Pseudo-Dirac Dark Matter Leaves a Trace

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Pseudo-Dirac dark matter is a viable type of dark matter which originates from a new Dirac fermion whose two Weyl states get slightly split in mass by a small Majorana term. The decay of the heavier to the lighter state naturally occurs over a detectable length scale. Thus, whenever pseudo-Dirac dark matter is produced in a collider, it leaves a clear trace: a visible displaced vertex in association with missing energy. Moreover, pseudo-Dirac dark matter behaves Dirac-like for relic abundance and Majorana-like in direct detection experiments. We provide a general effective field theory treatment, specializing to a pseudo-Dirac bino. The dark matter mass and the mass splitting can be extracted from measurements of the decay length and the invariant mass of the products, even in the presence of missing energy.

Dark matter is outstanding evidence of physics beyond the standard model and may well be within reach at colliders. The best known parameter about dark matter (DM) is its abundance in the Universe. Other than that, limits on its mass and interactions with nuclei are extracted from negative results of direct and indirect searches. The theoretical bias is toward a particle of mass of hundreds of GeV with weak-strength interactions. However, even if the DM particle could be produced at colliders, its presence is inferred indirectly, in the form of missing energy.

In this Letter, we propose a simple scenario in which DM leaves a clear trace at colliders. We extend the standard model (SM) to include a new fermion $\Psi$, having both Dirac and Majorana masses. The introduction of a Majorana mass has the effect of removing the degeneracy of the two Weyl spinors composing $\Psi$. The Dirac mass is assumed to be of the order of the electroweak scale, whereas the Majorana contribution is much smaller and controls the sum of the two Weyl spinors composing $\Psi$, having both singlet under the SM gauge group, with the most general structure of the model.

We consider a fermion $\Psi$, singlet under the SM gauge group, with the most general Dirac and Majorana masses:

$$L_0 = \bar{\Psi}(i\gamma^\mu \partial_\mu - M_D)\Psi - \frac{m_L}{2} (\bar{\Psi}^\dagger P_L \Psi + \text{H.c.})$$

$$-\frac{m_R}{2} (\bar{\Psi}^\dagger P_R \Psi + \text{H.c.}),$$

where $P_{RL} = (1 \pm \gamma^5)/2$. The mass eigenstates are a linear combination of $\Psi$ and $\Psi^\dagger$. In this study we are focusing on a pseudo-Dirac situation, where Majorana masses are suppressed with respect to the Dirac mass term. At zeroth order in $\delta \equiv (m_L - m_R)/M_D \ll 1$, the mass eigenstates are given by $\chi_1 \approx i(\Psi - \Psi^\dagger)/\sqrt{2}$ and $\chi_2 \approx (\Psi + \Psi^\dagger)/\sqrt{2}$. Note that the fields $\chi_{1,2}$ are self-conjugates at the zeroth order in $\delta$; i.e., $\chi_{1,2} = \chi_{1,2}^\dagger + O(\delta)$, with masses $m_{1,2} = M_D \mp \delta^2/2 [m \equiv (m_L + m_R)/2]$ are just split by the Majorana term. Instead of the three mass parameters of the free Lagrangian $(1)$, we will use the set of $m_1, \Delta m \equiv m_2 - m_1$ and $\delta$. To account for the stability of the DM particle, we introduce a new parity such that $\Psi$ is odd and the whole SM sector is even. In this case, the pseudo-Dirac fermion can interact with the SM fermions $f$ via nonrenormalizable interactions such as $\mathcal{L}_{\text{int}} = \frac{1}{\Lambda^2} \bar{\Psi} \gamma^\mu (c_L P_L + c_R P_R) \Psi f \bar{\gamma}_\mu (c_L P_L + c_R P_R) f$, or, in terms of the mass eigenstates $\chi_1$ and $\chi_2$,

$$\mathcal{L}_{\text{int}} = \frac{1}{4\Lambda^2} [2i(c_R + c_L)\bar{\chi}_1 \gamma^\mu \chi_2 + (c_R - c_L)(\bar{\chi}_1 \gamma^\mu \gamma^5 \chi_1 + \bar{\chi}_2 \gamma^\mu \gamma^5 \chi_2)][\bar{f} \gamma_\mu (c_L^\dagger P_L + c_R^\dagger P_R) f].$$

Other dimension-6 operators of the kind $m_f \bar{\Psi} \Psi f/\Lambda^3$ are

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suppressed with respect to the ones in Eq. (2) by \( m_i/\Lambda \ll 1 \), \( \Lambda \) above the weak scale, and hence they are negligible (the only exception may be for the top quark, but we exclude this possibility, as discussed later). There also exists a dimension-5 operator \( \bar{\Psi} \Psi H H/\Lambda \), coupling the fermion \( \Psi \) to the Higgs doublet \( H \) [2]. This operator leads to velocity-suppressed contributions to the annihilation cross section of the \( \chi_i \)'s, having small impact on our analysis; thus, we will ignore this operator in the following. Note also that, although we consider here only the possibility of a spin-1/2 new particle, a similar analysis applies to the case of a complex scalar which splits into two quasigenerate real scalars.

The dimensionless coefficients \( c_{R,L} \) and \( c_{R,L}^{(f)} \) are model-dependent. Generically, the four-fermion operators in Eq. (2) may be the result of integrating out a heavy particle of mass \( M \) and electroweak couplings: \( c_{R,L}^{(f)}/\Lambda^2 \sim g^2/M^2 \). Notice that assuming \( m \ll M_D \) is technically natural once one considers a symmetry which forbids a Majorana mass, such as a \( U(1) \) symmetry. A small violation of \( U(1) \) symmetry would lead to small Majorana masses.

Similar models have been proposed for inelastic dark matter [3,4], but with an important difference: Unlike inelastic DM, we do not require the mass splitting of the pseudo-Dirac fermion to be 10–100 keV; instead, we consider natural mass splittings of the order of a few GeV. Whereas inelastic DM models are designed to explain the DAMA modulation, pDDM focuses on the collider-cosmology interplay by relating the DM abundance of the Universe with a remarkable collider signature of measurable displaced vertices.

**The pseudo-Dirac bino case.**—Supersymmetry provides a natural scenario for pseudo-Dirac fermions. Dirac gauginos arise as a consequence of the \( U(1)_R \subset SU(2)_R \) symmetry in \( \mathcal{N} = 2 \) supersymmetry. In practice, Dirac gauginos can be thought of as an subset of an minimal supersymmetric standard model, where each Majorana gaugino marries a new particle in the adjoint representation of the gauge groups—see, for example, Ref. [5] for a discussion. One particularly interesting situation is bino dark matter, in which one would marry the \( U(1) \) gaugino \( B \) with a partner \( B' \) with terms such as \( M_B^2 \bar{B} B' \). Small breaking of \( U(1)_R \) would lead to Majorana splittings: \( (m_i/2) B B + (m_i'/2) B' B' \). In this case, the mass eigenstates would be \( \chi^0_{1,2} = [(1 \pm \epsilon) B + (1 \mp \epsilon) B']/\sqrt{2} \), with \( \epsilon = (m_i - m_{i'})/2 M_B^2 \).

**Relic abundance.**—Majorana-type \( \chi_1 \chi_1 \) and \( \chi_2 \chi_2 \) annihilations are velocity-suppressed, while the Dirac-type \( \chi_1 \chi_2 \) is in the \( s \) wave. To a good approximation we can neglect the velocity-suppressed self-annihilations and restrict to the leading contribution to the cross section coming from coannihilations. Therefore, the effective annihilation cross section [6] is well approximated by

\[
\langle \sigma_{\text{eff}} v \rangle \approx \frac{2 \alpha}{(1 + \alpha)^2} \frac{C^4 (2 m_1 + \Delta m)^2}{8 \pi \Lambda^4},
\]

where \( \alpha = [1 + (\Delta m/m_1)]^{1/2} \exp(-\Delta m/m_1) \) and we have defined \( C^4 = (1/4) \sum_i |c_i|^2 + c_R^R (c_L^L)^2 + (c_R^L)^2 (c_L^R)^2 \). The relic abundance is given by \( \Omega_{\text{DM}}h^2 = 8.9 \times 10^{-12} \text{ GeV}^{-2}/\int_0^\infty ds \ln(\sigma_{\text{eff}} v)/s^2 \), where the freeze-out temperature \( T_F = m_i/\lambda_F \) is determined implicitly by \( x_F = 13 \times 10^{6} \text{ GeV} m_i (\sigma_{\text{eff}} v)/\sqrt{s} \). The sum over fermions in \( C \) is restricted to those species which are relativistic at \( T_F \). Since we consider \( m_1 \) of the order of a few hundreds of GeVs, the third-generation quarks are excluded from the sum. As we can see, the cross section, and hence the leading contribution to the relic abundance, depends only on the masses of the two particles \( \chi_1 \) and \( \chi_2 \) and an unspecified mass scale \( \Lambda/C \), which encodes the model dependence. Indeed, we obtain a relation between the parameter \( \Lambda/C \) and the relic abundance:

\[
\frac{\Lambda}{C} \approx 0.8 \text{ TeV} \left( \frac{\Omega_{\text{DM}}h^2}{0.11} \right)^{1/4} \left( \frac{m_1}{100 \text{ GeV}} \right)^{1/2} e^{-(\Delta m/m_1)}.
\]

This approximation agrees with the numerical findings with an accuracy better than 10%.

**Direct detection.**—Besides the operators written in Eq. (2), there is also a small residual vector-vector interaction of the DM to the quarks, due to the nonpure Majorana nature of the mass eigenstates. The parameter \( \delta \) controls these interactions, which potentially lead to large cross sections with heavy nuclei and thus constraints from direct detection experiments.

In general, the operators relevant to direct detection are the vector-vector and the axial-vector couplings of the DM to the quarks \( q, q' \): \( \delta \left( \bar{q} \gamma \mu q \right) \left( \bar{q'} \gamma^\mu q' \right) + \left( \bar{q} \gamma \gamma^5 q \right) \left( \bar{q'} \gamma^\gamma q' \right) \times \left( \bar{q} \gamma \gamma^5 q' \right) \). Mixed axial-vector and vector-axial interactions give cross sections for direct detection suppressed by the small DM velocity.

Since the mass splitting between \( \chi_1 \) and \( \chi_2 \) is larger than the momentum transfer (\( \sim 10–100 \text{ keV} \)) of the DM-nucleus scattering, direct detection experiments are sensitive only to the lightest state. Thus, the usual Majorana DM predictions for spin-dependent cross sections apply here. The best experimental limits (from xenon [7]) are still above the expected cross sections for Majorana DM [8]. In addition, the vector-vector term proportional to \( \delta \) mediates coherent spin-independent DM-nuclei scatterings. The spin-independent total cross section of \( \chi_1 \) on a nucleus can be translated into a constraint on the parameter \( \delta \): \( \delta \approx 0.03 \times \left( \frac{\sigma_{\text{SI}}}{10^{-44} \text{ cm}^2} \right)^{1/2} \left( \frac{m_{\text{DM}}}{1 \text{ GeV}} \right) \). The best current direct detection limit on a spin-independent DM-nucleon cross section comes from CDMS-II [9]: \( \sigma_{\text{SI}}^\text{CDMS-II} \approx 3 \times 10^{-44} \text{ cm}^2 (m_{\text{DM}}/100 \text{ GeV}) \), for \( m_{\text{DM}} \approx 70 \text{ GeV} \). Thus, the region of parameter space dictated by the DM relic abundance together with a \( \delta \) at the percent level is comfortably consistent with direct detection constraints. Note that direct detection constraints apply to \( \delta \approx m_L - m_R \) and not \( \Delta m \) and can be relieved...
by imposing a parity symmetry relating the $L$ and $R$ sectors.

Decay length.—In the limit when $\Delta m \ll m_1$, the decay length of $\chi_2 \rightarrow f \bar{f} \chi_1$ is given by $L_0 \approx 4.6 \text{ cm}(\frac{\Lambda/C}{500 \text{ GeV}})^5 \times (\frac{1 \text{ GeV}}{\Delta m})^5$, where $C'$ is defined as $C$ after Eq. (3) but now the sum runs over SM fermions whose mass is less than $\Delta m/2$. For the range of $\Delta m$ under consideration, the $t$ quark is excluded. Decay into b quarks may be kinematically allowed in a narrow region at large $\Delta m$ but still suppressed with respect to decays to lighter particles, leading to a small branching ratio to b quarks. Therefore, we neglect the possible emission of b quarks, which implies $C' = C$. $L_0$ is related to the decay length in the laboratory frame by $L_{\text{lab}} = (p_2/m_2)L_0$, $p_2 = |\vec{p}_2|$ being the momentum of $\chi_2$, typically of the order of a few times $m_2$. Therefore, a mass splitting of the order of GeV naturally leads to a decay length of the order of a measurable displaced vertex.

The decay length depends on the strength of the coupling of pDDM to the SM fermions [see Eq. (2)], but it is intriguing that electroweak couplings and masses of the order of a few hundreds of GeV would lead to an observable displaced vertex: $100 \mu m \leq L_{\text{lab}} \leq 1 \text{ m}$ [10].

Leaving a trace at colliders.—To describe the pseudo-Dirac phenomenology at colliders, we need to specify the production mechanism. The four-fermion operator in Eq. (2) describes the interaction of pDDM with the SM fermions but does not capture interactions involving new heavy particles besides $\chi_1, \chi_2$. For example, in a supersymmetric scenario where $\chi_1, \chi_2$ are neutralinos, the main production mechanism is not given by Eq. (2) but instead by pair production of squarks $\tilde{q} \rightarrow \chi_0^0 + j$ and the subsequent decay of $\chi_0^0$ into 2 SM fermions and the lightest neutralino.

In the rest frame of $\chi_0^0$, a small $\Delta m$ implies that the $p_T$ distribution of the leptons or jets is small, typically $p_T^l; p_T^j < \Delta m$. Objects with very low $p_T$ would not be triggered [10]; hence, a sizable boost from the $\chi_0^0$ reference frame to the laboratory frame is a requirement for detection. LEP energies are too low to produce such a boost, as pointed out by Ref. [11], but at Tevatron or LHC $\chi_0^0$ would typically carry the $p_T$ of the heavy squark, and the leptons or jets could have a sizable $p_T$. To determine whether such a boost would render the leptons or jets detectable, and with what efficiency, we performed a Monte Carlo simulation with MADGRAPH/MADEVENTV4.3 [12]. In the rest of this section we consider decays to leptons, $f = e, \mu, \tau$, although the discussion can be generalized. In this case, the final state we are considering is 2 hard jets $+ 4$ leptons $+ E_T$. This signal contains many leptons, high-$p_T$ jets, and missing energy. Therefore the background is reducible [13], and the measurement is not very sensitive to a good determination of the standard model background. In pDDM, a large missing energy ($E_T \approx 200 \text{ GeV}$) and two high-$p_T$ jets would be the main handles for triggering.

In pDDM, two measurements, the dilepton edge and decay length, suffice to determine the overall DM scale and the splittings. The end point in the dilepton invariant mass distribution provides a measurement of $\Delta m, m_{1\ell_{\text{edge}}} = \Delta m$, as discussed in Refs. [10,14]. To measure this edge we asked for 4 well separated ($\Delta R_{\ell\ell} > 0.7$), central ($|\eta| < 3.5$) leptons with $p_T > 4 \text{ GeV}$ and paired them asking for the closest 2 opposite-sign leptons [13]. After this selection, the combinatorial background is very small, and the dilepton invariant mass has a clear edge at the position of $\Delta m$; see the right side of Fig. 1. The lepton momentum can be determined with a precision of a few percent for $p_T \geq 4 \text{ GeV}$; hence, a determination of $\Delta m$ is possible within the range of a few percent. In the example shown in Fig. 1, we chose 500 GeV squarks decaying into $\chi_0^0$ of mass $(100 + \Delta m) \text{ GeV}$, and then we varied $\Delta m$. Note, though, that there are no LEP bounds for neutralino masses when $\Delta m < 4 \text{ GeV}$, and smaller neutralino masses could be considered. The $p_T$ distribution depends on the energy of the collider, and we simulated the events assuming a 7 TeV running for the LHC. The efficiency of a $p_T$ cut of 4 GeV for at least two leptons is sizable: With a splitting between the two Majorana states $\chi_1$ and $\chi_2$ of 2 GeV, the efficiency is 20%, whereas for 10 GeV the efficiency is close to 100%. Although Tevatron has no kinematic access to 500 GeV squarks, we could consider lighter squarks, although the efficiency of the $p_T > 4 \text{ GeV}$ cut would be lower.

The other essential ingredient to pDDM at colliders is the measurement of a displaced vertex. The proper length $L_0$ and the length measured in the laboratory differ by a factor $p_T/m_2$, but one can measure $p_T^2$ as follows. Bounds on squarks—or any colored particle—at Tevatron indicate that $m_q \approx 400 \text{ GeV}$ [15]. Such heavy particles would be produced with very little boost at a 7 TeV collider, resulting in a back-to-back jet and $\chi_2$, i.e., $p_2 \sim p_j$. We have verified this statement with the Monte Carlo simulation.

The dark matter-collider connection.—Having discussed in the previous sections the DM bounds and the collider phenomenology of pDDM, we are now in a position to connect these very different pieces of information. We can eliminate $\Lambda/C$ and obtain an expression for the decay length of $\chi_2 \rightarrow f \bar{f} \chi_1$, which includes the relic abundance:

![FIG. 1 (color online). Dilepton invariant mass distribution for different choices of $\Delta m$ (LHC 7 TeV run).](image)
The relation (5) then makes a prediction for the dark matter mass we extract which may rule out the model. In fact, suppose that from cosmological quantities also allows us to make predictions the plane of the mass splitting with a singlet partner [16]. The translation from our effective pDDM scenario is provided by the supersymmetric bino [121802 (2010) PHYSICAL REVIEW LETTERS week ending 17 SEPTEMBER 2010] L0 \approx 30 \text{ cm} \left( \frac{\Omega_{\text{DM}} h^2}{0.11} \right)^2 \left( \frac{m_1}{100 \text{ GeV}} \right)^2 \left( \Delta m \right)^5 e^{-24 \Delta(m/m_1)}.

(5)

This relation is the main result of this Letter. It provides an intriguing connection between cosmological and collider measurements, which is independent of the details of the coefficients of the effective Lagrangian, and contains only readily measurable quantities. This remarkable property of pDDM is easy to understand: The processes leading to the relic abundance (pDDM is easy to understand: The processes leading to the relic abundance (\chi_1 \chi_2 \to \bar{f} f) and the decay length (\chi_2 \to \chi_1 f \bar{f}) come from the same term in the effective Lagrangian in Eq. (2). Still, it is an outstanding coincidence that the constraints from DM abundance and from having a mass splitting which is loop-suppressed with respect to the overall DM mass scale point to the region of parameter space corresponding to visible displaced vertices 10^{-2} \text{ cm} \leq L_0 \leq 10^2 \text{ cm}; see Fig. 2. It is worth noticing that the simple relation between collider and cosmological quantities also allows us to make predictions which may rule out the model. In fact, suppose that from measurements of the dilepton invariant mass distribution we extract \Delta m. The decay length is also easily measured. The relation (5) then makes a prediction for the dark matter mass m_1, which can be tested against other independent measurements from direct or indirect searches.

As mentioned above, a concrete realization of the pDDM scenario is provided by the supersymmetric bino with a singlet partner [16]. The translation from our effective Lagrangian to the bino case is straightforward: (C/\Lambda)^4 = \sum_f (1/4) (g' Y_f / m_f)^4, where g' is the U(1) Y gauge coupling and Y_f is the hypercharge of the fermion f. If the neutralinos \tilde{\chi}^0_{1,2} are pure combinations of \tilde{B} and \tilde{W}^0, and the decay occurs mostly through the exchange of a right-handed slepton of mass m_{\tilde{f}_R}, the decay length is easily obtained: L_0 \approx 1.8 \text{ cm} \left( \frac{m_{\tilde{f}_R}}{100 \text{ GeV}} \right)^4 \left( \frac{1 \text{ GeV}}{\Delta m} \right)^5 \text{ valid up to order } \mathcal{O}(\Delta m / m_{\tilde{f}_R})^2. \text{ Instead, for DM annihilations only into right-handed leptons, the analytical approximation in Eq. (4) translates into } m_{\tilde{f}_R} \approx 202 \text{ GeV} \left( \frac{\Omega_{\text{DM}} h^2}{0.11} \right)^{1/4} \times \left( \frac{m_1}{100 \text{ GeV}} \right)^{1/2} e^{-6(\Delta m / m_1)}, \text{ which can be regarded as a prediction for the slepton mass, once the bottom of the supersymmetric spectrum is known.}

In conclusion, we presented a predictive and testable scenario called pseudo-Dirac dark matter which possesses a virtue uncommon to DM theories: observable collider signatures in the form of displaced vertices. We have found a simple relation between collider and cosmological quantities. We have also shown how one could determine experimentally the decay length and the mass splitting and, based on that, obtain a prediction for the DM mass, which can be tested against other independent measurements from direct or indirect searches.

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