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Magnetic impurity in a $U(1)$ spin liquid with a spinon Fermi surface

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We address the problem of a magnetic impurity in a two-dimensional $U(1)$ spin liquid where the spinons have gapless excitations near the Fermi surface and are coupled to an emergent gapless gauge field. Using a large $N$ expansion, we analyze the strong coupling behavior and obtain the Kondo temperature which was found to be the same as for a Fermi liquid. In this approximation, we also study the specific heat and the magnetic susceptibility of the impurity. These quantities present no deviations from the Fermi liquid ones, consistent with the notion that the magnetic impurity is only sensitive to the local density of fermionic states.

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I. INTRODUCTION

A large number of theoretical proposals for the low-energy description of spin-liquid phases consider fractionalized fermionic degrees of freedom, the spinons, carrying spin $1/2$ but no electric charge, coupled to an emergent $U(1)$ gauge field, a gapless photon-like mode. The spinons are gapless having either nodal points\(^1\) or a Fermi surface. The latter case not only arises naturally in the slave particle approach to the $t - J$ model\(^2\) but also in other physical contexts such as the half-filled Landau level\(^3\) and the description of metals at a Pomeranchuk instability.\(^4\) It presents non-Fermi liquid behavior due to the strong interactions between the spinons and the gauge field that lead to a spectrum with no well-defined quasiparticles. This phase has a number of remarkable thermodynamical and transport properties,\(^5\) for example, at low temperatures, soft gauge modes contribute to the specific heat with a term proportional to $T^3$.\(^3\,6\)

Magnetic impurities embedded in a parent material provide an experimental probe to the bulk properties and can help to discriminate between possible candidate phases. Moreover, in order to be observed experimentally, the system itself should be stable to a dilute density of such impurities. In a Fermi liquid, an antiferromagnetically coupled spin impurity leads to the well-known Kondo effect\(^7\) characterized by a crossover from the low-temperature strong coupled regime, where the magnetic moment of the impurity is completely screened by the bulk quasiparticles, to the high-temperature regime, where the impurity susceptibility follows a Curie law.\(^8\) This crossover occurs near the Kondo temperature which is an example of a dynamically generated energy scale. The study of impurities in different bulk phases has attracted much attention,\(^8\,10\) in particular, for bosonic\(^10\) and algebraic spin liquids.\(^11,12\)

The purpose of the present work is to study the behavior of a magnetic impurity embedded in a $U(1)$ spin liquid with a Fermi surface. Being a charge insulator, this system still presents a Kondo-like behavior since the spin degrees of freedom are free to screen the magnetic impurity at low energies. We will pay special attention to the role of the gauge field, which is absent in the conventional Kondo problem. This paper is organized as follows: in Sec. II we describe the model and give some details of the $1/N$ expansion (Secs. II A and II B), the specific heat and the local spin susceptibilities are, respectively, computed in Secs. II C and II D. Finally, in Sec. III we conclude discussing the implications of our results.

II. METHODS

Starting from the $t - J$ model in 2D, the action describing the spin-liquid phase with a spinon Fermi surface coupled to a compact $U(1)$ gauge field can be obtained within the slave-boson formalism\(^9\) or using a slave-rotor representation,\(^13\) when fluctuations around the mean-field solution are considered. We assume that due to the presence of a large number of gapless fermions, the system is deconfined, i.e., one can consider a noncompact $U(1)$ gauge theory.\(^14\) The partition function is written as a path integral over the spinon Grassmannian fields $f_\sigma = \pm 1$ and the bosonic gauge fields $a_\mu = \langle a_0, a \rangle$ with action

$$S_{SL} = \int d^2 x \sum_{\sigma = \pm} \left\{ f_\sigma^\dagger \left( \partial_\tau + i a_0 \right) f_\sigma + \frac{1}{2m} \left[ (\partial_\tau - ia) f_\sigma \right] \cdot \left[ (\partial_\tau + ia) f_\sigma \right] - \frac{1}{b} i a_0 \right\},$$

where $b$ is the microscopic lattice volume and $m$ is the spinon mass. The integration over the temporal component of the gauge field, $a_0$, acts as an on-site chemical potential for the spinons enforcing $\sum_\alpha f_\alpha^\dagger f_\alpha = 1$. We use the notation $\int d^2 x = \int_0^\beta d \tau \int d^2 x$.

At $x = 0$, the interaction with the magnetic impurity is given by $S_K = S_{Berry} + J_K b \int d \tau \, S_f(0) \cdot \mathbf{S}$, where $S_{Berry}$ is the action of the free impurity spin, $J_K$ is the Kondo coupling, and $S_f(0) = f_\lambda^\dagger(\tau,0) \sigma_{\alpha,\beta} f_\alpha(\tau,0)$. Using a fermionic representation for the impurity spin $\mathbf{S} = c_\lambda^\dagger \sigma_{\alpha,\beta} c_\alpha$, this term becomes $S_K = \int d \tau \sum_\alpha c_\lambda^\dagger(\partial_\tau + i \lambda) c_\alpha - i \lambda - J_K b S_f(0) \cdot \mathbf{S}$, where $\lambda$ is an integration parameter inserted in order to enforce the constraint $\sum_\alpha c_\alpha^\dagger c_\alpha = 1$.

A. Large N expansion

Perturbative expansions for the Kondo problem are plagued with infrared logarithmic divergences signaling the fact that, for low energy, the system flows to a strong coupled fixed point where the impurity forms a singlet with the bulk electrons.
Even if resummation of the divergent terms is possible, this method is not well suited to describe the low-temperature phase. Alternatively, the large $N$ expansion reproduces the essential features of the Kondo effect in the strong coupling regime. However, for temperatures of the order of the Kondo temperature $T_K$, where a crossover to the asymptotic free regime is expected, this technique becomes unreliable due to the violation of the single occupancy constraint and instead predicts a continuous phase transition. Therefore, our results are restricted to the low-energy regime. For the $U(1)$ spin liquid, the large $N$ expansion corresponds to the random-phase approximation (RPA) used to obtain most of the physical predictions for this phase. Recently, the validity of this method applied to this specific problem was questioned since all planar diagrams were shown to contribute to the leading order. A possible resolution was proposed in Ref. 17 using a double expansion to control higher loop contributions and essentially recovering the RPA result.

In order to perform a saddle-point expansion, we generalize the above action to $su(N)$ following the standard procedure: the Pauli matrices $\sigma = \{\sigma_1, \ldots, \sigma_A\}$ are replaced by the generators of $su(N)$ $\tau = \{\tau_1, \ldots, \tau_N\}$ with the index $a = 1, \ldots, N^2 - 1$ and the coupling constant is rescaled $J_K \to \frac{J_K}{N}$. The representation of the impurity spin is taken to be conjugate to the spinons one. Using the Fierz-like identity, the Kondo term writes

$$S_K = S_{ab} = \frac{1}{N} \text{Tr} \left[ G_0 \right] - \frac{1}{N} \text{Tr} \left[ F^{-1} \right] + \int d\tau \left\{ \sum_a \epsilon_{\sigma} \left( \bar{\epsilon}_{\lambda} + i\lambda \right) \frac{\tau}{\sqrt{\Delta^2 + \Delta^2}} \right\},$$

and the last term of Eq. (1) is now multiplied by $Q_f$ defined such that $b \sum_{\sigma} f_{\sigma}^a(x) f_{\sigma}(x) = \sum_{\sigma} c_{\sigma}^a c_{\sigma} = Q_f$.

Following Ref. 18, the interaction term is decoupled inserting a bosonic Hubbard-Stratonovich field $\chi = \kappa e^{i\phi}$. The integration over $\phi$ can be performed and the partition function is given by $Z = \int D\chi D\kappa D\phi e^{-N s_b}$, where

$$s_b = \frac{1}{N} \text{Tr} \ln [-G^{-1}] - \frac{1}{N} \text{Tr} \ln [-F^{-1}] + \int d\lambda \frac{J_K}{N} \int d\lambda \frac{Q_f}{N b} + \frac{1}{N} \int d\lambda \kappa^2 - i\lambda \frac{Q_f}{N} - \int d\lambda \frac{Q_f}{N b}$$

is the action for the bosonic fields $a_\mu$, $\lambda$, and $\kappa$ only and $F^{-1}$ and $G^{-1}$ are the inverse of the full interacting propagators of the impurity and spinon fermions. We proceed performing a saddle-point expansion in the large $N$ limit imposing the a static ansatz

$$\kappa(\tau) = \kappa_0 \neq 0,$$

$$\lambda(\tau) = -i\varepsilon_c,$$

$$a_\mu(x) = \delta_{0,\mu} a_\mu,$$

At $T = 0$, the variations of the action in order to $a_{\mu}, \lambda,$ and $\kappa$ give, respectively,

$$\frac{1}{b} \text{Tr} \left[ G_0 \right] = \frac{V Q_f}{b N},$$

$$\frac{1}{\pi} \tan^{-1} \left( \frac{\Delta}{\varepsilon_c} \right) = \frac{Q_f}{N},$$

$$n(0)b \ln \left( \frac{\Lambda}{\sqrt{\Delta^2 + \Delta^2}} \right) = \frac{1}{J_K}.$$
Using the diagrammatic rules of Fig. 1, the bubble-like diagrams, including the transverse gauge as well as the $\kappa$ and $\lambda$ fluctuations, are given in Fig. 2 and are divided in impurity diagrams, mixed diagrams, and gauge diagrams. The transverse component of the gauge vertex is such that $\mathbf{q} \times \mathbf{j} = -\frac{1}{m} \mathbf{q} \times \mathbf{k}$.

The impurity diagrams, corresponding to the fluctuations of $\kappa$ and $\lambda$, were obtained in Ref. 18 and are given explicitly in Sec. II of the Appendix. It is easy to prove that, due to parity considerations, all the diagrams including the pieces of Fig. 3(a) vanish. (see Appendix). These include the mixed diagrams as well as the ones labeled by $Z$ in Fig. 2 and implies that, at this order in the $1/N$ expansion, the gauge and the impurity propagators decouple. The influence of impurity scattering on the gauge field enters only through the $X$ diagrams contribution:

$$X_{i,j}(i\Omega_\nu, \mathbf{q} - \mathbf{q}') = \frac{1}{\beta} \sum_n \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{b_k^2}{\pi} F_0(i\omega_n) G_0(i\omega_n, \mathbf{k} - \mathbf{q}) \times G_0(i\omega_n, \mathbf{k} - \mathbf{q}') \left[ \frac{1}{m} + \frac{\mathbf{k} \times \mathbf{q}}{m^2 |\mathbf{q}|} \right] j \times \left[ G_0(i\omega_{n+\nu} - \mathbf{q}) + G_0(i\omega_{n+\nu}, \mathbf{k}) \right].$$

Taking into account all nonzero terms, the bosonic action (3), developed at Gaussian order, writes $s_b = s_0 + \frac{1}{N} \sum_{\text{imp}} + s_\mu$, where

$$s_0 = -\text{Tr} \ln \left[-G_0^{-1}\right] - \text{Tr} \ln \left[-F_0^{-1}\right] + \frac{\beta}{J^2} k_0^2 - \frac{\mathcal{Q}^2}{N} \beta + \frac{\nu V}{b} \frac{Q_f}{N^2}$$

is the value of the action at the saddle point, $s_{\text{imp}}$, includes the fluctuations of the impurity degrees of freedom (given the original Read and Newns paper18 and in Sec. II of the Appendix) and $s_\mu$ is the action for the transverse component of the gauge field

$$s_\mu = \frac{1}{\beta} \sum_n \int \frac{d\mathbf{q} d\mathbf{q}'}{(2\pi)^3} \tilde{a} (i\Omega_n, \mathbf{q}) a(i\Omega_n, \mathbf{q}') \times [\delta(\mathbf{q} - \mathbf{q}')] \Theta (i\Omega_n, \mathbf{a} + X(i\Omega_n, \mathbf{q} - \mathbf{q}')),$$

where $\mathbf{a} = \mathbf{q}_{\perp} a$ and $X_{i,j} = (\mathbf{q}_{\perp j})(\mathbf{q}_{\perp i}) X$.

C. Specific heat capacity

We compute the specific heat considering the temperature dependence of the free energy defined as

$$F = -\frac{1}{\beta} \ln Z = \frac{N}{\beta} \left[ s_0 + \frac{1}{N} \left( \frac{1}{2} \text{Tr} \ln \Gamma + \frac{1}{2} \text{Tr} \ln [\Pi + X] \right) \right].$$

The $s_0$ term gives the contribution of the bulk fermionic spinons $c^{(b)}_\nu = N \frac{\nu}{\mathcal{V}^2} \frac{V n(0)}{\mathcal{F}^2}$ and the leading order impurity term $c^{(\text{imp})}_\nu = N \frac{\nu^2}{\mathcal{V}^2} \rho(0) T$. Here $\rho(0) = \rho(v = 0)$ and $\rho(v)$ is the Lorentzian impurity density of states defined earlier. The $1/N$ terms carry the contributions from the bosonic degrees of freedom. For low temperature, all internal (fermionic) propagators of Fig. 2 can be computed at $T = 0$ and the temperature dependence is given by the bosonic degrees of freedom.18 The first next-to-leading order correction due to the impurity bosons [proportional to $\mathcal{F} \text{Tr} \ln \Gamma$ in Eq. (13)] has been shown to give a correction to the impurity contribution to the specific heat.18 Defining $c^{(\text{imp})}_\nu = \gamma_{\text{imp}} T$, one obtains $\gamma_{\text{imp}} = (N - 1)^2 \rho(0)$ which can be interpreted as the suppression of one of the $N$ impurity degrees of freedom due to the presence of the constraint.

Since the fluctuations of the gauge and the impurity fields factorize, new phenomena can only arise from the $X$ corrections to the propagator of the gauge field. In a system, with a dilute number of magnetic scatterers, this term is of the order of the density of impurities, and in the present case of a single impurity, it is simply proportional...
the impurity-impurity at the impurity site and its different contributions coming from the bosonic impurity fields, case, one can use the results of Ref.18 where the fluctuations of the Kondo temperature. 

Consequently of the independence of the self-energy of the impurity [second term of \( \Sigma_1 f \)] vanishes. Thus, the only contribution to the specific heat due to the impurity is given by the correction to the bosonic propagators up to the order \( 1/N \); for the sake of clarity, symmetry related diagrams are not shown.

\[
\frac{1}{\beta} \text{Tr}[\Pi^{-1} X] = \frac{1}{\beta} \sum_n \int \frac{d\mathbf{k}}{(2\pi)^3} 2bk^2 G_0(i\omega_n) G_0(i\omega_n, \mathbf{k}) \times G_0(i\omega_n, \mathbf{k}) \Sigma_f(i\omega_n) = 0, \tag{14}
\]

where \( \Sigma_f(i\omega_n) \) is the spinnon self-energy, the impurity-impurity susceptibility is given, at leading order in \( 1/N \), by the bubble diagram of Fig. 3(b). It is easy to prove that such contribution vanishes, and one can rewrite it as

\[
\frac{1}{\beta} \text{Tr}[\Pi^{-1} X] = \frac{1}{\beta} \sum_n \int \frac{d\mathbf{k}}{(2\pi)^3} 2bk^2 G_0(i\omega_n) G_0(i\omega_n, \mathbf{k}) \times G_0(i\omega_n, \mathbf{k}) \Sigma_f(i\omega_n) = 0, \tag{14}
\]

Besides the self-energy term, the spinon propagator, given in Fig. 4(b), has also a \( 1/V \) contribution from impurity scattering processes. This correction can be safely ignored in the computation of the local susceptibility since it gives a contribution proportional to \( 1/V^2 \).

The impurity-impurity susceptibility is given, at leading order in \( 1/N \), by the bubble diagram of Fig. 5(a) (first term in the rhs). \( 1/N \) vertex corrections due to the gauge field arising in the impurity-impurity susceptibility also vanish (see Fig. 5) since they contain the terms such as the ones given in Fig. 3(a). One thus concludes that the impurity-impurity susceptibility \( \chi_{\text{imp, imp}} \) has no contribution from the gauge field at this order in \( 1/N \). So the impurity degrees of freedom only see the local density of the spinons. In particular, the result given in Ref. 18 for the static susceptibility holds: \( \chi_{\text{imp, imp}}(i\Omega_n = 0) = \frac{1}{N} J(J + 1)(2J + 1) \rho(0) \), where \( N = J + 1 \).

Gauge contributions are known to enhance Friedel-like oscillations in \( U(1) \) spin-liquids\(^{11,17,20} \). This is a consequence of the renormalization of the \( 2k_F \) component of the susceptibility vertex. One could thus expect that the local spinon-spinon susceptibility carried some trace of this behavior. Remarkably, no vertex corrections to the local susceptibility due to the gauge field are possible since simple parity arguments such as the ones used in the Appendix show that the contribution given by the second diagram in the rhs of Fig. 3(b) vanishes.

Finally, the crossed impurity-spinon susceptibility can also be shown to remain unaffected by the presence of the gauge field using the same simple arguments. Local measurements of the susceptibility at the impurity site are thus completely insensitive to the gauge degrees of freedom.

III. DISCUSSION

We considered the Kondo screening in a bulk system of spinons strongly interacting with a \( U(1) \) gauge field. While it is remarkable that Kondo screening can occur for a charge...
insulator, the results obtained here predict that no particular signature due to the presence of the gauge field can be measured if only the impurity degrees of freedom or local magnetic properties are monitored.

Let us put this seemingly negative result in context. First, to leading order in $1/N$, gauge fluctuations are responsible for singular corrections of the bulk specific heat and susceptibility, giving non-Fermi liquid signatures.\textsuperscript{5,6} Without a detailed calculation, it is not a priori obvious whether the Kondo effect of such a non-Fermi liquid will be modified. Second, it is known that the gauge fluctuations give rise to enhancement of the $2k_F$ spin susceptibility, again to leading order in $1/N$.\textsuperscript{20} Such singularity gives rise to enhanced Friedel oscillations of the spinon density.\textsuperscript{11,12} Perturbing in $J_K$, one finds that the local moment of the Kondo Hamiltonian is coupled to the local spin susceptibility of the spinons which is the sum of all Fourier components, including $2k_F$. Thus, naively, one might expect singular corrections to the leading order of $1/N$. In this context, our finding that no trace of such singular behavior was found at order $1/N$ in the impurity quantities is a significant one. Measurements such as specific heat and susceptibility will be unable to distinguish the bulk spinons from a Fermi liquid.

In particular, the Wilson ratio $R = \frac{\pi^2 T_{imp}(0)}{J/(J+1)T_{imp}} = \frac{N}{N-1}$ for this case is the same as the one found for a magnetic impurity embedded in a Fermi liquid.\textsuperscript{18}

Finally, we should emphasize that the results reported here are only valid up to order $1/N$. Higher order corrections can, in principle, introduce non-FL features that unveil the presence of the gauge field. Unfortunately, to our knowledge, the next order calculation has not been carried out even in the conventional Kondo problem, and we have not attempted that calculation here.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6.png}
\caption{Change of variables $k' = \frac{2q}{q^2}q - k$, implying that the diagram in Fig. 3(a) is zero.}
\end{figure}

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\section*{Appendix}
\subsection*{1. Vanishing of the diagram of Fig. 3(a)}
The diagram of Fig. 3(a) is given by

\[ W = b\kappa_0^2 \int \frac{dk}{(2\pi)^2} G_0(i\omega_n, k) \frac{(k \times q)}{m|q|} G_0(i\omega_{n-u}, k - q). \]

Changing variables $k \rightarrow k' = \frac{q}{q^2}q - k$, where $k'$ is obtained reflecting $k$ on axes $q$ (see Fig. 6), leaves the norms $|k| = |k'|$ and $|k - q| = |k' - q|$ invariant and changes the sign of $k \times q = -k' \times q$. Since $G_0(i\omega_n, k) = G_0(i\omega_n, k')$ for a spherically symmetric Fermi surface, it follows that $W = -W = 0$.

\subsection*{2. Impurity fluctuation's matrix}
The impurity action at Gaussian level is given by

\[ s_{imp} = \frac{1}{\beta} \sum_n \left[ \tilde{\kappa}(i\omega_n) \right] \Gamma(i\omega_n) \left[ \kappa(i\omega_n) \right], \]

where

\[ \Gamma(i\omega_n) = \begin{bmatrix} \delta x^2 & \xi x \xi \delta x^2 \\ \delta x \xi & \xi x \xi \delta x \xi \end{bmatrix}. \]

is the fluctuation matrix. Evaluating the fermionic Matsubara sums at zero temperature, one obtains:

\begin{align*}
\delta_x \delta_x s &= \frac{\Delta}{\pi |\Omega_n| (2\Delta + |\Omega_n|)} \ln \left[ \frac{\varepsilon_c^2 + (|\Omega_n| + \Delta)^2}{\varepsilon_c^2 + \Delta^2} \right], \\
\delta_x \delta_x s &= \frac{2bn(0)\kappa_0}{|\Omega_n|} \left[ \tan^{-1} \left( \frac{|\Omega_n| + \Delta}{\varepsilon_c} \right) - \tan^{-1} \left( \frac{\Delta}{\varepsilon_c} \right) \right], \\
\delta_x \delta_x s &= bn(0) \left( \frac{2\Delta}{|\Omega_n|} + 1 \right) \ln \left[ \frac{\varepsilon_c^2 + (|\Omega_n| + \Delta)^2}{\varepsilon_c^2 + \Delta^2} \right].
\end{align*}