Predicting Dislocation Climb and Creep from Explicit Atomistic Details

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Here we report kinetic Monte Carlo simulations of dislocation climb in heavily deformed, body-centered cubic iron comprising a supersaturation of vacancies. This approach explicitly incorporates the effect of nonlinear vacancy-dislocation interaction on vacancy migration barriers as determined from atomistic calculations, and enables observations of diffusivity and climb over time scales and temperatures relevant to power-law creep. By capturing the underlying microscopic physics, the calculated stress exponents for steady-state creep rates agree quantitatively with the experimentally measured range, and qualitatively with the stress dependence of creep activation energies.

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Dislocations are line defects that play a central role in crystalline plasticity [1]. Nonconservative motion of these defects via emission or absorption of lattice vacancies, termed dislocation climb, is a key mechanism of high temperature deformation (creep). As illustrated in Fig. 1, microscopic vacancy-dislocation interactions and energetic barriers to vacancy migration can depend strongly on the vacancy migration paths. However, these atomistic details are generally overlooked in the study and prediction of climb [1]. Moreover, incorporation of both vacancies and dislocations in a single computational framework is complex, as this requires accounting simultaneously for nonlinear vacancy-dislocation interactions (beyond the range of applicability of dislocation dynamics [2,3]) and long time scales associated with thermally activated processes such as vacancy diffusion (beyond the accessible time scales of molecular dynamics [4]). In this Letter, we introduce a bridging approach to include atomic fidelity within simulations over macroscopically relevant time scales. This approach involves constructing energy barrier databases, which include every plausible (microscopic) atomistic mechanism, and then statistically sampling the various pathways through kinetic Monte Carlo (KMC) methods to predict (macroscopic) dislocation mobility over relevant time scales.

Mott initially posited that motion of an edge dislocation segment normal to the slip plane proceeds at a velocity proportional to applied stress ($\dot{\varepsilon} \propto \sigma^2$) [1,5]. Application of this model predicts that, for crystalline materials comprising noninteracting, pinned dislocations that exhibit only climb, the steady-state creep rate exhibits a power-law stress dependence with an exponent of three ($\dot{\varepsilon} \propto \sigma^3$) [6]. However, this prediction has not been experimentally verified and thus, as noted by Weertman [6], has prompted more complex analytical solutions to include dislocation pile-up effects [7] and core diffusion [8,9]. In fact, the exponent is instead between 4 and 6 even under conditions in which power-law dislocation creep is expected to be the dominant mechanism [6,10]. Here, we explore whether explicit consideration of microscopic point-line defect interactions can address this discrepancy. Moreover, our goal is to develop a model that could accurately predict the general experimental trends (such as variations of stress exponent and activation barrier with creep conditions) observed for power-law creep deformation, and thereby capture the underlying microscopic physics. We show that for a model system (bcc Fe comprising a high density of pinned dislocations and vacancies), this general framework that accounts for atomistic effects in dislocation climb allows prediction of power-law creep stress exponents in quantitative agreement with experiments.

The unit process of dislocation climb is migration of a vacancy to or from a dislocation core [5,11]. We thus first determined the migration barriers of bcc Fe vacancies as a function of distance and orientation of the migration path with respect to a dislocation core. We have previously computed these migration barriers via the nudged elastic band method [12] for the $\{111\}$ $(110)$ $71^\circ$ mixed edge-type dislocation dipole of Burgers vector $b = \frac{1}{2}[111]$ and line direction along $[11\bar{1}]$ [Fig. 1(a)], using our many-body Finnis-Sinclair potential [13] and sampling the nonredundant pathways [Fig. 1(b)] (see Ref. [11] for details). Briefly, the vacancy-dislocation interaction $E_{v-d}$ deviates significantly from elastic theory [1] when the vacancy is within the core region ($\sim 4|\mathbf{b}|$) [11] and depends on the migration path details. Figure 1(c) shows that the vacancy migration energies $E_m$ inside the core region ($\sim 4|\mathbf{b}|$) are strongly influenced by the nonlinear core-vacancy interaction. The latter decreases with core-vacancy distance $r$ for a particular migration path and differs substantially along various migration paths defined by orientation $\theta$ to the dislocation line. Interestingly, vacancy migration barriers for a particular path do not depend on the jog structure of the core except for the last jump ($<0.1$ eV) [11], whereas $E_m$ for the last jump depends much more strongly on the path details [e.g., paths C and...
deformed materials, vacancy concentrations $\%v$ producing the number of distinct possible events. Substantially simplified subsequent KMC simulations by reducing the number of distinct possible events.

FIG. 1 (color online). (a) Simulation cell, in which a pair of edge dislocations ($T_1$, $T_2$) of opposite sign (dislocation dipole) is created by removing a half plane of atoms (yellow). Vacancies (red) are distributed randomly; two are indicated with red arrows. (b) Various possible vacancy migration paths towards the dislocation core. (c) $E^m_v$ and activation barrier for vacancy migration along various paths shown in (b).

D in Fig. 1(c) differ by $\sim$0.8 eV. This observation substantially simplified subsequent KMC simulations by reducing the number of distinct possible events.

We used these calculated barriers $E^m_v(r, \theta)$ within event tables for KMC simulations of heavily deformed bcc Fe, our case of interest. The total dislocation density of highly deformed metals such as cold-rolled or fatigued bcc Fe and its alloys can range from $10^{15}$ m$^{-2}$ to $10^{16}$ m$^{-2}$ [14]; the mobile dislocation density $\rho_d$ depends on applied stress $\sigma$ and will attain these magnitudes at sufficient $\sigma$. In such deformed materials, vacancy concentrations $q_v$ also significantly exceed thermal equilibrium values (by at least 4 orders of magnitude), due to processes such as dislocation jog drag [15], and under a constant applied stress this supersaturation attains a steady state [16]. Because of the annihilation of vacancies at the core, the vacancy concentration adjacent to the core instantly achieves its thermal equilibrium level such that $q_v$ at the core can be neglected. However, far from the core, the vacancy concentration remains at steady-state supersaturation corresponding to the applied stress. This sets a vacancy concentration gradient between the core and lattice, resulting in vacancy diffusion towards the core. Under this assumption of steady-state vacancy supersaturation, explicit consideration of vacancy emission in the unit process of climb is neglected here [17]. The KMC configuration was constructed for a monoclinic periodic supercell with periodic boundaries along $x = [111]$, $y = [111]$ and $z = [\overline{1}0\overline{1}]$ directions with $N_x$, $N_y$ = 50, and $N_z$ repeat layers. $N_x$ and $N_z$ control the dislocation density $\rho_d$ and separation distance between dislocations $1/\sqrt{N_x}$. This $\rho_d$ is directly related to the applied stress according to Taylor’s relation, $\rho_d = (\sigma/\alpha Gb)^2$, where $\alpha$ is an empirical constant of 0.4 and $G$ is the shear elastic modulus [9]. The point/line defect densities and temperature ranges were chosen to approximate conditions relevant to highly deformed bcc Fe exhibiting power-law creep: $10^{15}$ m$^{-2} < \rho_d < 10^{17}$ m$^{-2}$, $5 \times 10^{-5} < q_v < 10^{-3}$, and $T > 0.4T_m$, where $T_m$ is melting temperature [14,15,18].

As dislocation climb is mediated by free vacancy diffusion, we first studied the dependence of vacancy diffusivity on $\rho_d$. To maintain the steady-state vacancy supersaturation constant, a new vacancy was inserted (far from the core at a random Fe lattice position), each time a vacancy was absorbed to a core. In the temperature range of interest (at least 700 K below $T_m$), it is reasonable to invoke the harmonic approximation to the transition state theory. Here, vibrational entropy effects are included in the temperature-independent preexponential factor, and thus a constant prefactor (60 THz) is assumed to calculate vacancy diffusion rate. Figure 2 shows self-diffusivity $D^{\rho_d,T}$ normalized by diffusivity in a dislocation-free lattice, $D^{0,T}$, indicating that $D^{\rho_d,T}/D^{0,T}$ increases with increasing $\rho_d$ and decreases with increasing temperature $T$. Note that the severity of this temperature dependence also increases with increasing $\rho_d$. These features can be rationalized qualitatively as follows. The general diffusion equation reads as, $D^{\rho_d,T} = D_0 \exp(-E_m/k_B T)$, where $E_m$ is the vacancy migration barrier in a dislocation-free lattice and $k_B$ is Boltzmann’s constant. Near the dis-
Results for different levels of core can be written as \( \%v = \frac{kBT}{\text{climb of edge-type dislocations segments}} \). Thus, we did simulations. Although both dislocation climb and glide can increase faster-moving vacancies near the core concurrently.

Therefore, vacancy diffusivity within the core vicinity, \( D_{\text{climb}} = D_0 \exp\left(\frac{E_m - \langle E_m(r, \theta) \rangle}{k_BT}\right) \), exceeds the diffusivity of vacancies that are beyond the influence of the core: \( D_{\text{climb}} = D_0^{\text{climb}} \). The effective diffusivity in the presence of dislocations at a given density can be written as \( D_{\text{climb}} = f_v D_{\text{climb}}^{\text{core}} + (1 - f_v) D_0^{\text{climb}} \), where \( f_v \) is the volume fraction of vacancies within \( 4|b| \) of the core vicinity. Therefore, \( D_{\text{climb}} / D_0^{\text{climb}} \) at a fixed \( \rho_d \) decreases with increasing temperature. However, at fixed \( T \) and \( \rho_v \), diffusivity increases with increasing \( \rho_d \) because the corresponding fraction of faster-moving vacancies near the core concurrently increases.

Next, we extracted the climb velocity \( v_c \) from KMC simulations. Although both dislocation climb and glide can occur sequentially, power-law creep is controlled by the climb of edge-type dislocations segments [9]. Thus, we did not explicitly allow glide moves, but incorporated these effects through the maintenance of a vacancy supersaturation under a constant applied stress; see analytical treatment in Eq. (1) below. We calculated the number of vacancies binding to the core as a function of time, the slope of this linear regression yielding the vacancy-core binding rate \( \kappa \), and \( v_c = \kappa h/N_v \). Here, \( h \) is the dislocation jog displacement due to single vacancy adsorption or interplane spacing along \( z = [101] \) and \( N_v = 50 \), the number of \{111\} planes intersected along the dislocation line direction [111] for this periodic simulation cell. We also explicitly allowed vacancy diffusion along the dislocation core, which changes the local dislocation jog structure but does not contribute to the climb displacement. The KMC simulation enables us to separately investigate the effects of increasing dislocation density or vacancy concentration on climb velocity: \( v_c(\rho_d, \sigma, T) \). Figure 3(a) shows that for a constant \( \rho_d \), \( v_c \) increases monotonically with \( \rho_d \), and exhibits a power-law dependence, \( v_c = \mathcal{P}(T)\sigma^\beta(T) \); \( \mathcal{P}(T) \) is a temperature-dependent prefactor. Vacancy supersaturation is realized in heavily deformed metals [15]; under a constant applied stress and temperature, this supersaturation will be maintained at steady state and can be given by [16,17]

\[
\mathcal{Q}_v = \mathcal{Q}^\text{ref}_v \frac{\sigma}{\sigma^\text{ref}} \exp \left[ \frac{E_m}{k_BT} \left( \frac{1}{T^\text{ref}} - \frac{1}{T} \right) \right] \frac{\mathcal{P}(T)\sigma^\beta(T)}{\mathcal{P}(T^\text{ref})\sigma^\beta(T^\text{ref})}, \tag{1}
\]

where \( \mathcal{Q}_v^\text{ref} \) is the assumed supersaturation at a reference applied stress \( \sigma^\text{ref} \) and temperature \( T^\text{ref} \). Figure 3(b) shows that when the vacancy supersaturation level is varied with \( \sigma \) for constant \( \rho_d \), \( v_c \) also exhibits a power-law \( v_c \propto \sigma^\gamma(T) \).

Next, we simultaneously varied both the defect densities, \( \rho_d \) and \( \rho_v \), corresponding to \( \sigma \) and as shown in Fig. 3 complete stress dependence of \( v_c \), where both \( \rho_d \) and \( \rho_v \) change simultaneously and correspond to \( \sigma \). Statistical errors in \( v_c \) are smaller than the data points in log-scale. (d) Climb velocities in (a), (b), and (c) show power-law dependence on \( \sigma \). Exponents of these power-laws \( \beta, \gamma \) and \( \mu \), respectively, vary weakly with \( T \). Results for different levels of \( \mathcal{Q}_v \) are shown in supplementary material [17].

FIG. 3 (color online). Dislocation climb velocity as a function of applied stress for (a) varied \( \rho_d(\sigma) \) (via Taylor’s equation) and constant \( \mathcal{Q}_v = 10^{-4} \) for all \( \rho_d \) at temperatures \( T > 0.4T_m \); and (b) varied \( \mathcal{Q}_v(\sigma, T) \) according to Eq. (1) for a set of constant \( \rho_d \) and \( T = 800 \) K, where \( \mathcal{Q}_v^\text{ref} = 729.9 \) MPa, 1100 K = 10^{-4}. (c) Complete stress dependence of \( v_c \), where both \( \rho_d \) and \( \rho_v \) change simultaneously and correspond to \( \sigma \). Statistical errors in \( v_c \) are smaller than the data points in log-scale. (d) Climb velocities in (a), (b), and (c) show power-law dependence on \( \sigma \). Exponents of these power-laws \( \beta, \gamma \) and \( \mu \), respectively, vary weakly with \( T \). Results for different levels of \( \mathcal{Q}_v \) are shown in supplementary material [17].
FIG. 4 (color online). (a) Creep over a range of stresses and temperatures $T > 0.4 T_m$ for reference supersaturation $\gamma^\text{ref} = 729.9 \text{ MPa, 1100 K} = 10^{-4}$. (b) Stress exponents and effective activation energies at different temperatures and applied stresses, respectively. The solid horizontal line represents the activation barrier of self-diffusion (0.84 eV) in a dislocation-free lattice. The effective activation energy is a linear function of applied stress, $Q = Q_0 - V_v \sigma$, where $Q_0 = 0.87 \text{ eV}$. Calculated activation volume $V_v = (0.85 \pm 0.05) \Omega_0$, where $\Omega_0$ is the Fe atomic volume, is consistent with dislocation creep controlled by vacancy diffusion.

the microscopic dependence of $v_c$ on applied stress and thus on the abundance of point and line defects [17]. Both the temperature dependence and magnitude of $n$ in Fig. 4(b) are in good agreement with experiments for which $n \gtrsim 4.5$ [10,18]. The creep activation energy $Q$ decreases with increasing applied stress [Fig. 4(b)], as effective $E_m$ decreases and diffusivity near dislocation core increases with increasing $\sigma$. This observation is in agreement with experiments [18,19]. Although our case of interest is for $\sigma > 500 \text{ MPa}$, experimentally reported creep rates for bcc Fe are limited to lower stresses. Assuming the same creep mechanism (climb controlled) in the experimental stress regime, our extrapolation of $\dot{\varepsilon}$ (for $\gamma^\text{ref} = 10^{-4}$) agrees within 1 order of magnitude with experimental creep rates for $\sigma < 100 \text{ MPa}$ and $900 \text{ K} \leq T \leq 1100 \text{ K}$ [10,17,18]. The agreement is poorer at lower temperatures, although the trends for the stress exponent and activation energy are maintained. In understanding the microscopic origin of power-law creep deformation, these trends of stress exponent $n$ and activation energy $Q$ with temperature and applied stress, respectively [10,18,19], are arguably more important than the creep rate magnitudes alone; these trends are correctly predicted within the present microscopic model [Fig. 4(b)]. Further, we anticipate that explicit inclusion of magnetic interactions will systematically affect migration barriers [20], and thus the quantitative creep rate, but not the qualitative effects of applied stress and temperature.

In summary, we have demonstrated a general framework to predict the macroscopic, dislocation climb-mediated creep plasticity from consideration of microscopic interactions between point and line defects. The departure from the previous models ($n \sim 3$) lies in the stress dependence of the overall vacancy concentration of the present formulation. This assumption along with the recognized dependence of mobile dislocation density on applied stress not only gives rise to the predicted $n \sim 5$ but also predicts the trends in creep deformation, in agreement with experimental findings. We anticipate future studies of macroscopic creep with such microscopic fidelity will also consider solute-vacancy and solute-dislocation interactions in alloys.

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