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Planning the Reconfiguration of Grounded Truss Structures with Truss Climbing Robots that Carry Truss Elements

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Abstract — In this paper we describe an optimal reconfiguration planning algorithm that morphs a grounded truss structure of known geometry into a new geometry. The plan consists of a sequence of paths to move truss elements to their new locations that generate the new truss geometry. The trusses are grounded and remain connected at all time. Intuitively, the algorithm grows gradually the new truss structure from the old one. The truss elements are rigid bars joined with 18-way connectors. The paper also introduces the design of a truss-climbing robot that can execute the plan.

I. INTRODUCTION

Our long-term goal is to apply a reconfiguration paradigm to construction via self-assembly. We wish to create self-assembling robot systems consisting of passive structural modules, possibly manufactured on-demand and/or composed from elements present in the environment, combined with active robotic modules. The structural passive elements are rigid passive bars and general connectors capable of supporting multiple bars. The active elements are robotic modules that may travel on the structural components, pick up or disassemble a passive element from a known location in the structure, carry the element to a desired location on the structure and connect the passive element at the destination. For the case when the passive elements are rigid bars, the structures that can be created with this paradigm are self-assembling and self-reconfiguring trusses. If the robot elements are an integral part of the truss, the truss is a dynamic and controllable structure. The resulting structures form a large class of truss and linkage geometries. We introduced some of our initial ideas for designing robots and passive parts, as well as self-assembly and control algorithms for these types of trusses in [1], [2], [3].

In this paper we describe an algorithm for reconfiguring truss structures. We assume that the truss structure consists of passive elements. Truss-climbing robots are capable of (1) disconnecting a truss element, (2) carrying it along a path on the truss, and (3) re-attaching it to new locations on the truss. We develop an algorithm that takes as input a grounded target truss structure and a grounded goal truss structure. The algorithm computes an optimal set of truss element moves that reconfigure the target truss into the goal truss while ensuring that all truss elements stay connected at all times. The algorithm considers trusses that consist of rigid bars of a fixed number of lengths and connectors of one type. Figure 1 shows an example for changing the geometry of a truss structure.

The main contribution of this paper is algorithmic. We also propose a system design for truss elements (connectors and edges of two types), and robots capable of executing the reconfiguration algorithms proposed. This truss robot system is partially completed and will be the subject of a different paper.
A. Inspiration

In developing our problem formulation and solution, we were inspired by biological metabolism. An organism breaks down food into constituent building blocks (catabolism) and then uses those building blocks for its own growth (anabolism). This process has several properties of interest to our problem: extensive reuse and recycling of modular building blocks, autonomous deconstruction and reconstruction, autonomous design given a set of functional requirements, resilience to raw material variation, and self-repair. These properties have not generally been replicated in synthetic structures. Duplicating such properties in a robotic ecology could yield a number of practical benefits, from more robust manufacturing processes to improved recycling ability to space exploration.

The general replication of these properties is a distant computational goal. Many challenges exist. How many and what kind of building blocks could be used for effective metabolic processes? (Doyle et al. [4] suggest that a relatively small set of building blocks can yield a large number of source and target structures in a robust manner.) How can functional definitions be transformed into physical designs in an autonomous way? What kind of manipulation is required? How is the transformation process to be accomplished?

If one simplifies the problem to solely deal with single-diagonal cubic truss structures as shown in Figure 1, the problem becomes more tractable, but significant questions still remain.

B. Related Work

Many robots have been developed as climbers and/or manipulators of various structures, including general truss structures. An inchworm robot, using electromagnetic force to attach itself to ferrous surfaces, was developed in [5], but structural manipulation abilities are not included. Ripin et al. [6] and Tavakoli et al. [7] developed pole-climbing robots, and the latter has some capacity for manipulation. Amano, Osuka, and Tarn [8] developed a robot for climbing high-rise buildings. None of these three robot is capable of general truss traversal, however. TREPA [9], a parallel robot; ROMA [10], a caterpillar-like robot; and Shady3D[11], a modular robot utilizing a passive member, are capable of traversing a wide variety of structures, but do not have the ability to effect structural assembly. Nechya and Xu [12] developed a truss-walking inspection robot, SM2, for space station trusses. Skyworker was developed for orbital assembly tasks [13], and was demonstrated performing truss-like assembly tasks [14]. However, the truss structure was not specifically designed for robotic manipulation and required an independent vision robot to perform the assembly. Our proposed robot design (discussed briefly in section II and in more detail in [15]) is fashioned for traversal of general cubic trusses with face-centered diagonals and for physical manipulation and reconfiguration of such trusses. It utilizes a custom truss design discussed briefly in Figure 3 and in detail in [16].

C. Outline

This paper is organized as follows. Section II introduces the design for a truss climbing robot and for the truss elements (edges and nodes) it operates with. Section III-B describes and analyzes an optimal reconfiguration planning solution that is centralized and efficient but does not maintain truss connectivity. Section III-C describes and analyzes a reconfiguration planning solution that is guaranteed to maintain connectivity and compute the correct plan. Section IV presents simulation results.

II. DESIGNING A ROBOT TRUSS CARRIER

The robot design we have chosen for this work is one we refer to as a hinge robot, as shown in Figure 2. Two halves of the robot are connected together via a hinge. The angle between the halves can be adjusted to any angle between 35° and 180°. This is done with a pair of linear actuators and a rigid connecting link. Moving one or both of the linear actuators changes the angle between the halves. Each half, then, is capable of gripping a truss element, rotating itself and the entire robot around that truss element, and translating along that truss element using internal wheeled grippers.

If a single half of the robot is currently gripping a truss element, the robot can then move about a truss by translating, rotating, and adjusting the angle between halves in order to align the free half with another truss element adjacent to it. (This element may be at an angle of 45°, 90°, 135°, or 180° from the originating element.) The robot then grips the destination element, releases the originating element, and the process begins again.

If a strut needs to be carried from one location to another on the truss, a carrying pod (shown on the side of each half of the robot in Figure 2) can be actuated to retrieve a strut from the structure, hold it, and return it as needed.

The robot is also capable of manipulating the truss, given amenable truss element designs. The truss elements in Figure
Fig. 3. Truss elements designed for robotic manipulation. The hubs contain threaded holes in each principle axis as well as in the diagonal axes. The struts contain threaded studs on either end, and also have a threaded stud and mating insert in the middle. This allows the sides to be inserted or removed from the hub independently via rotation without requiring the movement of the hub.

are designed for ease of robotic manipulation via a twisting action in complex truss structures [16]. Since the robot is able to rotate itself about a truss element, it is also able to remove struts via rotation. Thus, the robot is capable of completely manipulating such truss structures.

III. PLANNING THE OPTIMAL RECONFIGURATION OF TRUSSES

In this section we formulate the specific truss reconfiguration problem we solve in this work and describe an efficient algorithm.

The truss structures we consider are general compositions and arrangements of rigid bars of different lengths connected at truss nodes using the connector design described in Section II. Our truss uses a meta-module with the geometry of a cube. Thus, without loss of generality, we focus on two types of truss elements: (1) sides, which are short bars for the cube sides, and (2) diagonals, bars that make up the diagonals in the basic cube meta-structure. All trusses are grounded.

Each truss structure is represented as a weighted graph $G = (V, E)$. Vertices $V$ correspond to the nodes of the truss. Edges $E$ show the connectivity of the truss nodes by truss elements. The weight of each edge indicates the type of edge (e.g. side or diagonal for the proposed design.)

Planning for the reconfiguration of the truss represented by graph $G_1$ into the truss represented by graph $G_2$ can be formulated as optimal matching between $G_1$ and $G_2$. We wish to keep the truss connected at all times as well as to guarantee the globally optimal matching solution. The intuition is as follows. First, we compare $G_1$ and $G_2$ to identify their overlap. The overlap corresponds to truss elements that do not have to move in the process of reconfiguring one object into another. Next, truss elements in $G_1$ that are not part of this overlap are assigned new locations to assemble $G_2$. This involves computing a truss trajectory for moving each element to the new location (by robots that can carry truss elements), and the order in which the moves have to be done. We wish to minimize the total number of steps required to complete the truss reassembly task.

A. Problem Formulation and Assumptions

The goals for the truss reassembly planning algorithm are as follows:

- find the graph $G_{in} = G_1 \cap G_2$ that yields the optimal matching between $G_1 - G_2$ and $G_2 - G_1$
- compute the trajectories for moving the edges in $G_1 - G_2$ to assemble $G_2$ subject to the constraint of maintaining connectivity of the entire structure

The cost function is the total traveling distance of the truss elements (e.g. for the edges in $G_1 - G_2$). The cost function can be extended with other criteria such as maintaining the integrity of the structure in the presence of gravity, etc.

The goals for the general reassembly planning problem of arbitrary trusses are challenging. The maximal common subgraph (MCS) isomorphism algorithm is NP-hard [17] in the general case. Though the bipartite matching in a graph can be solved, executing the matching may disconnect the graphs in our case because the matching physically moves a truss that includes an edge and a couple of nodes.

In order to find an efficient solution for truss reassembly planning, we make the following assumptions:

- $G_1$ and $G_2$ are restricted to two types of truss elements: sides and diagonals. Each edge is labeled by its type (side or diagonal). This assumption is relaxed straightforwardly to trusses with a finite number of types for their truss elements.
- The orientation of the side truss elements is orthogonal along one of the $x$, $y$, or $z$ axes.
- A truss node (vertex) has the ability to hold and store multiple truss elements.
- Trusses are grounded.

The first, second, and fourth assumptions restrict the geometric structure of the truss. This class of trusses admits polynomial-time algorithms for reconfiguration planning. The third assumption is needed to ensure that during the process of reconfiguration the structure remains connected. The prototype connector (see Figure 3) was designed with this goal in mind. Thus, a robot can connect a truss element to a node temporarily.

We do not impose any restriction on the size of $G_1$ and $G_2$. Specifically the size of the initial truss does not have to be identical to the size of $G_2$.

B. Finding the optimal matching by scanning

The truss elements have fixed lengths and orientations. Therefore, we may consider the truss structure as a set of the square cubes whose edges are the side truss elements and whose diagonals are diagonal truss elements. The optimal matching can be obtained by scanning $G_2$ over the $G_1$ as described in Algorithm 1.

Figure 4 illustrates Algorithm 1 in the context of a 2D example for ease of explanation. Algorithm 1 works with
Algorithm 1 Scanning algorithm for the optimal matching.
The algorithm returns the optimally merged graph $G_m$ and
the optimal matching $M_m$ in $G_m$.

1: Make $G_1$ and $G_2$ cornered at the origin
2: for orientation $O_{G_2} \in [0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}]$ do
3: Rotate $G_2$ by $O_{G_2}$ w.r.t z-axis
4: $X_1 = \max\{x(G_1)\}$, $Y_1 = \max\{y(G_1)\}$
5: $X_2 = \max\{x(G_2)\}$, $Y_2 = \max\{y(G_2)\}$
6: for $x_t \in [-X_2 \ldots X_1 + X_2], y_t \in [-Y_2 \ldots Y_1 + Y_2]$ do
7: Transform $G_2$ by $(x_t, y_t)$
8: $n =$ number of the overlapped elements in $G_1 \cap G_2$
9: $G_3 = G_1 \cup G_2$
10: $M =$ OptimalMatching$_{side}$ ( $G_1 - G_2, G_2 - G_1$ )
     + OptimalMatching$_{diagonal}$ ( $G_1 - G_2, G_2 - G_1$ )
11: end for
12: end for
13: $M_m =$ argmin$_M (\text{cost}(M))$
14: $G_m =$ argmin$_{G_3} (\text{cost}(M))$

The running time of Algorithm 1 is $O(nm^4)$, where $n$ is number of the nodes and $m$ is number of the truss elements.

**Proof:** The computation required for comparing the graphs is $O(m)$. The number of the scans is bounded by the $xy$-region of the graphs which can not be more than $n_1 + n_2$. The optimal matching is computed by the Hungarian algorithm [18], which has $O(m^3)$ runtime. Note that computing the cost matrix for the Hungarian algorithm requires the execution of Dijkstra’s algorithm $O(m)$ times; however, the running time of the Hungarian algorithm dominates.

C. Maintaining Connectivity
Algorithm 1 yields the optimal matching for transforming $G_1$ into $G_2$ but does not provide a correct sequence of moves to ensure that the structure (e.g. all the truss elements) will stay connected at all time. Figure 6 illustrates some snapshots from performing the reconfiguration from $G_1$ to $G_2$ for the example of Figure 4. Note that one of the black truss elements has to be moved to the uppermost edge. However, this location is not reachable before the other truss elements move to their matched locations. Additionally, the next black truss loses its shortest path because the first black truss is gone.

To maintain connectivity and the shortest paths, an additional computational step is needed. The order of the obtained optimal matching needs to be analyzed and processed so that all truss elements stay connected at all times. If a target location is not reachable by any means, that truss element is moved along its trajectory to the farthest available intermediate truss vertex and temporarily stored there. That is, the paths for the truss elements are divided so that the elements may move to an intermediate point along the path until full connectivity to the target location is available. Practically, elements can be buffered by temporarily connecting them to a joint with a free connector.

Algorithm 2 describes the analysis and computation required in order to generate trajectories with intermediate storage locations that connectivity at all time for all truss elements. Let $S$ be the set of the source truss elements of $G_1 - G_2$ and $T$ be the set of the target edges for $G_2 - G_1$. Initially $T$ is empty.

Algorithm 2 uses a dynamic graph $G$. $G$ has all the edges of $G_1 \cup G_2$, however only the edges of $G_1$ are activated at the
start. As the computation proceeds, the set of active edges changes. The algorithm ends if all the \( s_i \in S \) reach their target locations \( t_i \). The algorithm chooses a truss element \( s_i \) whose target is connected to the current structure \( S - T \). If the element does not belong to any paths of the other truss elements, it advances to its target. Otherwise, the algorithm picks another element the path of which includes the current element. If the two elements are the same trusses (diagonal or side), they exchange their target and adjust the paths according to the new targets. The exchange is reasonable since the two trusses are physically same and it does not hurt the optimality. If there is no same element of the same type among the path-overlapping elements, the algorithm searches for the deepest predecessor of the element and let it advance to its target. After the exchange, we repeat the process until \( S = T \).

Algorithm 2 guarantees no queue when the structures are made of only a single type of the trusses, since it can always fill the picked edge with the deepest predecessor as long as no cycle in the paths. The queue is necessary only if the only predecessor has the different type. The concept of exchanging is also useful when the work is extended to a distributed system where many robots collaborate [1], [3].

**Theorem 2:** An edge \( s_i \) that is not in \( p_j (j \neq i) \) can always be founded.

**Proof:** Suppose every \( s_i \) is in \( p_j (j \neq i) \). Then at least a pair of paths \( p_i \) crosses each other, which means there is a loop in \( P \). This is a contradiction because it means we can have a better matching by exchanging \( t_i \) with the intersection of the paths \( p_i \) or by cutting the loop.

**Theorem 3:** Algorithm 2 terminates.

**Proof:** By Lemma 2, the algorithm adds a part of \( P \) to the trajectory in every loop. Since \( P \) has a finite number of the paths, it will be completely traversed by \( S \).

Figure 7 shows the adjusted paths of Figure 6. The black edges move to the blue edge locations without breaking connectivity. In this example, no source truss edge has been paused and added to the queue of elements with unreachable destination.

### IV. Results

We have implemented the algorithms described in Section III and we evaluated them on six 3D canonical structures. In this section we describe the results.

**A. Solution examples**

Figure 8 shows an example set of truss structures[16]. The left structure consists of 6 cubes connected as a compact structure. The others are like a tower. \( G_1 \) to \( G_5 \) has a total of 83 truss elements, 31 of which are diagonal and the rest side. \( G_6 \) has only 75 truss elements. Figure 9 and 11 show how the planning algorithms in this paper morph \( G_1 \) into \( G_3 \) and \( G_6 \) for this example. Figure 10 shows transformation \( G_1 \) to \( G_5 \) with a programmatically generated animation. Between individual reconfiguration steps, the robot uses Dijkstra's algorithm to plan the shortest path to the start point of its next reconfiguration step. The exchange algorithm is used to generate the paths. Fortunately, the structure does not require a queue. Note that \( G_1 \) can transform to \( G_6 \) in the optimal way even though the numbers of the trusses are different.

**B. Performance analysis**

Table I summarizes the performance of the algorithm for all combinations of the given truss geometries. The source structure is successfully transformed into the target structure as minimizing the total displacements of the truss elements.

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**Algorithm 2 Exchange algorithm for trajectories to maintain the connectivity and the shortest paths**

1: \( S = \text{truss}(G_1 - G_2) \)
2: \( T = \text{edge}(G_2 - G_1) \)
3: \( P = \text{path}(S \rightarrow T) \) in \( G_m \)
4: deactivate \( T \) in \( G_m \)
5: \( Q = \emptyset \)
6: while \( S \neq T \) do
7: \( \text{pick } s_i \text{ such that } t_i \in T - S \) is connected to \( S - T \) and \( p_i \in S - T \)
8: \( \text{trussSelected} = \text{false} \)
9: while not trussSelected do
10: \( \text{if } s_i \notin p_j (j \neq i, j \in S - T) \) or \( s_i \in Q \) then
11: \( \text{move } s_i \text{ along } p_i \)
12: \( \text{delete } p_i \)
13: \( \text{activate the edges that are connected to } t_i \)
14: \( \text{pull out } s_i \text{ from } Q \) (if \( s_i \in Q \))
15: \( \text{trussSelected = true} \)
16: else
17: \( \text{choose } s_j \text{ such that } s_i \in p_j \)
18: \( \text{if } \exists s_j \text{ such that } s_i \in p_j \text{ and type}(s_i)=\text{type}(s_j) \) then
19: \( \text{exchange } t_i \) and \( t_j \)
20: \( p_i \leftarrow p_j(s_i \rightarrow t_j) \)
21: \( p_j \leftarrow p_j(s_j \rightarrow s_i) + p_i(s_i \rightarrow t_i) \)
22: \( i \leftarrow j \)
23: else
24: \( \text{pick } s_k, \text{ the deepest predecessor of } s_j \)
25: \( \text{move } s_k \text{ along } p_k \)
26: \( \text{if } s_k \neq t_k \) then
27: \( s_k \rightarrow Q \)
28: end if
29: \( \text{trussSelected = true} \)
30: end if
31: end if
32: end while
33: end while
We do not need any queue for the given structures. In the future, the required conditions for no queue will be considered.

On the other hand, the cost can be considered as time required for a robot to finish the transformations. In future, an algorithm for multi-robot processing may reduce the times by parallelism.

V. CONCLUSIONS

This paper presented and demonstrated an algorithm that solves the problem of how to optimally transform a given truss structure into another structure, piece by piece, subject to maintaining a variety of constraints. The algorithm is realizable using a future robot capable of traversing a truss and executing element removal and insertion operations at desired locations. We have also presented initial designs for such a robot, as well as physical elements that can be removed and inserted at any valid truss location.

Because of the regular nature of the truss lattice, the algorithms presented remain tractable in the size of the source and target graphs. Similarly, capabilities of the robot, such as the type of joints it can traverse and the number...
elements it can carry in one pass greatly affect the type and performance of the algorithm that can be achieved. Various aspects of the truss elements design also impact the algorithm performance, such as the ability to remove and insert elements at random access, and structural the ability of cantilevered elements to sustain the load of a working robot.

The coupled algorithmic and hardware centered nature of this problem open the door to many future challenges and opportunities. In particular, we are interested in performance of this system when multiple robots operate on the structure simultaneously, and when knowledge about the current global state of the structure, resource availability and inter-robot communications are imperfect. In addition, practical considerations related to structural integrity, stress, and vibration, as well as assembly failures and non-regular lattices are also of interest. An additional interesting challenge is the inverse problem of designing useful target structures that a given structure can be easily transformed into [16].

VI. ACKNOWLEDGEMENTS

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