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Distributed Detection of Multi-Hop Information Flows With Fusion Capacity Constraints

Ameya Agaskar, Ting He, Member, IEEE, and Lang Tong, Fellow, IEEE

Abstract—The problem of detecting multihop information flows subject to communication constraints is considered. In a distributed detection scheme, eavesdroppers are deployed near nodes in a network, each able to measure the transmission timings of a single node. The eavesdroppers must then compress the information and transmit it to a fusion center, which then decides whether a sequence of monitored nodes are transmitting an information flow. A performance measure is defined based on the maximum fraction of chaff packets under which flows are still detectable. The performance of a detector becomes a function of the communication constraints and the number of nodes in the sequence. Achievability results are obtained by designing a practical distributed detection scheme, including a new flow finding algorithm that has vanishing error probabilities for a limited fraction of chaff packets. Converse results are obtained by characterizing the fraction of chaff packets sufficient for an information flow to mimic the distributions of independent traffic under the proposed compression scheme.

Index Terms—Information-theoretic limits, intrusion detection, network surveillance, traffic analysis.

I. INTRODUCTION

C onsider a wireless ad hoc network as illustrated in Fig. 1. We want to analyze traffic in the network to detect the presence of information flows. If every packet is re-encrypted and padded at each relay node, the only information we can work with is the timing of transmissions. The problem, then, is to infer the presence of flows from correlations among the transmission timing patterns of nodes. To do this, eavesdroppers may be deployed near several nodes in the network. No decision can be made by any eavesdropper since it can only observe the signal from a single node, but each eavesdropper may communicate with a fusion center under a rate constraint. The detector at the fusion center, given the limited data arriving from each eavesdropper, can then decide whether the observed path contains an information flow.1

There are many challenges in such a distributed traffic analysis problem. The network can mask information flows through random timing perturbations, packet shuffling, or the insertion of extra packets that are not part of the flow. These extra packets, as well as packets not part of the flow in question, constitute chaff noise. The rate constraint for the communications channels poses another challenge; each eavesdropper must quantize its measurements in a manner that preserves the detectability of flows at the fusion center.

The problem considered here has applications in network surveillance and intrusion detection. With the proliferation of advanced encryption techniques, it can be useful to glean as much information about data flow in a network as possible, even if decryption of the data is out of reach. An eavesdropper may want to monitor a network using several low-cost sensors that communicate wirelessly with a fusion center; the limited power of the sensors motivate the rate constraints in this problem.

A. Related Work

Staniford-Chen and Heberlein [2] were the first to consider the problem of detecting stepping-stone attacks in networks. These are attacks wherein a malicious user routes data and commands through several compromised nodes that do not belong to him before reaching the final, targeted node. Solving this

1Part of this paper was presented in [1].
problem involved finding information flows. Stanford-Chen and Heberlein used traffic content to make a decision, as did Wang et al. [3]. Zhang and Paxson [4] first considered dealing with encrypted traffic using timing information instead of content. Donoho et al. [5] considered encrypted traffic with timing perturbations, and found that it was possible to exploit a known delay constraint to differentiate a flow from independent traffic. The authors of [6] showed that similar results could be obtained if an intruder were known to have a memory constraint. They later found the first timing-based detectors with vanishing error probabilities that work even when the chaff noise grows proportionally to the total traffic [7]. In [8], the authors showed that as the number of hops in an information flow grows large, an information flow buried in chaff noise can no longer be hidden from a detector. This paper is a follow-up of [9], where the problem of distributed detection of 2-hop information flows was first considered. Under a similar formulation as that in [9], we extend the results of [9] in several aspects. First, we consider general multihop (more than 2) information flows. Second, we propose a new algorithm. Unlike [9], our algorithm operates directly on the quantized processes, rather than first reducing the problem to an equivalent unquantized problem. The new algorithm is faster than the old except in cases where the rate constraint is loose. Third, we present experimental results of our algorithm applied to actual network traces.

B. Summary of Results and Organization

We consider the distributed detection of multihop information flows by timing analysis. As in [9], we characterize the performance of a detector by the largest fraction of chaff noise under which a flow is still detectable, and we break the problem down to quantization at local traffic sensors and detection at the fusion center. In [9], algorithms previously developed for the unquantized version of the problem were applied to preprocessed realizations of the quantized processes. In this paper, a new algorithm is developed that applies directly to the quantized realizations and provides some speedup when the rate constraint is not too loose. This algorithm finds the minimum possible fraction of chaff noise that could have been mixed with a delay-constrained information flow to generate the observed, quantized realizations. This value is then compared to a threshold, yielding detection if the threshold is not exceeded. Under the assumption that, in the absence of information flows, nodes transmit according to independent Poisson schedules, we provide a characterization of the threshold. We show that if the flow size is above a certain level, the miss detection and false alarm probabilities vanish as the number of packets used in the detection increases. We then provide an analytical upper bound on the fraction of chaff noise under which our detector provides a consistent detection. We compare the performance under various rate constraints and flow lengths. We then loosen the Poisson assumption and determine the performance of the distributed detection scheme when the interarrivals follow the Pareto distribution, which is said to more closely model packet transmission timestamps in networks [10]. Finally, we test our algorithm on real data, demonstrating the performance of the detection scheme on TCP traces.

The rest of the paper is organized as follows. Section II is the problem formulation. In Section III, we define a performance measure. In Section IV, we define the quantization scheme. In Section V, we define an algorithm for finding the minimum fraction of chaff noise. In Section VI, we show that the value computed by this algorithm converges almost surely (a.s.) when the sources are Poisson and we define a detector that takes advantage of this fact. We also derive an analytical upper bound on performance as well as a lower bound that is computed numerically. In Section VII, we show the results of these experiments and interpret them. In Section VIII, we consider the Pareto interarrival model and compare results to those under the Poisson model. In Section IX, we demonstrate the performance of the detection scheme on actual TCP traces. Finally, in Section X we conclude with remarks.

II. PROBLEM FORMULATION

A. Notation

We use the following notation. Uppercase letters denote random processes; lowercase letters denote their realizations. Boldface letters represent vectors, and plain letters represent scalars. We use parentheses to indicate indexing. Script letters represent random processes; lowercase letters denote their realizations. A. Notation

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Given these quantized realizations, the detector $\delta_i((\mathbf{Q}_i^{(f)})_{i=1}^N)$ chooses a hypothesis. This is a partially nonparametric hypothesis test, as the correlations among the processes under $\mathcal{H}_1$ is not specified. We require that the marginal distribution of each node’s process be the same under each hypothesis; otherwise, each eavesdropper could independently test the hypothesis and send its one bit result to the fusion center [9].

Given a per-node rate constraint $R$, the problem is to design the quantizer functions $h_i^{(f)}(\cdot)$ and the detector $\delta_i(\cdot)$ to optimize the overall detection performance.

III. PERFORMANCE MEASURE

A measure of performance of a detector must take into account the detector’s tolerance of chaff noise. To this end, we need to quantify the chaff noise contained in a process. We use the concept of the chaff-to-traffic ratio (CTR) defined in [8] and [9].

Definition 3.1: Given realization of an information flow $(\mathbf{f}_i)_{i=1}^N$ and chaff noise $(\mathbf{w}_i)_{i=1}^N$, the CTR is defined as

$$\text{CTR}(t) \equiv \frac{\sum_{i=1}^N |\{\mathbf{f}_i \cap [0,t]\}|}{\sum_{i=1}^N |\{\mathbf{f}_i \cup \mathbf{w}_i \cap [0,t]\}|}$$

$$\text{CTR} \equiv \lim_{t \to \infty} \text{CTR}(t) \quad (2)$$

where the operator $|\cdot|$ gives the number of elements in its argument. So $\text{CTR}(t)$ is the fraction of packets that are chaff before time $t$, and the CTR is the asymptotic fraction of chaff packets. The CTR is a function of the realizations, so it is itself a random variable.

To measure the performance of a detector, we use the following metric defined in [8], which is derived from the notion of Chernoff-consistency [11].

Definition 3.2: A sequence of detectors $(\delta_i)_{i>0}$ is called $r$-consistent $(r \in [0,1])$ if the false alarm probability $P_T(\delta_i)$ and the miss probability $P_M(\delta_i)$ satisfy

1. $\lim_{t \to \infty} P_T(\delta_i) = 0$ for any $(\mathbf{S}_i)_{i=1}^N$ under $\mathcal{H}_0$;
2. $\sup_{\mathcal{P}(\mathbf{S}_i)_{i=1}^N} \lim_{t \to \infty} P_M(\delta_i) = 0$, where

$$\mathcal{P} = \{(\mathbf{S}_i)_{i=1}^N : (\mathbf{S}_i)_{i=1}^N \text{ contains an information flow and } \text{CTR} \leq r \text{ a.s.}\}.$$  

The measure of consistency of a sequence of detectors $(\delta_i)_{i>0}$ is the supremum of $r$ such that $\delta_i$ is $r$-consistent.

Given a per-node capacity constraint $R$ and a distributed detection scheme with consistency $r$ that obeys the constraint, we have an achievable pair $(R, r)$. We can take the supremum of achievable consistencies as a function of the rate constraint to partition the consistency-rate space into achievable and unachievable pairs. This gives us the following definition.

Definition 3.3: Given a per-node capacity constraint $R$, the consistency-rate function (extended from the consistency-rate function in [9]) is defined as

$$\alpha(R) \equiv \sup \left\{ r \in [0,1] : \exists (\delta_i^{(f)}(\cdot), i = 1, \ldots, N, (\delta_i(\cdot))_{i>0} \right\}$$

such that $(\delta_i)_{i>0}$ is $r$-consistent and

$$\lim_{t \to \infty} \frac{1}{t} H(\mathbf{Q}_i^{(f)}) \leq R, i = 1, \ldots, N \right\}. \quad (3)$$

In this paper, we will consider only deterministic quantizers. By the source-channel separation theorem [12], the constraint on
the entropy of the quantized process in Definition 3.3 is sufficient for lossless transmission over a channel with capacity \( R \). So for CTR below \( \alpha(\mathcal{R}) \), there must be a detection scheme that satisfies the capacity constraint \( \mathcal{R} \) and will detect all flows. In general, as \( \mathcal{R} \) increases and the rate constraint is loosened, \( \alpha \) should increase monotonically and approach the nonquantized case \( (R = \infty) \). The ultimate goal of this work is to find the consistency-rate function and design quantizers and detectors to achieve it.

IV. DATA COMPRESSION SCHEME

We now find a lower bound on the consistency-rate function for various numbers of nodes by defining an actual detection scheme. In particular, we employ a slotted quantizer as in [9] and originally used to compress Poisson point processes in [13].

Definition 4.1: Given a point process \( \mathbf{S} \), a slotted counter with slot length \( T \) is defined as \( \gamma(\mathbf{S}) \equiv (Z_1, Z_2, \ldots) \), where \( Z_j = |S|^T \cup ([j - 1]T, jT) \) is the number of packets in slot \( j \).

The slotted quantizer so defined gives us a sequence of random variables representing the number of epochs in successive bins of width \( T \). If \( \mathbf{S} \) is a Poisson process with intensity \( \lambda \), then the \( Z_k \) are i.i.d. Poisson random variables with mean \( \lambda T \). For large block length, we can use a Lempel–Ziv encoder to transmit the \( Z_k \) with no errors at a rate [12]

\[
R(T) = \frac{1}{T} H_p(\lambda T)
\]

where \( H_p(\cdot) \) is the entropy of a Poisson random variable with a given mean. As we have already required that the sensors be synchronized to a timing reference, we also require that their quantization slots be synchronized. This aids the development of an algorithm to operate on the quantized values only.

V. THE QUANTIZED BOUNDED DELAY RELAY ALGORITHM

Given the unquantized observations at the traffic monitors, algorithms developed in [8], [14] can be used to obtain an optimal partition (cf. [8, Section V-B]) of the observed point processes into flow with delay constraints and chaff processes. Specifically, the multithop bounded delay relay (MBDR) algorithm proposed in [8] gives the minimum CTR(\( t \)) of a process generating a given realization with duration \( t \). The resulting CTR is then used as the test statistic for flow detection.

We now consider the case when observations at traffic sensors must be quantized using, for example, slotted quantizers discussed in Section IV. As an extension to the detection algorithm for unquantized observations, the key idea is to obtain an optimal partition algorithm that finds the minimum CTR that could have generated the quantized observations. The first such algorithm was presented in [9], where the problem was first reduced to an equivalent unquantized case by performing a worst-case reconstruction of the original point processes. We present here an alternative algorithm that operates directly on the slot-quantized values.

This new algorithm is referred to as quantized bounded delay relay (QBDR). It operates on the quantized sequences \( Q_{n}^{\Delta} \), where \( n = \lfloor t/T \rfloor \). Specifically, it decomposes the sequences into traffic \( h(\mathbf{F}_i) \) and chaff \( h(\mathbf{W}_i) \) sequences such that \( S_i = F_i \oplus W_i \), \( h(S_i) = Q_{i} \), placing as few packets as possible into the chaff process while satisfying the delay and causality constraints. The algorithm requires the slot width of each quantizer to be equal, which is why we operate under equal rate constraints. In practice, one could analyze a system by assuming that each node has a rate constraint equal to the tightest one present.

The algorithm starts at the beginning of the sequences and progresses through candidate flow paths, eliminating as much traffic as possible. Whatever remains is classified as chaff. The candidate flow path is represented by a set of pointers, one per node, to a slot in that node’s sequence. These pointers are initialized to the beginning of each sequence. Four steps are performed repeatedly until the end of any process is reached. First, any possible flows along the candidate path are eliminated by subtracting the minimum value of any slot in the path from all slots in the path. Second, any pointer to a slot with no packets remaining is incremented. Third, the causality constraint is enforced by incrementing any pointer to a slot earlier than its source node’s pointer. Fourth, the delay constraint is enforced by incrementing any pointer that is not within \( D = \lfloor \Delta/T \rfloor \) slots of its destination node’s pointer. The algorithm is detailed in Table I with a specific example given in Table II.

Like its predecessor MBDR in the nonquantized case, QBDR is the optimal flow-finding algorithm in the quantized case, as stated in the following proposition.

Proposition 5.1: For any realization \( (q_1^{\Delta}, \ldots, q_N^{\Delta}) \), QBDR finds the minimum CTR of an information flow with bounded delay \( \Delta \).

Proof: It was shown in [8] that an algorithm that greedily finds the earliest possible relay sequences, or sequences containing one epoch from each point process, will find the most possible relay sequences. The proof, which is omitted here, shows that there exists a largest set of order-preserving relay sequences (since packets can be shuffled without breaking...
TABLE II

QBDR DEMONSTRATION

| Initialization (lines 4-6): | 2 2 5 3 0 |
| All slot pointers (represented by underlines) are initialized to slot 1. For this example, suppose \( D = 1 \). |
| Eliminate flows (lines 9-12): | 0 2 5 3 0 |
| The minimum value pointed to is 2. Subtract 2 from each value. |
| Find non-empty slots (lines 14-16): | 0 2 5 3 0 |
| Increment every pointer whose slot contains 0. |
| Enforce causality (lines 18-20): | 0 2 5 3 0 |
| Increment every pointer that is earlier than its predecessor’s pointer. |
| Enforce delay (lines 22-24): | 0 2 5 3 0 |
| No action necessary. |
| Find non-empty slots (lines 9-12): | 0 2 5 3 0 |
| The minimum value pointed to is 1. Subtract 1 from each value. |
| Find non-empty slots (lines 14-16): | 0 2 5 3 0 |
| Pointer 2 is incremented twice to get to the first nonzero value. |
| Enforce causality (lines 18-20): | 0 2 5 3 0 |
| Pointer 3 is incremented until it no longer occurs before pointer 2. |
| Enforce delay (lines 22-24): | 0 2 5 3 0 |
| Pointer 1 is incremented until it is within \( D \) of pointer 2. |

This process continues until all flows are found. All leftover packets are deemed chaff.

the constraints to ensure that relay sequences do not cross). Next, if this largest set is not the one found through the greedy algorithm, then there exists a way to find a relay sequence not in the largest set. As this is a contradiction, the assertion is proved.

Next, it remains to be shown that QBDR finds the earliest possible relay sequence at each step. We use an inductive argument. First note that the maximum delay bound in number of slots on average have at least one packet. This corresponds, for intensities of \( \lambda = 1 \), to a rate constraint less than 2–3 nats. For low rates, the improvement can be several orders of magnitude.

VI. A THRESHOLD DETECTOR AND ITS PERFORMANCE

Algorithm QBDR returns \( \hat{C}(t) \), which we can use as a statistic for our detector, which we define as follows:

\[
\delta_t(q_1^n, \ldots, q_N^n; \tau_n) = \begin{cases} 1 & \text{if } \hat{C}(t) \leq \tau_n \\ 0 & \text{otherwise} \end{cases}
\]

To determine the detector threshold \( \tau_n \), we use the following proposition.

Proposition 6.1: If the \( \hat{C}(t) \) obtained from QBDR converges almost surely to \( c \) under \( H_0 \) as \( t \to \infty \), then the measure of consistency (cf. Definition 3.2) of the detector given in (5) is \( c \).

Proof: Suppose that for large enough \( t \), the estimated CTR from QBDR under \( H_0 \) converges to some value \( c \) with probability one. Then we can fix any \( \tau_n = \tau < c \) for the threshold detector and be sure that the false alarm probability will go to zero. Meanwhile, under \( H_1 \), \( \hat{C}(t) < CTR \), where CTR is the true CTR of the process (this follows from Proposition 5.1). So long as \( CTR < \tau \), \( \hat{C}(t) < \tau \), and we will have detection with probability 1. So as long as \( \tau < c \), our detector is \( \tau \)-consistent, so the measure of consistency for this detector is \( c \).

We can show that under independent Poisson processes, the estimated CTR from QBDR does converge, as stated in the following proposition.
Theorem 6.3: If $S_t^i, i = 1, \ldots, N$ are independent Poisson processes of maximum rate $\lambda$, and $Q_t^i, i = 1, \ldots, N$ are the slot-quantized versions of the $S_t$ with slot length $T$, then

$$
\lim_{t \to \infty} \text{CTR}_{\text{QBD}}(t) \geq 1 - \kappa \text{ a.s.,}
$$

where

$$
\kappa = \min \left( (\lambda \Delta')^{N-2} \frac{1}{N} \Delta', \prod_{i=1}^{N} \left( 1 - e^{-i\lambda \Delta'} \right) \right),
$$

$$
\Delta' = T \left( \left[ \frac{\Delta}{T} \right] + 1 \right).
$$

Proposition 6.2: The minimum CTR of a process that produces the quantized realizations $q^i_t, i = 1, \ldots, N$ under $H_0$ with Poisson marginals converges a.s. to a constant $C$.

Proof: First consider $T \geq \Delta$ for simplicity; the argument can be extended to the other cases. As QBDR’s slot pointers move along the process, they modify the value in each slot by selectively removing packets corresponding to flows. Define $C^j_k(j)$ as the number of packets remaining in slot $j$ of process $k$ when the pointer to the first process $x_1$ first transitions from $j - d$ to $j - d + 1$ (before the packet removal for that stage).

Because of causality, $C^j_k(j) = C_k(j)$, the final number of chaff packets remaining in slot $j$ of process $k$. Once the process 1 pointer has passed a particular slot, there can be no more flow packets in that slot for any process. Because of the delay constraint, a flow can only move forward $k - 1$ slots from process 1 to process $k$. So $C^j_k(j) = Q_k(j)$.

Now consider the following collection of random variables, illustrated in Fig. 4:

$$
G(j) \equiv \left( \left( C^j_{k+1}(l) \right)_{l=1}^{N-j+1} \right)_{j=1}^N.
$$

In other words, when the pointer $x_1$ jumps from $j - 1$ to $j$, $G(j)$ has the number of remaining packets in processes 1 through $N$ in slot $j$, processes 2 through $N$ in slot $j - 1$, and processes $k$ through $N$ in slot $j + k - 1$ for $k$ up to $N$.

For the $N = 2$ case, $G(j)$ contains $C^j_1(j) = Q_1(j)$, $C^j_2(j)$, and $C^j_3(j + 1) = Q_2(j + 1)$. Given these variables

$$
C^j_2(j + 1) = \max \{ \min(Q_2(j + 1), 0) \}.
$$

In addition, $C^j_1(j + 1) = Q_1(j + 1)$, and $C^j_3(j + 2) = Q_2(j + 2)$, and we have a complete formula for $G(j + 1)$ given $G(j)$.

In general, $G(j)$ contains every possible slot that a flow starting in slot $j$ of process 1 can pass through. So obtaining $G(j + 1)$ involves running QBDR on $G(j)$ until $x_1$ jumps from $j$ to $j + 1$, then copying over the top diagonal from $(Q_1(j + 1), Q_2(j + 2), \ldots)$. So $G$ is a Markov process.

Process $G$ is aperiodic and irreducible. The realization of $G$ is bounded by the number of packets in each slot, which is a collection of independent Poisson random variables. The return time for the 0-state of this collection (where each element is zero) is finite, so the return time for the 0-state of $G$ is finite. So $G$ is positive recurrent. This property, combined with its aperiodicity and irreducibility, means that $G$ is ergodic. So $G$ converges to a limiting distribution. The amount of chaff in slot $j$, $C(j)$, is the number of packets remaining in slot $j$ after QBDR is applied to $G(j)$. Because $G$ converges to a limiting distribution, $(1/n) \sum_{j=1}^{N} C(j)$ converges a.s. to a constant (that we call $C$) by ergodicity. The CTR, then, converges a.s. to $(C)/(N\Delta T)$.

Although we do not have an analytical expression for $C$, the Proof of Proposition 6.1 suggests a numerical procedure:

1. Invert (4) to find $T(R)$.
2. Generate epochs of $N$ independent Poisson processes with parameter $\lambda$ over a period of time $t$, where $t$ is long so the estimated CTR is near the convergent value.
3. For each $R$, pass each process through a slot quantizer with slot length $T(R)$.
4. Use QBDR with delay bound $\Delta$ to find the minimum possible CTR to generate the observations.
5. $\alpha^*(R) = \text{CTR}$.

This gives us the achievable consistency value for each rate. In [8], it was shown that under certain conditions, a lower bound on the estimated CTR of independent Poisson traffic converges to one as $N$ increases. We can use this to obtain a similar result.

Theorem 6.3: If $S_t^i, i = 1, \ldots, N$ are independent Poisson processes of maximum rate $\lambda$, and $Q_t^i, i = 1, \ldots, N$ are the slot-quantized versions of the $S_t$ with slot length $T$, then

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$$

$$
\Delta' = T \left( \left[ \frac{\Delta}{T} \right] + 1 \right).
$$

Proof: First consider the meaning of $\Delta'$. In QBDR, $D = \lfloor (\Delta)/(T) \rfloor$ is the maximum difference in slots for two packets to be matched. Each packet can be matched to another packet in one of $D + 1$ different slots. For a packet at the very beginning of a slot, the maximum delay to the next packet is thus $T(D + 1) = \Delta'$.

Since all the processes have the same arrival rate, and a flow takes one packet from each process, the estimated CTR tends to $1 - P_N$, where $P_N$ is the probability that a packet in the $S_1$ is matched to packets in $S_2$ through $S_N$, which each successive packet satisfying the delay bound. We therefore wish to upper bound $P_N$. Since the first packet in $S_1$ claims packets in the remaining processes that become unavailable to future flows, $P_N$ is upper bounded by the probability that the very first packet in $S_1$ (call its arrival time $t$) is part of a flow. This is upper bounded by the probability that each $S_i, i = 2, \ldots, N$ has at least one packet somewhere in $t + i \Delta' (\text{a necessary but not sufficient condition for a flow})$. So, since arrivals are Poisson, $P_N \leq \prod_{i=1}^{N} \left( 1 - e^{-i\lambda \Delta'} \right)$. Using inductive arguments as in [8], we can also show that $P_N \leq \lambda \Delta' P_{N-1}$ for $N \geq 3$. Substituting our expression for $P_2$, we therefore have also that $P_N \leq (\lambda \Delta')^{N-2} \Delta'$. So $P_N$ is upper bounded by the minimum of the two expressions. Thus, we have a lower bound on the CTR.

It is easy to see, then, that if $\lambda \Delta' < 1$, then the CTR of QBDR goes to one exponentially with $N$. It is also clear that as $T \to 0$, $\Delta' \to \Delta$, meaning that the perfect information case agrees with the lower bound computed in [8].
Given a quantizing scheme and a null hypothesis, it may be possible to construct an information flow with chaff embedded in such a way that it is statistically indistinguishable from independent traffic. It can be shown [8] in the nondistributed version of this problem, with a Poisson null hypothesis, that the fraction of chaff required to do this is no greater than the consistency of a CTR threshold detector. This is not the case here because the optimal quantizer is unknown, but we can use it to find an upper bound on the consistency of our detector.

**Theorem 6.4:** Under the slot quantization scheme, the consistency obeys

\[ \alpha(R) \leq 1 - \frac{E(\min_{k=1,...,N} X_k)}{\lambda T} \]  

(8)

where \( X_k \sim \text{Poisson}(\lambda T) \).

**Proof:** We use the fact that in a Poisson point process, conditioned on the number of epochs \( Q \) in a slot, the location of the epochs has the same distribution as \( Q \) i.i.d. uniform random variables over that slot. We consider the following construction:

1. Generate sequences \( X_k(j), k = 1, \ldots, N; j = 1, \ldots, \) of i.i.d. Poisson random variables with mean \( \lambda T \).
2. In each slot \( j \), find \( X_{\min}(j) = \min_{k=1,...,N} X_k(j) \).
3. Generate the point process \( F_1 \) by choosing \( X_{\min}(j) \) epochs uniformly over \([j-1]T, jT] \) for all \( j \), and let \( F_i = F_{i-1} \cup \{X_k, k = 1, \ldots, N\} \).
4. Generate the point processes \( W_k, k = 1, \ldots, N \) by choosing \( (X_k(j) - X_{\min}(j)) \) epochs uniformly over \([j-1]T, jT] \) for all \( k \) and \( j \).
5. Generate the final point processes \( S_k = F_k \cup W_k \).

Now each \( S_k \) is a Poisson process with rate \( \lambda \), and the quantized sequences are still \( X_k, k = 1, \ldots, N \), which are i.i.d. Poisson random variables with mean \( \lambda T \). The average number of flow packets per node will be \( E(\min_{k=1,...,N} X_k) \), where \( X_k, k = 1, \ldots, N \) are i.i.d. Poisson random variables. From this we can compute the CTR and state that under slot quantization

\[ \alpha(R) \leq 1 - \frac{E(\min_{k=1,...,N} X_k)}{\lambda T}. \]  

(9)

This upper bound holds for all detectors that use the slot quantization scheme. For large \( N \), the upper bound is close to 1, and is thus not very instructive. For \( N = 2 \), it is equivalent to the upper bound derived in [9].

**VII. SIMULATIONS**

Experimental traces were generated in MATLAB, and the function \( \alpha^a(R) \) was computed according to the procedure outlined above. We generated new processes for each \( R \), and set \( t = 10^4 \cdot T/J \). This was long enough to produce consistent behavior between trials, which led us to believe that the convergent value was being found by the QBDR algorithm.

To assess the relationship between the number of nodes in the hypothesized flow and the consistency of the detection scheme, we let the number of nodes vary from 2 to 26, and for five rates plotted the consistency versus the number of nodes. The results are shown in Fig. 5.

![Consistency vs. Number of Nodes](image)

**Fig. 5.** The QBDR CTR \( \alpha^a(R) \) versus \( N \), the number of nodes, for various per-node rate constraints. In this case, (a) \( \lambda = 1/25 \) and (b) \( \lambda = 1 \). The cases do not meet the lower bound from (7) go to 1 as \( N \rightarrow \infty \).

We also found the consistency-rate curves for 2 through 6 nodes, and plotted the curves on the domain \( R \leq 1 \). The results are shown in Fig. 6.

The upper bounds from (8) were computed by generating length \( N \) sequences of Poisson random variables and finding the average of the minimum value. The upper bound for a particular case is plotted and compared to the achieved consistency in Fig. 7.

These plots yield several observations. For very low rates, the consistency does not grow much with the number of hops. In some cases where the condition for the lower bound is not met, the consistency still approaches one as \( N \rightarrow \infty \), but in other cases it appears to approach a smaller asymptote. This indicates that the slotted quantization introduces a fundamental limitation that is not present in the perfect information...
slots are used for each experiment to ensure convergence. Intuitively, this can be understood as transmitting a random variable with entropy proportional to $\log T$ being transmitted every period of length $T$. Since $T$ is large in this region, $\log T$ grows much more slowly than $T$. Changing the rate, then, is equivalent to changing the rate at which measurements are sent. So it is reasonable to expect that doubling this rate doubles the chaff we can tolerate, as the same chaff is now split over twice the measurements. The performance of our detection scheme deteriorates as the packet arrival rate increases, because the increase in $\lambda$ causes an increase in information entropy and a higher density of packets leads to more false matches between packets in successive nodes. As $\lambda$ increases, the achievable region shrinks. Fig. 7 shows that except at large rates our detector may be unable to achieve vanishing error probabilities even if there is a distinction between the measurable distributions under $\mathcal{H}_0$ and $\mathcal{H}_1$.

VIII. OTHER INTERARRIVAL MODELS

For tractability, we have thus far clung to the Poisson assumption—that interarrivals fit an exponential distribution. In actual networks, this assumption is not valid at the packet level. For example, TELNET packets have been shown [10] to be better modeled as i.i.d. Pareto distributed with the shape parameter $\beta \approx 1$.

In [15], we showed that it is more difficult to hide an information flow in traffic under Pareto interarrival times with shape parameter within a certain range (which includes, in particular, the shape parameter used in [10]) than traffic with exponential interarrival times. An equivalent statement is that it is easier to detect an information flow under such conditions. The intuition is that such traffic is "burstier" than Poisson traffic, and so under the independent hypothesis fewer packets would be falsely labeled as flow packets.

Because the distribution of the slot quantized values under Pareto interarrival times is not known analytically, we resort to simulation to test the performance of our detection scheme under this hypothesis. In addition, we will not be able to accurately determine the entropy of the quantized process, and so will need an upper bound.

The distribution of the slot-quantized sequences will have mean $\lambda T$, where $\lambda$ is the average rate of the original process and $T$ is the slot length, and will be over all nonnegative integers. It can be shown that of all discrete probability distributions with mean $\lambda T$ and nonnegative integer support, the one with the maximum entropy is the geometric distribution. The entropy rate under this distribution is $R = (1/(1 + \lambda T)) \log(1 + \lambda T) - \lambda T \log(\lambda T)$. We will use this formula to compute the desired entropy rate of the sequence, knowing that the actual entropy rate of the sequence is smaller than the maximum allowed rate.

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Fig. 6. An achievable consistency-rate function for detecting flows of various lengths using slot quantization. In this case, (a) $\lambda = 0.25$ and (b) $\lambda = 1$. $\Delta = 1$, and $10^4$ slots are used for each experiment to ensure convergence.

Fig. 7. An upper bound on the consistency-rate function of a scheme using slot quantization compared to the achievable consistency-rate function found by simulation. In this case, $\lambda = 0.25$, $\Delta = 1$, and $N = 4$.  

\[ F_X(r) = 1 - \left( \frac{r}{x_m} \right)^\beta \]  

where $x_m$ is the scale parameter and $\beta$ is the shape parameter.
Pareto $\beta = 1.3$

Pareto $\beta = 1.5$

Exponential

Fig. 8. Minimum CTR under independent traffic computed by QBDR after slot quantization under Pareto and exponential interarrival distributions. An upper bound on the rate is used for the Pareto distributions, so the estimated CTR is actually a lower bound for a particular rate. Pareto distributions have shape parameters $\beta = 1.5$ and $\beta = 2$. The results are closer to the Poisson case for larger shape parameters. In this experiment, $N = 4$, $\Delta = 1$, $\lambda = 0.5$ (where $\lambda$ is the mean arrival rate), $10^4$ slots are used to ensure convergence under Poisson interarrivals, and five trials with $2 \times 10^5$ slots are used in the Pareto interarrivals case.

A. Consistency Experiment

This experiment was similar to the one described in Section VII. We simulated two independent renewal sources over a long period of time with interarrivals obeying Pareto distributions, performed slot quantization with slot length determined by the geometric entropy rate, and ran the QBDR algorithm to find the minimum fraction of chaff. Because the interarrival process is not memoryless, we cannot assume that the slot quantized sequences are i.i.d. or even Markov, so we cannot analytically show convergence of QBDR. Instead, we ran the experiment several times with the same parameters and found that the results do appear to converge to a limiting CTR. We then compared these results to the results under exponential interarrivals (the Poisson process). The results are shown in Fig. 8.

From the figure, it is clear that at a given arrival rate, the Pareto interarrival distribution gives a higher CTR under $\mathcal{H}_0$ than the Poisson process, with the CTR getting smaller as the shape parameter $\beta$ increases. This means that a detector designed under the Poisson assumption would also be Chernoff consistent if the arrival rate is the same but the interarrivals are Pareto distributed. This fits the intuition of a process with Pareto interarrivals being more “bursty” than those with Poisson interarrivals. Since, in the absence of an information flow, the interarrivals being more “bursty” than those with Poisson interarrivals, the estimated CTR is higher under the Pareto interarrivals than the Poisson.

B. ROC Experiment

In this experiment, we compared the receiver operating characteristics of the QBDR threshold detector under a Poisson process source with those under a Pareto interarrival source. For each threshold value between 0 and 1, the false alarm rate was found by generating $N$ independent processes with mean arrival rate $\lambda$ over 45 seconds (much smaller than the processes used in the consistency experiments), quantizing them with a slot quantizer of slot length $T$ (the same length was used for both kinds of processes), and running the QBDR algorithm. If the estimated CTR was below the threshold, the trial was counted as a false alarm. 500 trials were used for each threshold value. To find the probability of detection of flows with a particular CTR $\gamma$, an information flow in chaff had to be generated. A process with mean arrival rate $(1 - \gamma)\lambda$ was generated, and each packet was delayed by a uniform random value between 0 and $\Delta = 1$ to generate $N$ processes. Then, each process was contaminated by superposing independent noise processes of rate $\gamma\lambda$ (so that the total rate was $\lambda$) QBDR was run, and a trial was ruled a detection if the estimated CTR was below the threshold. Again, two hundred trials were used. The shape parameter for the Pareto distribution was 1.2. Fig. 9 shows the results. In all tests, the ROC curves under the Pareto interarrival distribution were slightly better than those under the Poisson process distribution. Since, under $\mathcal{H}_0$, the estimated CTR is higher with Pareto interarrivals than Poisson, the false alarm rate is lower at a given threshold. The detection rate, on the other hand, will be lower under Pareto interarrivals because less of the chaff packets will be incorrectly considered as parts of a flow. However, it is clear from the graphs that the improvement in false alarm probability outweighs the loss.
in detection rate at a given threshold. It is also clear that the detector performance improves with decreasing CTR (which is to be expected) and with decreasing packet arrival rate.

IX. PERFORMANCE ON NETWORK TRACES

The final step from models to reality involved considering actual network data. We applied our quantization scheme followed by the QBDR algorithm on the LBL-PKT-4 trace generated by Paxson and first used in [10]. This trace is a record of packet timing and routing information for all wide-area traffic in and out of the Lawrence Berkeley Laboratory. It was also used in [8] for an experiment similar to ours. We extracted 41 traces that appeared upon inspection to be independent. For each trial, we randomly picked a pair of traces. We applied the slot quantizer to the pair and performed QBDR to find the estimated minimum CTR. Then we measured the average packet rate for the two traces and generated realizations of Poisson processes of those rates. We applied the slot quantizer to the Poisson pair and perform QBDR to find the estimated minimum CTR. The results are shown in Fig. 10. Each point corresponds to a single trial; the Y-coordinate is the CTR of the traces, and the X-coordinate is the CTR of the corresponding Poisson process pair. In the experiment shown, 93% of the traces have a higher CTR than the Poisson processes of the same rate. This indicates that the false alarm probabilities will be lower for actual network traffic than our Poisson model, which gives us increased confidence in the performance of a system designed under the Poisson assumption.

X. CONCLUSION

In this paper, we developed a practical distributed detection scheme for arbitrary-length information flows. As in the corresponding problem with perfect information, under certain conditions the performance of the scheme is near perfect for large $N$. Simulations show that if these conditions are not met, it is possible for flows embedded in chaff to remain undetected even for a large number of hops. Thus, given a rate constraint, traffic rate, and delay bound, it is possible to determine whether perfect detection of large information flows is guaranteed or unlikely. Preliminary results indicate that even if the detection scheme is designed under the assumption of exponential interarrival times, actual network traffic will perform at least as well as predicted under the faulty interarrival assumption.

REFERENCES

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