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Nonequilibrium Electromagnetic Fluctuations: Heat Transfer and Interactions

Matthias Krüger,1 Thorsten Emig,2 and Mehran Kardar1

1Massachusetts Institute of Technology, Department of Physics, Cambridge, Massachusetts 02139, USA
2Laboratoire de Physique Théorique et Modèles Statistiques, CNRS UMR 8626, Bâtiment 100, Université Paris-Sud, 91405 Orsay cedex, France

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The Casimir force between arbitrary objects in equilibrium is related to scattering from individual bodies. We extend this approach to heat transfer and Casimir forces in nonequilibrium cases where each body, and the environment, is at a different temperature. The formalism tracks the radiation from each body and its scatterings by the other objects. We discuss the radiation from a cylinder, emphasizing its polarized nature, and obtain the heat transfer between a sphere and a plate, demonstrating the validity of proximity transfer approximation at close separations and arbitrary temperatures.

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where \( C_0 = \alpha_0(\omega) \frac{c^2}{\omega^2} \text{Im} \mathcal{G} \) is the zero point term. The finite temperature contribution is thus a sum of \( N + 1 \) terms:

Each \( C_\alpha^q(T) \) contains an implicit integral over sources within \( \alpha \) and is identified with the field sourced by this object [1]; the scattering of this radiation by all other objects is accounted for by multiplying \( \text{Im} e_\alpha \) on both sides with the full Green’s function. The last term in Eq. (2), \( C^\text{env}(T) = -\alpha_T(\omega) \frac{c^2}{\omega^2} \text{Im} \mathcal{G}_0^{-1} \mathcal{G}^* \), is hence identified with the contribution sourced by the environment.

A key assumption of FE is that in a nonequilibrium situation, the thermal current fluctuations inside each object are described by the FDT at the corresponding local temperature, and are independent of the impinging radiation from the other objects. Having identified the different sources in Eq. (2), we can change their temperatures to arrive at the desired nonequilibrium generalization

\[
C^\text{eqi}(T, \{ T_\alpha \}) = C_0 + \sum_\alpha C_\alpha^q (T_\alpha) + C^\text{env}(T, \{ T_\alpha \})
\]

\[
= C^q(T_\text{env}) + \sum_\alpha \left[ C_\alpha^q (T_\alpha) - C_\alpha^q (T_\text{env}) \right].
\]

The second form is obtained by considering the difference of \( C^\text{eqi}(T, \{ T_\alpha \}) \) from \( C^q(T_\text{env}) \) due to the deviations of the object temperatures \( T_\alpha \) from \( T_\text{env} \). This form is useful because the equilibrium correlation can be regarded as known, and the number of sources is reduced from \( N + 1 \) to \( N \). Applying the formalism, e.g., to derive Casimir forces, the first term on the right-hand side of Eq. (3) yields the equilibrium force at temperature \( T_\text{env} \).

The next step is to compute the radiation field of object \( \alpha \) when isolated, i.e., before this field is scattered by the other objects, and with \( T_\text{env} = 0 \). This is given by \( C_\alpha^q (T_\alpha) = \alpha_T(\omega) \mathcal{G}_\alpha \text{Im} e_\alpha \mathcal{G}^*_\alpha \) where \( \mathcal{G}_\alpha \) is the Green’s function of object \( \alpha \) in isolation, and thus involves an implicit integration over the interior of object \( \alpha \). To employ multiple scattering techniques [18], it is considerably more convenient to express \( C_\alpha^q (T_\alpha) \) in terms of the \( T \) operator or scattering amplitude \( \mathbb{T}_\alpha \) of the object. In equilibrium, the electric field correlator for the isolated object \( C^\text{eqi}(T_\alpha) = \alpha_T(\omega) \frac{c^2}{\omega^2} \text{Im} \mathcal{G}_\alpha \), contains radiation sourced (i) by the environment and (ii) by the object itself. The latter can be obtained by subtracting the contribution from the environment, which can be regarded as an additional material with \( e_{\text{env}} \to 1 \), occupying the space complimentary to \( \alpha \) [17]. Towards this calculation, we introduce a Green’s function \( \mathcal{G}_\alpha \) with \( e_\alpha \) inside object \( \alpha \) and \( e_{\text{env}} \) outside.
\[ \frac{|H_r|}{L} = - \int_0^\infty \frac{d\omega}{(2\pi)^2} \omega T(\omega) e^{-d \omega/c} P_{E,M} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{a_T(\omega)}{2\pi^2 \omega^2} J_0(k_0 R) \times \int_{-\omega/c}^{\omega/c} dk_j |(\text{Re}[T_{n,kj}^{PP}]) + |T_{n,kj}^{PP}|^2 + |T_{n,kj}^{PP}|^2|, \]

where \( \dot{P} = M \), if \( P = E \) and vice versa. In Fig. 1, we compare the heat radiation of a plate (semi-infinite body), a sphere, and a cylinder (both of radius \( R \)), all evaluated with optical data of silicon dioxide (SiO\(_2\)), as used in experiments [6]. The radiation is normalized to the Stefan-Boltzmann law \( H = \sigma T^4 A \), with \( \sigma = \pi^2 k_B^4/(60h^3 c^5) \) and surface area \( A \) of the object.

For thin cylinders and small spheres \( H \) is proportional to the volume, while in the opposite limit, it is proportional to the surface area, reflecting a finite skin depth (absorption length) \( \delta(\omega) = c/(\text{Im} \sqrt{\varepsilon(\omega)}) \). Thermal fluctuations at frequency \( \omega \) within the object emit radiation which may be reabsorbed on its way out. If \( \delta \ll R \), only thermal fluctuations near the surface lead to emerging radiation, while for \( \delta \gg R \), the entire volume contributes to \( H \). An interesting feature of Fig. 1 is the intermediate range, where the sphere and cylinder emit more strongly than a plate of equal area, related to Mie resonances for the sphere [11]. For \( R \to \infty \), i.e., when the wavelengths involved (roughly peaked around the thermal wavelength \( \lambda_T = hc/(k_B T) = 7.6 \mu m \), as well as skin depths, are much smaller than the smallest dimension of the object, the classical (plate) result is approached. The asymptotic value is in these units denoted as emissivity \( \varepsilon(T) < 1 \). Interestingly, the radiation from a cylinder is polarized, its parallel and perpendicular polarizations obtained from the \( P = E \) and \( P = M \) terms in Eq. (7), respectively. The predominant radiation of a thin cylinder is parallel and changes to perpendicular for \( R = \lambda_T \). Both polarizations become equal asymptotically as \( R \to \infty \). Polarization effects have indeed been observed experimentally for wires [12,20] and carbon nanotubes [21], for which other descriptions have been offered [22].

Now we consider multiple objects. To compute \( C_{\text{eq}} \) involving spheres or cylinders, we also need to convert among bases appropriate to the different objects. For example, for the experimentally most relevant configuration of a sphere and a plate [6], the radiation from the plate, given in a plane wave basis, must be transformed to the spherical basis [18], reflected by the sphere, transformed back and so on. For simplicity, we focus on a plate held at a finite temperature \( T_p \neq 0 \), while the sphere and environment are at zero temperature \( T_s = T_{\text{env}} = 0 \). This suffices to describe also situations with \( T_s \neq 0 \) as the transfer vanishes for \( T_p = T_s \), and hence for \( T_s \neq 0 \) we subtract our result evaluated at \( T_s \). We express the correlations in Eq. (3) in a plane waves basis, and \( H_s \), the energy absorbed by the sphere, is obtained by integrating \( S \) in Eq. (6) over two infinite parallel planes enclosing the sphere and separating it from the plate.

Figure 2 shows the results for the heat transfer from a SiO\(_2\) plate at room temperature to a SiO\(_2\) sphere of \( R = 5 \mu m \) at zero temperature, with surface-to-surface separation \( d \), normalized by the Stefan-Boltzmann law \( H_s = \sigma T^2 \pi R^2 \) (only half of the sphere is exposed to the plate). For large \( d \), \( H_s \) is roughly 0.5 in these units, whereas for \( d \to 0 \), \( H_s \) diverges due to the increased tunneling of evanescent waves, eventually exceeding the Stefan-Boltzmann value. The figure shows the numerical solution of Eq. (3) together with a one reflection approximation, where we set \( \Theta_{\alpha,\beta} = (1 - \Theta_0 T^\beta) \) in Eq. (5), neglecting higher order reflections between sphere and plate. We see that the two curves approach each other for large \( d \), as most rays are scattered outward and will not hit the sphere a second time. The reflection expansion is hence helpful for getting analytical results for \( d \gg R \). Our numerical

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**FIG. 1 (color online).** Heat radiation of a cylinder and a sphere of SiO\(_2\) as a function of \( R \), normalized by the Stefan-Boltzmann result, at \( T = 300 \) K. The horizontal line shows the radiation of a SiO\(_2\) plate. \( \lambda_T \) and the smallest skin depth \( \delta \) in the relevant frequency range are marked on the \( R \) axis. For the cylinder, the contributions of the different polarizations are shown.

**FIG. 2 (color online).** Heat transfer rate (in units of Stefan-Boltzmann’s law) from a room temperature plate to a sphere at \( T = 0 \) of radius \( R = 5 \mu m \) (both SiO\(_2\)), as a function of separation. The horizontal bar shows the limit of \( d \to \infty \). The left inset shows the approach to PTA for the divergent terms in a one reflection approximation. The right inset shows the result at large separation as a function of \( R \).
solution involves an expansion in spherical multipoles: For \( R/\lambda_T \) large or \( d/R \) small, increasingly more multipoles are needed. In practice, we restrict to a maximal multipole order of \( l_{\text{max}} = 20 \), for accurate results up to \( d \approx R/2 \). Since closer separations are also interesting and relevant experimentally, but difficult numerically, we demonstrate in the inset of Fig. 2 the approach to a proximity transfer approximation (PTA), equivalent to the proximity force approximation (PFA) used in Casimir physics,

\[
\lim_{d/R \to 0} H_s(d) = 2\pi R \int_0^{d/R} S^{pp}(s) ds,
\]

where \( S^{pp}(d) \) is the Poynting vector for parallel plates at separation \( d \). We identify the divergent terms as \( d \to 0 \) for both the sphere-plate and plate-plate configurations (the \( E \) modes originating from evanescent waves), and evaluate their ratio in the one reflection approximation (allowing us to use \( l_{\text{max}} = 200 \)). As demonstrated in Fig. 2, this ratio approaches unity for \( d \to 0 \), suggesting \( \Upsilon \) of the sphere approaches \( \Upsilon \) of the plate in the PTA sense. From this, we anticipate that multiple applications of these matrices (leading to the full solution) will also approach the ratio unity (independent of the accuracy of the one reflection approximation as \( d \to 0 \)). We investigated different \( R \) and confirmed that PTA in Eq. (8) is valid in general, with \( H_s(d) \propto d^{-1} \) as \( d \to 0 \). While a similar point is discussed in Ref. [9] for the case of two spheres and used in experimental studies [6,7], to our knowledge the validity of PTA in Ref. [9] for the case of two spheres and used in experiments as discussed before. In this limit, \( H_s \) is given by (with magnetic permeability of the sphere \( \mu_s \) and Fresnel reflection coefficients \( r^F \) and \( r^M \) of the plate for angle \( \theta \)),

\[
\lim_{d \to 0} H_s = \frac{cR^3}{16\pi} \int_0^\infty d\omega \alpha_T(\omega) \Im\left(\frac{\mu_s - 1}{\mu_s + 2} + \frac{\varepsilon_s - 1}{\varepsilon_s + 2}\right) \int_0^{\pi/2} d\theta \sin\theta \sum_{P=E,M} (1 - |r^P(\theta, \omega)|^2). \tag{9}
\]

For \( R \gg \lambda_T \), we may expect the result to approach a classical limit, given by \( \sigma T^4 e^2 \pi R^2 \), with \( \varepsilon \) from Fig. 1. While the data points come close to this value, one does not expect exact approach \[3\], in contrast to Fig. 1, because the Fresnel coefficients depend on the angle of incidence. If we additionally let \( (\varepsilon_p, \varepsilon_s) \to 1, H_s \) will approach the classical limit since the Stefan-Boltzmann law applies to all convex black bodies.

While we highlighted applications to simple shapes, the formalism presented here is more general, and combined with a numerical scheme for the computation of scattering matrices \[23\] can deal with collections of objects at different temperatures. Indeed, such a formalism is needed to properly deal with near-field effects in device and fabrication at the micron scale. The formalism yields also Casimir forces between objects at different temperatures—examples of which we leave for future work. We note, however, that in the final stages of this project we became aware of two independent, partly related, studies of nonequilibrium effects \[24,25\].

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