evaluated at $\phi = \phi_-$. Additionally, since we assume no flow into or out of the cells, (9) integrated across each cell gives

$$\int_{\phi_1}^{\phi} \int_{z_b(\phi)}^{h} (\theta - \theta_e) \, dz \, \cos \phi \, d\phi = 0,$$  

with a similar expression integrated between $\phi_-$ and $\phi_+$. Equation (14) is a statement of no net diabatic heating within the Hadley cells.

The Hadley cells are characterized by weak horizontal temperature gradients; neglecting advection along the bottom boundary, (9) says that $\theta \approx \theta_e$ there, so that (12) can be written

$$\frac{g}{\theta} \int_{z_b}^{h} \theta \, dz = \dot{\Phi} - \frac{\Omega^2 a^2}{2} \left( \cos^2 \phi + \frac{\cos^4 \phi}{\cos^2 \phi} \right) - \frac{g}{\theta} \int_{\phi_1}^{\phi} \theta(\phi', z_b) \frac{dz_b}{d\phi'} \, d\phi'.$$

Substituting into (13) and (14) then prescribes the mathematical problem to be solved:

$$\dot{\Phi} - \frac{\Omega^2 a^2}{2} F(\phi_1, \phi_+) = X_c(\phi_+),$$

$$\dot{\Phi} - \frac{\Omega^2 a^2}{2} F(\phi_1, \phi_-) = X_c(\phi_-),$$

$$\int_{\phi_1}^{\phi} \left[ \dot{\Phi} - \frac{\Omega^2 a^2}{2} F(\phi_1, \phi) \right] \cos \phi \, d\phi = \int_{\phi_1}^{\phi} X_c(\phi) \cos \phi \, d\phi,$$

$$\int_{\phi}^{\phi_1} \left[ \dot{\Phi} - \frac{\Omega^2 a^2}{2} F(\phi_1, \phi) \right] \cos \phi \, d\phi = \int_{\phi}^{\phi_1} X_c(\phi) \cos \phi \, d\phi,$$

where we have used the shorthand

$$F(\phi_1, \phi) = \cos^2 \phi + \frac{\cos^4 \phi_1}{\cos^2 \phi}$$

and defined the forcing function:

$$X_c(\phi) = \frac{g}{\theta} \int_{z_b(\phi)}^{h} \theta(\phi, z) \, dz + \frac{g}{\theta} \int_{\phi_1}^{\phi} \theta(\phi', z_b) \frac{dz_b}{d\phi'} \, d\phi'.$$

Thus, (16)–(19) constitute a set of equations in the four unknowns: $\Phi, \Phi_1, \Phi_+, \text{ and } \Phi_-$. For a flat surface, $X_c$ reduces to the form $\frac{g}{\theta} \int_{\phi_1}^{\phi} \theta(\phi) \, d\phi$ as in Held and Hou (1980) and Lindzen and Hou (1988). Variations in the height of the lower boundary impact the forcing function both explicitly (since both terms involve $z_b$) and implicitly (through their influence on equilibrium temperature). The relative importance of these contributions will be discussed below.

The choice of $h$, taken to be the height of the tropopause on Earth, is not obvious for Mars. If the tropopause is defined as the level where the static stability changes drastically, no clear transition exists analogous to the terrestrial troposphere and stratosphere. The tropopause could also be defined as the level to which convection penetrates in radiative-convective equilibrium, but this definition excludes the pure radiative case. While work with MGCMs has shown that the Martian circulation extends to the mesopause, the mesopause hardly seems like a good definition since most of the mass transport in the Hadley cells is well below this level. Since in our simple MGCM (section 4) 50% of the mass flux is typically located below 15 km, we adopt that value for $h$.

Since the theory is assumed to be axisymmetric, only two of the topographies described above are relevant here: flat and zonal mean MOLA topography. The value of $\theta_e$ in (21) is evaluated using either the pure radiative equilibrium temperature given by (1) or the radiative-convective equilibrium temperature given by (3), recalling that the temperature must remain above the CO$_2$ frost temperature.

While the solutions to (16)–(19) depend solely on $X_c(\phi)$, it is only gradients of $X_c$ that drive the circulation. (If $X_c$ is constant, the trivial solution is a cell of zero width.) From (21),

$$\frac{dX_c}{d\phi} = \frac{g}{\theta} \int_{z_b}^{h} \frac{\partial \theta}{\partial \phi} \, dz,$$

from which we deduce the rather obvious conclusion that (as in the case of a flat surface) the circulation is driven by the vertically integrated temperature gradients. Topography enters both explicitly, in the integration limit, and implicitly, through its impact on $\theta_e$. Under the conditions of our calculations, both the surface temperature and the depth of the convecting layer, while varying substantially with latitude, are remarkably insensitive to the surface height variations. This insensitivity, a consequence of the weakness of the greenhouse effect, can be exploited to
arrive at a simple understanding of the role of topographic height variations on the forcing of the Hadley circulation.

Now,

$$\int_{z_b}^{h} \frac{\partial \theta}{\partial \phi} dz = \int_{h}^{h} \frac{\partial \theta^R}{\partial \phi} dz + \int_{z_b}^{h} \frac{\partial \theta^C}{\partial \phi} dz,$$

(23)

where $\theta^C(\phi, z)$ and $\theta^R(\phi, z)$ are the equilibrium temperatures within and above the convecting layer, respectively (and note that the latter is radiatively determined and independent of $z_b$), and $h_c$ is the altitude of the top of the convecting layer. Within the convecting layer, and consistent with the Boussinesq approximation,

$$\theta^C(\phi, z) = \theta^C(\phi) - \Gamma[z - z_b(\phi)],$$

where $\theta^C(\phi)$ is surface temperature and $\Gamma$ the adiabatic lapse rate; hence, the gradient

$$\frac{\partial \theta^C}{\partial \phi} = \frac{\partial \theta^S}{\partial \phi} + \Gamma \frac{\partial z_b}{\partial \phi}$$

(24)

is independent of $z$. Therefore

$$\int_{z_b}^{h} \frac{\partial \theta^C}{\partial \phi} dz = D \left( \frac{\partial \theta^S}{\partial \phi} + \Gamma \frac{\partial z_b}{\partial \phi} \right),$$

(25)

where $D = h_c - z_b$ is the depth of the convecting layer. As we have seen, both $\theta^C$ and $D$ are, to an excellent approximation, independent of $z_b$, and so the surface topography enters (25) only through the term $\partial z_b/\partial \phi$.

Above the convecting layer, the contribution to (23) is

$$\int_{z_b}^{h} \frac{\partial \theta^R}{\partial \phi} dz = \int_{D}^{D} \frac{\partial \theta^R}{\partial \phi} dz + \int_{h}^{D} \frac{\partial \theta^R}{\partial \phi} dz.$$  

(26)

Accordingly, dependence of (26) on $z_b$ is encapsulated in the second term. If the radiative lapse rate does not vary significantly with latitude, then $\partial \theta^R/\partial \phi$ is independent of $z$ (Fig. 2), so

$$\int_{D}^{D} \frac{\partial \theta^R}{\partial \phi} dz \approx \int_{D}^{D} \frac{\partial \theta^R}{\partial \phi} dz = \frac{\partial \theta^R}{\partial \phi}(\phi, D)(D - h_c) = -\frac{\partial \theta^R}{\partial \phi}(\phi, D)z_b.$$

In total, then, substituting into (22) and (23),

$$\frac{\theta_0}{g} \frac{dX}{d\phi} \approx \left( D \frac{\partial \theta^S}{\partial \phi} + \int_{D}^{D} \frac{\partial \theta^R}{\partial \phi} dz \right) + \Gamma D \frac{\partial z_b}{\partial \phi} - \frac{\partial \theta^R}{\partial \phi}(\phi, D)z_b.$$  

(27)

Of the two contributions that depend on $z_b$, the first depends directly on the local topographic slope, while the second does not (but note that both contributions are zero with flat topography, since then $z_b = 0$). For the most part, $\partial z_b/\partial \phi < 0$, so the first term is generally negative, and hence tends to influence the forcing function in the same way as displacing the $\theta_e$ maximum into the Southern Hemisphere. At equinox, the second term is small in the tropics and so the net effect of the topography is to shift the effective thermal equator into the Southern Hemisphere. This is illustrated in Fig. 8, for which the forcing function and its derivative have been calculated by direct substitution of the radiative–convective equilibrium temperature distribution, with the topography of Fig. 1, into (21). The impact of the topography is modest, but the analysis of Lindzen and Hou (1988) leads us to expect that a small asymmetry in the thermal forcing will produce a much larger asymmetry in the circulation. At the solstices, the possibly competing effect of the two terms makes simple statements on the basis of (27) more elusive. However, Fig. 8 shows that the net effect is weak at $L_s = 90^\circ$, with only a slight weakening of the gross latitudinal gradient. At $L_s = 270^\circ$, the impact of the surface topography is somewhat greater, with an intensification of the gross gradient.

Returning to (16)–(19), Fig. 9 shows the solutions for $\phi_1, \phi_2, \phi_3$ for each of the two topographies with pure radiative or radiative–convective forcing and physical constants appropriate for Mars. The solutions for flat topography for pure radiative and radiative–convective forcing are nearly equal, such that they lie on same curve in Fig. 9. Hadley cells are most sensitive to the meridional gradient in equilibrium temperature, which is purely
4. Experiments with a simple MGCM

a. Description of the MIT MGCM

We have converted the Massachusetts Institute of Technology (MIT) GCM to physical constants appropriate for the Martian atmosphere, which is assumed to be entirely CO$_2$ and to contain no dust. The dynamical core of the MIT GCM solves the fundamental equations of geophysical fluid dynamics in the hydrostatic approximation using the finite volume method on an Arakawa C grid (Marshall et al. 1997). The default configuration has no viscosity or vertical diffusion but uses an eighth-order Shapiro filter to remove gridscale noise. The horizontal configuration is a cube–sphere grid (Adcroft et al. 2004) with $32 \times 32$ points per cube face, equivalent to a resolution of $2.8^\circ$ or $166$ km at the equator. Note that the resolution of the MOLA dataset is $1/64\text{''}$ latitude $\times 1/32\text{''}$ longitude (Smith et al. 2001), which exceeds the horizontal resolution for the simple MGCM.

The vertical grid uses an $\eta$ coordinate (Adcroft and Campin 2004) with 30 levels, and the grid spacing increases approximately logarithmically with height. Zero elevation corresponds to a pressure of 6 hPa (the observed annual and global average value), which is also used as a reference pressure elsewhere in our calculations. The top level is centered at a pressure of $0.000117$ hPa, which corresponds to a log pressure height of $119$ km (using the reference pressure above and a scale height of 11 km). This is above the level where nonlocal thermodynamic equilibrium effects may become important (López-Valverde et al. 1998). We do not treat the top levels as realistically modeling the atmosphere in that region, since a sponge layer is also located in the upper levels (described below). Within each vertical level intersecting the surface, the resolution of the topography is increased by inserting subgrids spaced at 10% of the full vertical grid spacing at that level (Adcroft et al. 1997).

The external thermal forcing is specified by Newtonian relaxation to a prescribed equilibrium temperature following Held and Suarez (1994). Explicitly, this term in the energy equation is

$$\frac{\partial T}{\partial t} = \cdots - k_T(T - T_{eq}),$$

where $T$ is temperature, $t$ is time, $k_T = \frac{1}{2}$ sols$^{-1}$ is the radiative relaxation rate, and $T_{eq}$ is the equilibrium temperature, discussed in section 2. The choice of $k_T$ is based on a calculation using Eq. (6) of Showman et al. (2008) and is the same as the value calculated by Haberle et al. (1997) for clear-sky conditions. The assumption of

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2 A sol is a Martian solar day.
fixed $k_f$ implies that radiation and convection (if implicitly present in $T_{eq}$) operate on the same time scale.

Boundary layer friction is specified in the horizontal momentum equations by

$$\frac{\partial u}{\partial t} = \cdots - k_v(p)u,$$  \hspace{1cm} (29)

$$\frac{\partial v}{\partial t} = \cdots - k_v(p)v,$$  \hspace{1cm} (30)

where $k_v$ is the wind damping rate and is defined by

$$k_v = k_f \max \left( 0, \frac{p - p_b}{p_o - p_b} \right),$$  \hspace{1cm} (31)

where $k_f$ is the wind damping rate of the lower atmosphere and $p_b = 0.7p_o$ is the top of the boundary layer. We set $k_f = 1$ sol$^{-1}$, which is consistent with values in the literature (e.g., Lewis et al. 1996; Nayvelt et al. 1997). Terms similar to the right side of (29) and (30) are also included in the horizontal momentum equations in the top three model levels as a sponge layer to avoid reflection off the model lid, with $k_v$ set to 9, 3, and 1 sols$^{-1}$ from uppermost to lowermost level, respectively.

b. Experiments

Each experiment was run at constant $L_s$ for the following seasons: $L_s = 0^\circ$, $L_s = 90^\circ$, and $L_s = 270^\circ$. Given the short radiative time scale of the Martian atmosphere (2 sols) and the lack of oceans with high heat capacity, the atmosphere is expected to respond quickly to changes in radiative forcing compared with the length of a season (one quarter of the Martian year or 167 sols). As such, it was unnecessary to integrate the model over several years to achieve a seasonal equilibrium. The model was run for a total of 180 sols or 6 Martian months, where a Martian month is defined to be 30 sols, and was fully spun up from an initial rest state with $T_{eq} = 200$ K everywhere by the end of the second Martian month. A visual inspection of the zonally and monthly averaged results (i.e., potential temperature, horizontal and vertical velocities, and surface pressure) showed little variation from month to month after the initial spinup.

Figures 10 and 11 show the simple MGCM results for zonally averaged mass streamfunction time averaged for sols 60–180 when forced with the pure radiative equilibrium temperatures (Fig. 2) and radiative–convective equilibrium temperatures (Fig. 3), respectively. With pure radiative forcing, equinox conditions ($L_s = 0^\circ$), and flat topography (Fig. 10a), the cells are quite symmetric about the equator. When the north–south sloping component of topography is added (i.e., full and mean topographies; Figs. 10d and 10g, respectively), the dividing streamline protrudes slightly into the Southern Hemisphere, but its maximum extent is not past $-5^\circ$. A slight asymmetry between the strengths of the northern and southern cells develops. For $L_s = 90^\circ$ and pure radiative forcing (Figs. 10b,e,h), no significant difference arises between the three topographies. The same occurs at $L_s = 270^\circ$ (Figs. 10c,f,i).

When convective forcing is allowed, a substantial difference arises at equinox between experiments without the north–south sloping component of topography (i.e., flat; Fig. 11a) and with it (i.e., full and mean; Figs. 11d and 11g, respectively). The flat topography produces symmetric cells, whereas the full and mean produce a stronger northern cell and shift the latitude of the dividing streamline to about $-15^\circ$ in the lower atmosphere (cf. about $-20^\circ$ in Takahashi et al. 2003). At $L_s = 90^\circ$, the cross-equatorial Hadley cell is stronger when the north–south sloping component is removed (cf. Fig. 11b with Figs. 11e,h). At $L_s = 270^\circ$ the correlation with the presence of the north–south sloping component of topography is less apparent (cf. Figs. 11c,f,i).

c. Comparison of Hadley cell strengths

With pure radiative forcing, the strength of the Hadley cells at a given season varies little among the different topographies and does not appear to be correlated with the presence of the north–south slope. A much different scenario arises for radiative–convective forcing. Figure 12 shows the maximum magnitude of the zonally and time-averaged streamfunction from each of the simple MGCM experiments with radiative–convective forcing. Specifically, the maximum magnitude within each thermally direct cell above the boundary layer is plotted for each topography (black symbols).

For flat topography, the strengths of the northern and southern cells are symmetric at equinox, and the strengths at the opposite solstices are equal. For mean topography, at equinox the northern cell is enhanced, while the southern cell is reduced. It is generally recognized from theory (e.g., Lindzen and Hou 1988) and terrestrial observations (e.g., Peixoto and Oort 1992, their Fig. 7.19) that moving the latitude of maximum equilibrium temperature off the equator (as occurs because of changing seasons) produces a weak cell in the same hemisphere as the maximum and a strong cell that spans both hemispheres. The farther the maximum equilibrium temperature moves off equator, the more pronounced this effect becomes. If the north–south slope is acting to shift the

3 Compared with Earth, the difference between the equinoctial and solstitial circulations is much more dramatic, since the heat capacity of Martian soil is less than that of Earth’s oceans (Haberle et al. 1993). Without the mediating influence of the ocean, the latitude of maximum equilibrium temperature can move well off the equator.
maximum equilibrium temperature upslope, as indicated by Figs. 3g and 5a, then at equinox a stronger northern cell and weaker southern cell are expected for the mean topography case. For flat topography at $L_s = 90^\circ$, the maximum equilibrium temperature is shifted far northward (Figs. 3b and 5b), producing no summer cell at all and a strong winter cell. For mean topography, the strength of the cell is reduced as compared with flat topography. For flat topography at $L_s = 270^\circ$, the latitude of maximum equilibrium temperature is shifted far southward, producing a strong “winter” cell in the opposite sense as at $L_s = 90^\circ$. However, the addition of zonal mean topography at $L_s = 270^\circ$ does not change the strength of the cell as significantly as at $L_s = 90^\circ$.

To further illustrate this seasonal asymmetry, several more experiments were performed with radiative–convective forcing at $L_s = 90^\circ$ and $L_s = 270^\circ$, which had the following idealized zonal mean topography:

$$z_{bi} = -2 \tanh \left( \frac{\phi - 2}{\beta} \right),$$

where $z_{bi}$ is the height of the idealized topography (in km), $\phi$ is in degrees, and $\beta$ controls the steepness of the slope (lower values correspond to a steeper slope). Figure 13 shows the simple MGCM results for $L_s = 90^\circ$ for flat topography and the following three values of $\beta$: 40, 20, and 10. As slope steepness increases, the strength of the Hadley cell decreases. Figure 14 shows results for $L_s = 270^\circ$ with the same topographies. In this case, the steepness of the slope has little effect on the cell strength.

Evidently, a circulation that flows downslope near the surface causes the Hadley cells to lose strength, but that strength is not regained for an upslope flow. This phenomenon further comes into play in comparing full topography and mean topography. In Fig. 12, the strengths for full topography are always less than the strengths for
mean topography at the corresponding season. As the circulation flows up and down the mountains and valleys, strength is lost as the air flows downward along the topography but not regained when it flows upward. This diminishing is observed when comparing flat and “wave” (i.e., full topography minus zonal mean; not shown) topographies. For full topography, this mountain effect is superimposed onto the effect (if any) of the mean sloping component. The effect is associated with the radiative–convective forcing, as runs with pure radiative forcing show little difference in Hadley cell intensity for any topography.

d. Comparison with the GFDL MGCM

To demonstrate that our simple radiation scheme captures the effect of topography on the Martian Hadley cell, we compare our simple MGCM results to the GFDL MGCM, which uses a full (nongray) radiation scheme. The GFDL MGCM was originally based on the GFDL SKYHI terrestrial GCM and an early version of the model has been described in Wilson and Hamilton (1996). Subsequent descriptions appear in Richardson and Wilson (2002) and Hinson and Wilson (2004). This model has been used to examine tides and planetary waves (Wilson and Hamilton 1996; Hinson and Wilson 2002; Wilson et al. 2002; Hinson et al. 2003), the water cycle (Richardson and Wilson 2002; Richardson et al. 2002), the dust cycle (Basu et al. 2004, 2006), and cloud radiative effects (Hinson and Wilson 2004; Wilson et al. 2007, 2008). More recently, the physical parameterizations have been adapted to the GFDL Flexible Modeling System (FMS), which includes a choice of dynamical cores [finite difference, finite volume (FV), and spectral] and associated infrastructure. The Mars physics has been tested with all three dynamical cores and we have elected to use the finite volume model.

The model uses the radiation code developed and used by the NASA Ames Mars modeling group (Kahre et al. 2006, 2008). This code is based on a two-stream solution to the radiative transfer equation with CO2 and water vapor opacities calculated using correlated-$k$ values in 12 spectral bands ranging from 0.3 to 250 microns. The two-stream solution is generalized for solar and infrared radiation, with scattering based on the $\delta$-Eddington
The triangles are for the simple radiation scheme (section 2) are not centrally important. Additionally, experiments (not shown) with the GFDL MGCM were performed with the same idealized topography used in Figs. 13 and 14. These experiments again showed that the Hadley cell strength at \( L_s = 90^\circ \) is sensitive to the steepness of the topographic slope but shows little response to the slope at \( L_s = 270^\circ \).

Quantitative differences in Hadley cell strengths between the simple MGCM and the GFDL MGCM are due to fundamental differences in the radiation schemes. The Hadley cell strengths in the simple MGCM are sensitive to the values used for \( \tau_{oo} \) and \( k_r \). The simple radiation code also fails to produce a difference in the Hadley cell strengths at the opposite solstices for flat topography. The eccentricity of the Martian atmosphere creates a large difference in the solar constant; with more energy input into the atmosphere, the Hadley cells ought to be stronger. On the other hand, the boundaries of the cells are in quantitative agreement between the simple and GFDL MGCMs and are less sensitive to the particular aspects of our radiation scheme. This fact, along with qualitative agreement in the behavior of Hadley cell strengths between the two models, lends sufficient credibility to our simple radiation scheme.

e. Comparison with previous simulations

As wind data for the Martian atmosphere above the boundary layer are extremely sparse compared to those for Earth, we are left to compare the simple MGCM streamfunction results with previous models. The results for streamfunction strength from the MGCM of Takahashi et al. (2003) are also plotted in Fig. 12 (gray symbols). Comparison of mean and full topographies at equinox shows that the northern cell is stronger for the mean topography case, in agreement with our results. Quantitatively, our simple MGCM predicts a weaker cell for full topography at \( L_s = 270^\circ \), a stronger cell for full topography at \( L_s = 90^\circ \), and a weaker northern cell for full and mean topographies at equinox. Quantitative disagreement between the two models arises from differences in the radiation schemes between our model and that of Takahashi et al. (2003).

Finally, Joshi et al. (1995) stated that the Hadley cell at \( L_s = 270^\circ \) was a factor of 1.5 stronger than at \( L_s = 90^\circ \) in their “intermediate” global circulation model (based on the Oxford MGCM), which most closely corresponds to our radiative–convective forcing and full topography. Our simple MGCM also yielded a factor of \( \sim 1.5 \) for this setup, whereas Takahashi et al. (2003) obtained a factor of \( \sim 3 \). Joshi et al. (1995) also noted that the effect was still observed even when solar insolation was kept constant for both seasons. However, they speculated that an enhanced circulation at \( L_s = 270^\circ \) occurred because the...
rising branch is located in an area of lower pressure (i.e., higher elevation) and did not attribute the difference to the thermal affects of the slope.

5. Comparison of MGCM and modified Lindzen and Hou (1988) models

Figure 16 shows the solution to the modified Lindzen and Hou (1988) model replotted so that $f_1$, $f_1$, and $f_2$ for each topography and radiative forcing are displayed together. It is now apparent that as $L_s$ approaches 90° and 270°, $f_1$ approaches the boundary of the summer cell in the summer hemisphere. The physical implication is that the summer cell disappears, which is what is observed in the simple MGCM results (Figs. 10 and 11). Also plotted in Fig. 16 are the values for $f_1$, $f_1$, and $f_-$ determined from the MGCM results for the appropriate topographies, seasons, and radiative forcings, including some runs at seasons not discussed in section 4.

The boundaries of the Hadley cells in the simple MGCM results are computed as follows. For each thermally direct cell (one near solstice, two near equinox), the maximum magnitude of the zonally averaged mass streamfunction (above the boundary layer) is found. Then, at the same pressure level as each maximum, the latitude where the streamfunction falls to within 1% of its maximum value is located. The northernmost 1% latitude is $f_1$ and the southernmost is $f_-$. When there is one thermally direct cell, $f_1$ is equal to whichever of $f_1$ or $f_-$ is in the summer hemisphere. When there are two thermally direct cells, the average of the two innermost 1% latitudes is taken to be $f_1$. Interpreting the boundaries of the cells from the simple MGCM results can be difficult because in some cases the latitude of the boundary varies with height (e.g., at the boundary in the winter hemisphere at solstice). Also, particularly near equinox on the poleward sides of the cells, multiple 1% latitudes may exist when closed areas of circulation in the opposite direction appear within the thermally direct cell. In this case the innermost 1% latitude is used. Near the solstice, the streamfunction in the summer hemisphere decreases very gradually, which causes the value derived for the
boundary to vary slightly depending on the cutoff percentage used (e.g., 1% versus 0.1%).

From Fig. 16, the agreement of $f_1$ between the simple MGCM results and the modified Lindzen and Hou (1988) model is generally good. The solutions for the poleward boundary of the summer cell agree for pure radiative forcing, but the simple MGCM results are offset in the equatorward direction for radiative–convective forcing. The solutions for the poleward boundary of the winter cell do not agree. The simple MGCM results are offset equatorward, and they do not follow the same shape as the modified Lindzen and Hou (1988) model solution. One possible explanation is that in the simple MGCM results the latitudinal boundary of the winter cell varies with height, and in fact extends further poleward at higher altitudes. Another explanation is that the simple MGCM contains eddies, whereas the Lindzen and Hou (1988) model does not, on account of its axisymmetric nature.

Since the modified Lindzen and Hou (1988) model is axisymmetric, it is perhaps more sensible to compare it to an axisymmetric MGCM. We ran our simple MGCM in axisymmetric mode, where the cube–sphere geometry was replaced by grid points evenly spaced at 2.8° of latitude along a single strip of longitude of width 2.8°. When vertical diffusion of momentum was not included, inertial instabilities developed in the tropics that made it impossible to determine $f_1$ near equinox. Adding a constant diffusion of 0.0005 Pa² s⁻¹ everywhere resolved $f_1$ but induced a Ferrel cell.

Figure 17 shows a plot of the modified Lindzen and Hou (1988) model solutions as in Fig. 16 but with the axisymmetric simple MGCM results instead of the full 3D simple MGCM. The discrepancy in the poleward boundary of the winter cell remains, possibly due to the Ferrel cell created by adding diffusion. In runs without diffusion, the poleward boundary of the winter cell extends nearly to the winter pole, in agreement with the modified Lindzen and Hou (1988) results.

To further assess the role of eddies and the mean circulation in the 3D simple MGCM simulations, we decomposed the momentum budget as follows. Ignoring the effects of the boundary layer (i.e., mountain torque and friction) on the momentum budget, the total derivative of the absolute angular momentum given by (6) may be set equal to zero,

$$\frac{\partial M^a}{\partial \tau} + \frac{u}{a \cos \phi} \frac{\partial M^a}{\partial \lambda} + \frac{v}{a} \frac{\partial M^a}{\partial \phi} + \frac{\partial M^a}{\partial p} - fa \cos \phi v = 0,$$

(33)
where $\lambda$ is longitude, $\omega$ is the vertical velocity in pressure coordinates, and $M_\varphi = a \cos \phi u$ is the relative angular momentum. Averaging zonally and in time and making use of the continuity equation,

$$\frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left( \cos \phi \left[ v M_\varphi \right] \right) + \frac{\partial}{\partial p} \left( \left[ \omega M_\varphi \right] \right) = fa \cos \phi [v],$$

(34)

where the square brackets indicate a zonal average and the overbar indicates a time average. The term $\left[ v M_\varphi \right]$ may be further written as

$$\left[ v M_\varphi \right] = a \cos \phi \left( [\pi] [v] + [u] [v] + [u^* v^*] \right),$$

(35)

where primes indicate a departure from the time average and asterisks indicate a departure from the zonal average. The first term on the right-hand side of (35) is the meridional transport of angular momentum by the steady mean circulation, the second is the transport by the transient circulation, and the third is the transport by spatial eddies. We found that the second term was much smaller than the other two; thus, it was neglected. Similarly, (35) may be written for the vertical transport of angular momentum, where $v$ is replaced by $\omega$. The vertical transport by the transient and spatial eddies is small, leaving

$$\frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left( a \cos \phi [u^* v^*] \right) + \frac{1}{a \cos \phi} \frac{\partial}{\partial p} \left( a \cos \phi [\pi] [v] \right) + \frac{\partial}{\partial p} \left( a \cos \phi [\pi] [\omega] \right) = fa \cos \phi [v],$$

(36)

where the term on the right-hand side is the planetary term.

Figure 18 shows the three terms in (36) calculated from the simple MGCM results with radiative–convective forcing and full topography at $L_\varphi = 0^\circ$, $90^\circ$, and $270^\circ$. Within the Hadley cells, the planetary term is balanced by the meridional transport by the steady mean circulation during the solstices. At equinox, all three terms
are small within the Hadley cells. Outside the Hadley cells, the meridional transport by spatial eddies nearly balances the planetary term. The presence of eddies near the poleward boundary of the Hadley cell, together with the fact that the axisymmetric simple MGCM runs without diffusion (i.e., no spatial eddies), shows better agreement with the Lindzen and Hou (1988) model and suggests that by driving the extratropical Ferrel cell, eddies may be important in determining the poleward boundaries of the Hadley cells at equinox and in the winter hemisphere at the solstices.

Walker and Schneider (2006) showed in idealized GCM simulations that the strength of the Hadley cell is related to the eddy momentum flux divergence. In Fig. 18, the spatial eddy term is greater in the northern (winter) hemisphere at $L_s = 270^\circ$ than in the southern (winter) hemisphere at $L_s = 90^\circ$. This effect is also seen in our simple MGCM simulations with flat topography. Eddies may also be important in addition to the topography in determining the seasonal asymmetry in Hadley cell strength.

6. Discussion and conclusions

We have continued the investigation of the effect of a latitudinally varying slope on the Martian Hadley cells initiated in detail by Richardson and Wilson (2002) and Takahashi et al. (2003). The problem was approached by revising the Hadley cell model of Lindzen and Hou (1988) to include topography, which allowed us to solve for the latitudinal boundaries of the cells. This theory was compared with the results from a simple MGCM. Both models were thermally forced using Newtonian relaxation to one of two equilibrium temperature states: one that included the effects of convection and one that did not. In the case of the latter, almost no dependence on the topography was observed in either model. When convective forcing was allowed, both models predicted that the latitude of the dividing streamline is shifted upslope at equinox. The effect of the slope on the radiative-convective equilibrium temperature aloft was found to dominate over the geometrical effects of the slope.
From the simple MGCM results, an analysis of the strengths of the Hadley cells was carried out. At equinox with radiative–convective forcing, the presence of the slope strengthens the southern cell and weakens the northern cell as the latitude of maximum equilibrium temperature moves upslope. At $L_s = 90^\circ$ with radiative–convective forcing, the cross-equatorial cell is weakened with sloping topography; at $L_s = 270^\circ$, the strength of the cell is independent of slope. Further experiments with idealized zonal mean topography showed that as the steepness of the slope is increased, the strength of the cell decreases at $L_s = 90^\circ$.

A discrepancy arose between our simple MGCM and the modified Lindzen and Hou (1988) model results regarding the latitude of the poleward boundary of the Hadley cell in the winter hemisphere around the solstice seasons. This difference could be caused by eddies, since the simple 3D MGCM allows for their presence but the Lindzen and Hou (1988) model does not. Axisymmetric simple MGCM runs without diffusion and hence without eddies produced better agreement with the modified Lindzen and Hou (1988) model. A breakdown of the momentum budget showed that the spatial eddies are relatively weak within the Hadley cells at the solstices but that eddies are relatively strong outside the Hadley cells. Eddies are stronger during northern winter ($L_s = 270^\circ$) than southern winter ($L_s = 90^\circ$). Eddies may play a role in determining the strengths and poleward boundaries of the Hadley cell, as also suggested by Walker and Schneider (2006) in idealized GCM runs.

Molnar and Emanuel (1999) considered the radiative–convective equilibrium case when absorbers, namely water vapor, are present within the atmosphere. In the limit in which the atmosphere is opaque to solar radiation, the temperature at any level aloft becomes independent of the height of the surface. The Molnar and Emanuel (1999) study may be applicable to periods when the Martian atmosphere is experiencing very dusty conditions (Viking Lander 1 measured optical depths in excess of 4 during a dust storm; Colburn et al. 1989), since atmospheric dust is a strong absorber. Basu et al. (2006) showed with the GFDL MGCM that as dust optical depth increases, the ratio of the streamfunction maximum at $L_s = 270^\circ$ to $L_s = 90^\circ$ decreases, which implies that dust inhibits the effect of the slope. It would be interesting to use the modified Lindzen and Hou (1988) model to study the effects of increased dust on the Hadley cell boundaries, provided the appropriate changes could be made to our radiation scheme.
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Fig. 18. Simple MGCM results with radiative–convective forcing for the momentum budget derived in (36). Units are m² s⁻². Latitude is plotted on the x axis, pressure is plotted on the left y axis, and log pressure height is plotted on the right y axis, assuming a scale height of 11 km and a reference pressure of 6 hPa. Only results below ~0.2 hPa (40 km log pressure height) are shown. From left to right, the columns are $L_s = 0°$, 90°, and 270°. From top to bottom, the rows are the meridional transport by spatial eddies [first term on the lhs of (36)], combined meridional and vertical transport by the steady mean circulation [second and third terms on the lhs of (36)], and the planetary term [rhs of (36)]. Negative contours are shaded. Note the different contour intervals (100 m² s⁻² for the top row and 200 m² s⁻² otherwise).