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Laboratory observations of electron energization and associated lower-hybrid and Trivelpiece–Gould wave turbulence during magnetic reconnection

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This work presents an experimental study of current-driven turbulence in a plasma undergoing magnetic reconnection in a low-$\beta$, strong-guide-field regime. Electrostatic fluctuations are observed by small, high-bandwidth, and impedance-matched Langmuir probes. The observed modes, identified by their characteristic frequency and wavelength, include lower-hybrid fluctuations and high-frequency Trivelpiece–Gould modes. The observed waves are believed to arise from electrons energized by the reconnection process via direct bump-on-tail instability (Trivelpiece–Gould) or gradients in the fast electron population (lower-hybrid).

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I. INTRODUCTION

Magnetic reconnection,[1,2] is an important physical process in magnetized plasmas ranging from the laboratory to astrophysical scales, governing the storage and explosive release of magnetic energy and thereby powering solar flares[3] and magnetospheric storms.[4] More crucially, it also controls coupling of plasma between regions of different magnetic topology. This “opens” the magnetosphere to the solar wind and, in fusion devices, allows macroscopic tearing and sawtooth instabilities to transport plasma across the minor radius.[5]

The starting point for much reconnection study is the Sweet–Parker[6] reconnection framework, which describes steady state reconnection through an extended current sheet. Notably, estimates based on standard collisional resistivity yield extremely narrow current sheets in most contexts. The resulting small aspect ratio of a Sweet–Parker current sheet throttles the mass flow, and this geometrical effect is the major reason why Sweet–Parker reconnection is so slow. However, current sheets as thin as predicted by this theory will be vulnerable to a number of additional effects. First, tearing instabilities (resistive[7] or collisionless[8]) can break the thin current sheet into a series of magnetic islands. Second, the Sweet–Parker width can be narrower than fundamental plasma length scales, such as the ion inertial length $d_i = c/\omega_{pi}$ or the ion “sound” gyroradius $d_s = c/\omega_{ci}$. (The latter is the relevant quantity for the low-$\beta$, strong-guide-field reconnection regime studied in this work.)[9] At these length scales electrons and ions decouple so that the relative flow velocities become large.

The current sheet thinness has a profound effect for both laminar and turbulent reconnection mechanisms. In terms of laminar theories, a major breakthrough has been the discovery of the Hall effect reconnection mechanism for current sheets of thickness near $d_i$.[10,11] The extra currents due to the differing electron and ion flows at this scale are found to change the geometry of the dissipation layer to an open “x” geometry compatible with fast inflow and outflow. At the same time, if the current sheet thins to these levels, it can also become unstable to a host of microinstabilities, which may speed the reconnection process by imbuing the plasma with “anomalous resistivity.” In fact, reconnection simulations employing an ad hoc anomalous resistivity are found to achieve fast reconnection through enhanced dissipation and, more subtly, because a space-dependent resistivity can also open the current sheet geometry to an x configuration.[12] Central questions in magnetic reconnection research therefore revolve around the necessity of turbulence and fluctuations in the reconnection process.

Fluctuations can also interact with high-energy particles, which are ubiquitously observed to be a consequence of reconnection processes. Solar flares energize electrons, inferred both from hard x-ray emission[13] and from “type III” solar radio emission.[14] Within the reconnecting current sheet in the Earth’s magnetotail, 300 keV electrons have been observed directly by spacecraft.[15] Runaway electron production is also a well-known consequence of sawtooth events in tokamaks.[16] Finally, basic laboratory experiments on reconnection have also reported the creation of anisotropic, superthermal tails to the ion[17] and electron distributions, and the associated anisotropy-driven instabilities.[18] Other theoretical work has found that instabilities can play a role in energizing particles;[19] so much work remains toward understanding the interplay of fast particles, fluctuations, and reconnection.

A series of experiments has been conducted on the Versatile Toroidal Facility (VTF) at the Massachusetts Institute of Technology Plasma Science and Fusion Center, focused on basic study of the magnetic reconnection process. At present, experiments are conducted in a low-$\beta$, large-guide-field regime. Recent experiments[20] have identified a “spontaneous reconnection” regime, where the current sheet was...
found to be stable for hundreds of $\mu$s before suddenly undergoing a reconnection event, which released the magnetic energy on a time scale of about 10 $\mu$s. Interestingly, the current sheet thinned down to about 1 $\mu$m before an explosive onset of reconnection.

The observed time-variation of the reconnection rate is very useful for experimentally discerning which mechanisms play an important role in controlling the reconnection process. For instance, if current-driven turbulence is the cause of fast reconnection in VTF, then we expect to establish tight temporal correlation between the relevant fluctuations and the reconnection rate. On the other hand, time delay between reconnection and fluctuations or persistence of fluctuations long after the reconnection events would imply that the reconnection process gives rise to the fluctuations (perhaps by generating short gradient lengths or creating populations of energetic particles prone to kinetic instability) but that ultimately these fluctuations do not play a crucial role in the reconnection process.

This work studies electrostatic turbulence, which arises in the plasma during the reconnection events. Previous work on VTF in this vein has reported an observation of a nonlinear state of the turbulence consisting of “electron hole” structures, which arise when instabilities are strong enough to trap electrons in the wave trough. Electron holes have also been observed in conjunction with reconnection in the magnetosphere and have been found to play an important role in generating resistivity in computer simulations. However, the conclusion from our studies was that it was not likely that the holes played an essential role in the reconnection process in VTF; naively, at least, they move too quickly, $v_{he} = (2k_B T_e/m_e)^{1/2}$, to efficiently couple electron and ion momentum and thereby provide anomalous resistivity. Rather, they arose as a result of strong electron-electron instability, reining in an unstable population of energized electrons.

Work presented here will focus instead on the broadband aspects of the plasma turbulence observed during reconnection. Two basic classes of modes are found: lower-hybrid (LH) modes ($f \gtrsim f_{LH}$) and higher-frequency, Trivelpiece–Gould (TG) modes ($f \sim f_{ce}/2$). The low-frequency, LH modes appear to be driven by strong gradients that arise in the plasma, and a likely possibility is the gradient in electron temperature or, similarly, spatial gradients (“filamentation”) of energetic electrons. The higher-frequency Trivelpiece–Gould modes appear to be driven by a population of high-energy runaway electrons energized by the reconnection process. Similar to the electron holes, these modes appear connected to the production of energetic electrons. Interestingly, the observed phase velocities of the TG modes imply that a population of electrons is energized to $\sim 1$ keV by the reconnection events; the fluctuations are the first “diagnostic” on the device to suggest their existence.

Based on the measurements presented here, we cannot conclude that the observed modes feedback strongly on or control the macroscopic aspects of the reconnection process. Fluctuation power systematically lags the peak of the reconnection events. Further, an estimate of anomalous resistivity (calculated based on quasilinear theory) is not large enough to account for the observed reconnection rates. This is largely connected to the fast parallel phase velocity of the modes, which limits their ability to couple momentum from electrons to the ions. Instead, similar to the electron holes discussed above, these modes are believed to arise in the plasma to re-in the energetic-electron population.

This paper is organized as follows. Section II reviews the setup of VTF for magnetic reconnection experiments and the probes used to study the interplay between high-frequency fluctuations and the macroscopic dynamics of the current sheet. Section III presents our experimental measurements of plasma fluctuations and electron energization during the reconnection events, including the observation of strong spatial gradients (filamentation) in the fast electron population. Sections IV and V discuss in detail the LH and TG components of the spectrum, respectively, including phase-velocity measurements necessary to identify the waves, and measurements of correlation of the observed waves with the reconnection events. Theoretical models are presented of the driving mechanisms for each. Section VI presents a discussion of the results and conclusions. In the Appendix, we derive the dispersion relation for LH waves driven by strong temperature gradients.

II. EXPERIMENTAL APPARATUS

The VTF device studies magnetic reconnection in the regime of strong “guide” magnetic field, which is applied by toroidal field coils. For the experiments reported here, the typical toroidal guide field is 70 mT at the reconnection current sheet.

An additional set of toroidal conductors fixed within the vacuum vessel generates the poloidal magnetic field, which in vacuum has a figure-8 geometry. Figure 1 shows a schematic of the device at a single poloidal cross-section, displaying internal conductors and poloidal projections of the magnetic field lines (i.e., magnetic surfaces). The magnetic field has an x line on the midplane where the poloidal field is zero. Initially, plasma is created by microwave breakdown followed by typically about 1 ms of Ohmic heating. The moderate inductive toroidal electric field that drives this...
plasma current also drives equal currents in the four internal coils. Next, by quickly changing the currents in the internal coils (decreasing currents in the inner pair and increasing in the outer), flux is pulled away from the x line, and the plasma forms a current sheet whose subsequent reconnection is then studied. The typical poloidal field “upstream” of the current sheet is 3–5 mT. This is much less than the toroidal field, hence the strong-guide-field regime.

The main goal of experiments reported here is to connect observation of the evolution and reconnection of the plasma current sheet with measurements of “fast,” high-frequency plasma fluctuations. Magnetic equilibrium, currents, and toroidal electric fields all derive from measurement of the flux function \( \Psi(R, z) = \int_0^B \vec{B}_x(R, z) dR. \) \( \Psi \) measures the quantity of field that has reconnected, and \( \Psi \) evaluated at the x line or on the current sheet is therefore the reconnection rate. From Faraday’s law it is related to the inductive component of the electric field by \( E_x = -\partial \Psi/\partial R. \) The flux function is measured with two-dimensional (2D) arrays of novel magnetic flux probes.22

In addition to flux probes, we employ 2D arrays of Langmuir probes and an interferometer to measure density line-integrated along a vertical chord. For reference, Table I presents typical plasma parameters.

### A. Fluctuation probes

Plasma fluctuations are observed with a set of fast Langmuir probes designed to work at high frequencies. Critical here is the use of impedance-matched coaxial lines to carry the signal from the Langmuir probe tip to a high-bandwidth oscilloscope. The Langmuir tips are simply the 0.3 mm diameter, silver-plated center conductor of 0.047 in. semirigid coax (UT-047C/LL, Micro-Coax, Pottstown, PA), of which 2 mm is exposed in the plasma. The coax outer conductor is covered by a Teflon sleeve and does not contact the plasma. The total length of coax from tip to digitizer is about 2 m.

Note that we do not employ resistors at the probe tips, a common technique to achieve high input impedance to the probe. Instead, we typically use a 4 nF rf capacitor (model BKL-18+, Mini-Circuits, Brooklyn, NY) at the digitizer. This achieves high input impedance on long time scales and allows the probe tip to self-bias to the floating potential. The rf capacitor does not play a role (compared to the 50 \( \Omega \) termination) on fast time scales relevant for impedance matching. The rf blocking capacitor also supplies useful high-pass filtering of the large floating potential swings, which occur on the “slow” time scale of the reconnection events in VTF (\(-10\ \mu s, -\tau_x\)) allowing the oscilloscope full-scale range to focus on high-frequency fluctuations. Simple theory shows that the plasma-probe coupling over most of the range is resistive, \( R_p = T_{ei}/I_{ei} \), i.e., the inverse of the slope of the Langmuir probe \( I(V) \) curve evaluated at the floating potential. More detailed theory also shows that the probe has a rising response at high frequencies due to additional capacitive coupling \( C_p \) through the sheath. This impedance \( Z_p = R_p/1/i\omega C_p \) forms a divider with the 50 \( \Omega \) termination impedance of the oscilloscope; therefore to convert from measured voltage to fluctuation in the plasma, one multiplies by \( Z_p/50 \). This ratio is estimated to be \(-40 \) by measuring the Langmuir \( I(V) \) curve with the same probes. However, this ratio does vary with plasma conditions, and its uncertainty represents a drawback of the simple probe design employed here. For reasons such as this, data presented here are taken over ensembles of discharges, focusing on robust, qualitative features.

Fluctuation traces are digitized by a Tektronix DPO72004 oscilloscope. Typically 1 ms traces are acquired for four channels, spanning a large fraction of the plasma discharge, of course, including the reconnection events. The oscilloscope’s specified analog bandwidth is 16 GHz, and for data presented here, the sample frequency is 12.5 GS/s. This resolves the electron cyclotron frequency \( f_{ce} \approx 2 \) GHz, and indeed fluctuations up to \( f_{ce} \) are observed.

### III. OBSERVATION OF FLUCTUATIONS AND ENERGETIC ELECTRONS DURING RECONNECTION

#### A. Fluctuations

To date, two primary types of fluctuations have been observed and identified in VTF during reconnection events: LH waves and high-frequency TG waves. The former are the strongest component of the fluctuations and have peak fluctuation power near \( f_{LH} \approx 10 \) MHz. The TG waves, on the other hand, comprise the high-frequency band of the spectrum and typically have peak power near 1 GHz (\(-f_{ce}/2\)).

Figure 3 shows a sequence of plots from our studies of the relationship of reconnection events and development of fast plasma fluctuations. This particular discharge was selected because it contains a clear picture of phenomena that will be discussed in detail below.

Figure 3(a) shows the plasma current density, and Fig. 3(b) shows the electric field evaluated at the reconnection x line. The plasma discharge was initiated at \( \tau = 0 \), and density was built up through 1.2 ms of Ohmic heating. At

### Table I. Typical plasma parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ( n_e )</td>
<td>( 1 \times 10^{18} ) m(^{-3} )</td>
</tr>
<tr>
<td>Temperature ( k_B T_e )</td>
<td>( \geq 15 ) eV ( \approx kT_e )</td>
</tr>
<tr>
<td>Gas fill (argon)</td>
<td>( 1 \times 10^{-4} ) torr</td>
</tr>
<tr>
<td>Toroidal magnetic field ( (R=0.92 ) m)</td>
<td>( B_\phi ) 72 mT</td>
</tr>
<tr>
<td>Poloidal magnetic field ( B_z )</td>
<td>(&lt; 5 ) mT</td>
</tr>
<tr>
<td>Plasma beta ( \beta )</td>
<td>( 2 \mu \text{m}T/B^2 )</td>
</tr>
<tr>
<td>Plasma frequencies</td>
<td>( \omega_p = (ne^2/e\mu_m)^{1/2} ) 2 m( \times 10 ) GHz</td>
</tr>
<tr>
<td>Gyrofrequencies</td>
<td>( \omega_e = eB/m_e ) 2 ( \pi \times 2 ) GHz</td>
</tr>
<tr>
<td>Lower-hybrid ( \omega_{lh} = (\omega_p/\omega_e)^{1/2} )</td>
<td>( 2 \pi \times 7 ) MHz</td>
</tr>
<tr>
<td>Electron thermal speed ( v_{th} = (2k_B T_e/m_e)^{1/2} )</td>
<td>( 2.5 \times 10^6 ) m/s</td>
</tr>
<tr>
<td>Ion sound speed</td>
<td>( c_i = (k_B T_e/m_i)^{1/2} ) 5 ( \times 10^1 ) m/s</td>
</tr>
<tr>
<td>Alvén speed</td>
<td>( v_A = (B^2/\mu_0 m_n)^{1/2} ) 2.5 ( \times 10^1 ) m/s</td>
</tr>
<tr>
<td>Inertial lengths</td>
<td>( d_c = c/\omega_p ) 5 mm</td>
</tr>
<tr>
<td></td>
<td>( d_e = c/\omega_e ) 1.5 m</td>
</tr>
<tr>
<td>Electron gyroradius</td>
<td>( \rho_e = v_{th}/\omega_e ) 200 ( \mu )m</td>
</tr>
<tr>
<td>Sound gyroradius</td>
<td>( \rho_s = c/\omega_s ) 4 cm</td>
</tr>
<tr>
<td>Debye length</td>
<td>( \lambda_D = (e_0 k_B T_e/\pi n_e)^{1/2} ) 25 ( \mu )m</td>
</tr>
</tbody>
</table>
Signals are color-coded green, blue, red, and yellow, centered on the time indicated in the associated flux frame.

The four panes of Fig. 3(c) show the contours of magnetic flux (lines) and profile of toroidal electric field (colors) at four times indicated by green vertical lines in Figs. 3(a) and 3(b). Black lines track constant flux surfaces, whereas the white contour denotes the magnetic separatrix. The chosen times illustrate characteristic times during the discharge: current sheet formation, reconnection, and postreconnection.

Finally, associated fluctuation observations are shown in Figs. 3(d) and 3(e). Figure 3(d) shows the time signals observed on four electrostatic probes for the 10 μs window centered on the time indicated in the associated flux frame. Signals are color-coded green, blue, red, and yellow, corresponding to the probes in the experiment cross-section shown as the colored symbols in Fig. 3(c). (They are in the “fan” configuration from Fig. 2.) Figure 3(e) then shows the power spectra, found from standard spectral estimation techniques, for these 10 μs time windows.

The essential observations are as follows: away from the reconnection events, for instance, at \( t=1300 \mu s \), fluctuations are small, close to the bit-noise level. (Some rumbling visible near \( f=1 \) MHz is due to noise from the reconnection drive firing circuits; it is actually visible on the flux probe diagnostic as well.) During the reconnection event at \( t=1375 \) and \( 1400 \mu s \), strong fluctuations arise. The fluctuations are flat out to or have a peak near 10 MHz, which is approximately the LH frequency. Above this frequency, the fluctuations maintain a broadband character, with power extending all the way to the electron cyclotron frequency.

In the 10–100 MHz frequency band, the fluctuations typically show strong spatial variation, apparent in both the time traces of fluctuation power in Fig. 3(e) and in the spectra in Fig. 3(g), with the red and yellow probes initially picking up much stronger fluctuations than the green and blue. In general the spatial distribution of fluctuation power in this lower-frequency band has been found to be highly complex and structured.

Continuing on, at frequencies near 800 MHz, generally a new peak appears (visible on blue and green at \( t=1375 \mu s \) and on all four at \( t=1400 \mu s \)). In this high-frequency range, all probes typically measure similar power. This new peak at 800 MHz is notable because it persists long after the reconnection event—it is still visible on the power spectrum at \( t=1420 \mu s \). Note that at this time, the low-frequency modes have mostly disappeared, and the reconnection event has ended. Such high-frequency modes are also observed away from the main reconnection event, for instance, during other types of relaxations of the plasma current during the initial Ohmic heating phase of the experiment.

Figure 4 shows the evolution of a statistical collection of spectra from 25 discharges and all four probes (in the same fan configuration) at three times relative to the reconnection event. The black curve indicates the geometric mean of the spectra, and the colored band denotes the \( 1-\sigma \) group so that 2/3 of the measured spectra lie within this band; this gives a measure of the shot-to-shot and probe-to-probe variations in observed fluctuation power. (These are substantial, more than an order of magnitude in the 10–100 MHz frequency range.)

The location of the lower-hybrid frequency \( f_{LH} = (f_{ce} f_{ci})^{1/2} \), ion plasma frequency \( f_{pi} = 1/2 \pi \times (n e^2 / e \mu m_i)^{1/2} \), and electron cyclotron frequency \( f_{ce} \) are indicated in the figure as well. The electron plasma frequency \( f_{pe} = 1/2 \pi \times (n e^2 / e \mu m_e)^{1/2} \simeq 10 \) GHz is at or off the end of the abscissa.

These curves illustrate that the fluctuation phenomena discussed above are generic: Before the reconnection events, fluctuations are near the noise level. Subsequently, strong fluctuations in the lower-hybrid regime, extending out to \( f_{ce} \), are driven during the reconnection events. Finally, after the reconnection events, a band of high-frequency modes peaking near 1/4 to 1/2 \( f_{ce} \) persists.

To assemble this data set, reconnection events are identified by the flux probe diagnostic by the time of maximum \( \partial \Phi / \partial t \); discharges are only included if the peak reconnection electric field is at least 10 V/m. (This excludes about 1/3 of the discharges.) The reconnection electric field is measured at a magnetic flux array relatively close to the fluctuation probes (approximately 20° away toroidally). Spectra are calculated using standard spectral estimation techniques \(^{23,24} \) for the 10 μs window bracketing the reconnection event. This is partitioned into ten segments, which are separately fast Fourier transformed, and the results are averaged to reduce statistical variance in the periodogram estimate. The individual segments are windowed using a Bartlett window before fast Fourier transform. The data were decimated by 2 from the initial 12.5 GS/s sample frequency to decrease processing requirements for this ensemble; however signals from 3.125 to 6.25 GS are observed to be at the noise floor.

To summarize, two distinct species of fluctuations are observed during the reconnection events, a collection of lower-frequency modes peaking near the LH frequency and a high-frequency band peaking from 500 to 1000 MHz.

FIG. 2. Two fast Langmuir probe array geometries.
These modes can be distinguished by their time and space behavior. In particular, the low-frequency modes are more tightly coupled to the reconnection events in space and time, while the high-frequency modes are more uniform in space and persist for tens of microseconds after the completion of the reconnection events. Based on wavelength (dispersion) measurements, in the coming sections we will show that the low-frequency modes are LH modes and the high-frequency modes are electrostatic TG modes. We believe that the former are driven by gradients in the fast electron population, while the latter are driven by high-energy (runaway) electrons energized by the reconnection process.

B. Fast electrons

During the course of these investigations, results suggested that many of the instabilities could be driven by a population of fast electrons energized by the reconnection events. To study these, a series of gridded electron energy analyzer probes has been constructed. These probes measure the total flux of electrons with energy greater than the applied retarding potential. Compared with a simple Langmuir probe, the use of extra grids allows ions to be rejected, so therefore the signals correspond simply to the electron tail population.
We constructed a multieheaded energy analyzer probe, which integrates seven grid/collector pairs into a small (1.5 × 1.5 cm²) area. (Details of the probe construction and operation will be described in a separate paper.) Initially, a goal of including the seven channels was to see details of the electron distribution, namely, unstable (i.e., positively sloped) features. In theory, this is achieved by biasing the seven channels to different retarding biases, then using finite differences between adjacent channels to find f(E).

However, scans with all channels biased to the same retarding voltage quickly found that the fast electron populations were not uniform over the scale of the probe. First, this means that details in the electron distribution cannot be measured since separate channels do not necessarily observe the same population of electrons. Naïvely taking finite differences as described above yields unphysical negative values for f. Instead, we have more success averaging the results across the probe to estimate an average “temperature” of the tail, which is derived from a linear fit of the log of current versus bias. Note, however, that these measurements were not uniform over the scale of the probe. First, this means that details in the electron distribution cannot be measured since separate channels do not necessarily observe the same population of electrons. Naïvely taking finite differences as described above yields unphysical negative values for f. Instead, we have more success averaging the results across the probe to estimate an average “temperature” of the tail, which is derived from a linear fit of the log of current versus bias. Note, however, that these measurements have been found to be noisy, and currents to individual channels varied by up to a factor of 2 or 3 away from the linear fit.

Figure 5 shows temperature measurements over a large collection of discharges. Figure 5(a) shows measured reconnection electric fields (evaluated using a flux array 20° toroidally away from the energy analyzer, which is within a few centimeters of the reconnection x line in the poloidal plane.) Time is measured relative to the reconnection event, which is identified as the time of peak toroidal electric field. For each discharge, the tail temperature is measured by fitting as discussed above. Results are shown in color histogram form in which color indicates the fraction of discharges that land in a given time-temperature bin. The results robustly show heating of the tail of electrons from near 20 eV before reconnection events to a typical 30–35 eV immediately after.

The filamentation, or short perpendicular length scales in the fast electron population, is inferred from scans with all seven grids biased to the same retarding voltage. Even with equal biases, it is found that the collectors measure highly differing time traces. Figure 6(a) shows the time traces of all seven collectors, showing a burst of fast electrons onto the collectors as a result of a reconnection event. Data to these collectors have been normalized by probe areas and nominal...
grid transparency to yield fast electron current density. For this discharge, the grids are biased at $-120$ V with respect to ground, which is approximately $180$ V below the plasma potential. Figure 6(b) shows the color coding of collector channels used in the plotting.

As can be seen in Fig. 6(a), a few “waves” of fast electrons wash over the probe, but they are not measured on all channels equally. Moreover, there are clear correlations between adjacent collectors. For example, near $1.39$ ms, the green, blue, and violet channels pick up the largest currents, indicating localization on the bottom quadrant of the probe. Similarly, near $1.44$ ms, the green, yellow, and orange channels are strongest, indicating that the electrons predominantly crossed the top half of the probe. Finally, Fig. 6(c) zooms in on the $10$ μs region from $1.39$ to $1.40$ ms and normalizes each collector by its peak current for clarity in comparing the time-behavior. Here, the fast electron current crossed all the probe faces, roughly in order from right to left, reaching its maximum first on the yellow and green collectors, followed by orange, gray, and blue, and finishing with red and violet. From the time traces one can also see that by the time the current is maximum over red and violet, it is down by about 50% on yellow and green, and vice versa. This indicates that the characteristic size of these current filaments is approximately the probe size (1 cm). (A similar conclusion can be reached from the amplitude-variation between adjacent channels on the probe, which are separated by about 0.4 cm.) At the same time, the current filament can be seen to propagate roughly 1 cm in $3 \times 10^3$ m/s; this is of the same order as the characteristic speed of the reconnection outflow.

We will now present a more detailed analysis of the two wave regimes.

IV. LOWER-HYBRID REGIME

A. Measurements

We have conducted more detailed investigations of the waves using cross-correlation techniques between probes in the “correlation” configuration displayed in Fig. 2. The probes are sampled simultaneously by the oscilloscope, and correlation analysis is applied off-line on the saved data traces. The probe shaft can be rotated to measure the cross-correlation properties of modes as a function of angle with respect to the magnetic field. (In the machine geometry, the probe rotates in the $Z-\phi$ plane.) Recall that the magnetic field is almost entirely toroidal, especially near the reconnection x point where the poloidal component is zero.

Of primary interest here is the cross-spectrum $C_{XY}$, which is the Fourier transform of the cross-correlation of signals $x(t), y(t)$, defined as $c_{xy}(\tau) = \frac{1}{T} \int_x y(t) y(t-\tau) dt$, where $T$ is an averaging time. Experimentally this is evaluated from the Fourier transforms $X(f)$ and $Y(f)$ of the signals as

$$C_{XY}(f) = \langle X^*(f) Y(f) \rangle.$$

Here $\dagger$ denotes complex conjugation, and the angle brackets indicate an ensemble average, which is achieved experimentally by further segmenting the time window of interest. $C_{XY}$ is complex and therefore contains both amplitude and phase information. The cross-phase, defined as $\arg(C_{XY})$, can supply the average phase relationship between the two signals at each frequency. To relate back to the waves, this is interpreted as $k \cdot \Delta$, where $k$ is the dominant wave-vector and $\Delta$ is the vector separation between probe tips. The coherency, which is the magnitude of $C_{XY}$ normalized by $\langle x^2 \rangle^{1/2} \langle y^2 \rangle^{1/2}$ so that it is dimensionless and lies between 0 and 1, is a measure of the degree to which the two probe signals are correlated. When the coherency is 0, the cross-phase is random.

In VTF, it is often found that the signals are strong but the coherence is low. This can arise if multiple waves exist at the same frequency but have different wave-vectors $k$; then they will register different phase shifts between the probe tips. This “wave-bath” effect can quickly destroy coherence; a rough quantitative criterion is that coherence is destroyed once $\Delta \langle k \rangle \sim \pi$, where $\langle k \rangle$ is a characteristic width in $k$-space occupied by the spectrum (at a given frequency). Similarly, the decoherence can arise if the two probes are simply observing two separate, uncorrelated wave packets. This leads to a similar criterion for $k$ so that one can construct a sufficiently narrow wave packet. This decorrelation criterion will be used below to infer a lower bound on $k$ when direct measurement is unsuccessful.

Figure 7(a) shows color histograms of the observed cross-phase for frequencies in the lower-hybrid regime of $5–200$ MHz, for probes at 0.3, 1, and 1.3 cm separations, with the probe aligned with the magnetic field. An ensemble

![FIG. 7. (Color) (a) Phase measurements vs probe separation for the LH frequency regime $f < 200$ MHz. The plot is shown in color histogram form, showing fraction of discharges which measured a given phase at each frequency. (b) Coherency vs separation and exponential model with parallel correlation length of $\sim 2$ cm.](image-url)
have varied the angle \( \theta \) of the probe with respect to the magnetic field for the 0.3 cm probe separation. The phase quickly becomes random with angle, as is found in Fig. 8(b), which displays the coherency versus angle. Fitting an exponential to coherency versus \( \Delta \sin \theta \), one estimates a perpendicular correlation length of about 1 mm. The fluctuations are strong here, much larger than the bit noise, so the decorrelation should arise directly from the presence of a bath of waves with differing \( \mathbf{k} \cdot \Delta \) at each frequency. Because we have already found that \( k_\perp \Delta_\perp \) is small, it is therefore the bath of waves with different \( k_\perp \) that gives the spread. Using the above wave-bath estimate of decorrelation, one estimates that \( k_\perp \sim \pi/L_\perp \), or roughly 3000 m\(^{-1}\). In plasma units, \( k_\perp \rho_e \sim 0.5 \).

In conclusion, we find that \( k_\perp \gg k_\parallel \) for waves in this regime and quickly find poor coherence between probes as they are separated by a small distance (~1 mm) perpendicular to the field, even as they are coherent over longer distances (~1 cm) parallel to the field. While \( k_\perp \) could not be directly measured, its magnitude is estimated from the short perpendicular correlation lengths of the waves.

One important observation to highlight is the strong perpendicular inhomogeneity of the fluctuations, evident, for example, in Fig. 3. Probes separated from each other by as little as 1–2 cm can measure an order-of-magnitude different fluctuation power. We have also scanned the probe radially and vertically near the x line but found little trend observable on top of these large natural variations, over the limited (10 cm) range explored. With the probe all the way to the edge of the machine, however, little LH wave activity is observed. Further exploration could be undertaken in the future, but substantially more statistics and improved experimental control of the current sheet location and reconnection events would be necessary to make definitive progress studying the averaged spatial profiles.

Finally, Fig. 9 shows the time correlation of these LH modes and the reconnection events, evaluated for a large ensemble of discharges, with the probes in the fan configuration as used above. The reconnection electric field is evaluated from \( \partial \mathbf{V} / \partial t \) at the x line using the flux probe array separated by about 20° toroidally from the fluctuation probe. An important observation here is that while there is good overall correlation of the reconnection events with fluctuation observations, there appears to be a systematic delay of ~10–20 \( \mu s \) between peak fluctuations and peak reconnection electric field. This point will be discussed further below.

B. Discussion

As found from wavelength measurements, \( k_\perp \leq 300 \text{ m}^{-1} \), and \( \omega/k_\parallel \) is found to be of order the electron thermal speed or higher. These measurements, while crude, are sufficient to conclude that the waves cannot be acoustic waves driven by ion-acoustic instability. These would have \( \omega/k_\parallel \sim c_s \ll v_{te} \) and would have registered a substantially faster winding of the phase than was observed and much shorter parallel correlation lengths. It is not automatically clear why ion-acoustic instability has not been observed.
Based on observed current densities near 50 kA/m² and density of $1 \times 10^{18}$ m⁻³, we find electron-ion drifts, $v_{de} = j/ ne \approx 50 c_{s}$. Assuming that the current is carried by a drifting, Maxwellian bulk of electrons, this should be large enough to trigger the ion-acoustic instability (for argon/electron mass ratio) as long as $T_{e}/T_{i} > 5$. At present the ion temperature is not known experimentally, though ion temperature measurements on the open configuration of VTF (using laser-induced fluorescence) found ion temperatures of up to 2 eV during reconnection.

Instead, the results better match expectations for LH waves, electrostatic waves which satisfy the dispersion relation,

$$\omega^{2} = \frac{\omega_{pe}^{2} + \omega_{ce}^{2} \cos^{2} \theta}{1 + \omega_{pe}^{2}/\omega_{ce}^{2}},$$

$$\approx \omega_{LH}^{2} + \omega_{ce}^{2} \cos^{2} \theta,$$

where $\omega_{LH}^{2} = \omega_{pe}^{2}/(1 + \omega_{pe}^{2}/\omega_{ce}^{2}) = \omega_{ce} \omega_{idi}$ as VTF is in the regime $\omega_{pe}^{2} \gg \omega_{ce}^{2}$. (The dispersion relation and propagation characteristics of LH waves have been thoroughly investigated in laboratory plasmas, e.g., in Ref. 27.) Near the LH frequency, these modes are quite anisotropic with $k_{i} \ll k_{\perp}$, in agreement with observations here. Finally, standard LH waves derive from cold electron response, in accordance with the observed bound $\omega / k_{i} \approx v_{ce}$. Overall, the measurement of $k_{i}$ is quite noisy, but this may be expected due to projection effects from slight misalignment of the probe with the magnetic field: near $\omega_{LH}$, $k_{i} / k_{\perp} \approx 1/270$ for argon. Observations here do not decisively measure $k_{\perp}$, but its effect is manifested in the decorrelation of the probe signals as the probe is barely rotated away from being parallel to the magnetic field.

Narrow, reconnecting current sheets typically imply strong gradients in plasma parameters, which have long been understood to potentially drive LH instabilities. The criterion here is that the current sheet width and thus the gradient length scale [e.g., the density scale length $L_{n} = n/(dn/dx)$] be of the order of the ion or sound gyroradius ($\rho_{i}$ or $\rho_{s}$). With such strong gradients, the “drift frequency” $\omega_{drift} = k_{i} T_{e} / e B L_{n}$ can approach the LH frequency when $k_{i} \rho_{i} \rightarrow 1$ and $L_{n} \rightarrow \rho_{i}$. Interaction of the drift and LH modes leads to a strong instability.

These types of instabilities have previously been studied in basic laboratory plasmas, e.g., in Ref. 29. Their role in reconnection has also been extensively studied since their early identification as a potential method of producing anomalous resistivity. Recent experiments on the Magnetic Reconnection Experiment (MRX) device by Carter et al. observed the “lower-hybrid drift instability” (LHDI), which is the version driven unstable by a strong density gradient. That work concluded, however, that the observed LHDI did not correlate well with enhanced reconnection. They also found that the LHDI was confined to the low-β half of their current sheet (which is in-out asymmetric due to toroidal effects). This agreed with the theoretical predictions that the electrostatic LHDI modes are stabilized by finite plasma beta. The LHDI has now also been observed in the magnetospheric current sheet, work that found the same finite-β effect and similarly found that LHDI could not explain the enhanced reconnection.

Both of those observations were in an antiparallel reconnection geometry with $\beta \sim 1$. Experiments reported here, on the other hand, are in a low-β regime due to the strong-guide magnetic field, and so this stabilization mechanism is absent. Furthermore, VTF also has a current sheet that is of the order of $\rho_{i}$ wide, and therefore in principle the same gradient mechanism can act in VTF as in MRX. The potential role of LHDI and the similar “modified two-stream instability” (MTSI) for the LH waves in VTF will be discussed in more detail at the end of this section.

However, a more thorough consideration finds other instability mechanisms that may be more important in VTF than the density-gradient-driven LHDI. Instead, we find that the LH modes can be driven by the filamented, fast electron population. This driving mechanism matches the experimental observations in a few key ways. First, recall that the fast electron population was found to be filamented on a fairly short length scale (about 1 cm); this will be shown below to already be a sufficient criterion for driving these modes. (In contrast, the detailed density measurements necessary for inferring the existence of LHDI are not yet available.) Next, recall that cross-correlation measurements find $k_{i}$ pointing preferentially in the electron parallel drift direction (Fig. 7). This suggests a connection to the energization of electrons produced by the reconnection event. Note that while we find overall good correlation experimentally between the LH modes and the fast electrons and that both are observed to be finely structured in the poloidal plane, it has not been possible yet to directly link individual filaments with fluctuation bursts. Finally, the observation that fluctuations slightly lag the reconnection events fits very well with instabilities of energized electrons based on the finite time it takes to accelerate the electrons.
The filamentation of the fast electrons is an important part of driving modes in the LH range. This has long been known in the context of beam-driven instability. For instance, drive by a spatially uniform beam is found to primarily drive (TG) modes at high frequency, with $\omega / \omega_{pe} \sim 1/2$, since the temporal growth rate is found to increase with frequency over most of the unstable range. (This is discussed in detail in the Sec. V.) However, as discussed by Papadopolous and Palmadesso,\(^{34}\) narrow beamlets might preferentially excite LH waves since the perpendicular group velocity ($c_{\perp}$) becomes small near $\omega_{\text{LH}}$, so the convective growth rate $k_{i\perp}$ actually maximizes at $\omega_{\text{LH}}$.

Besides these considerations, it is also possible that LH modes can be driven by a drift instability, i.e., due to a spatial gradient in the fast electron population. This is the best model for the LH waves in VTF at present. Such an instability is found to be operative for the observed experimental parameters and primarily drives modes for real frequency near $\omega_{\text{LH}}$ without having to assume that the distribution function has beam- or positively sloped components.

Instability due to a gradient of fast electrons derives from adding a small imaginary part to the standard dielectric for LH waves,

$$\frac{d^2}{d^2 \omega_{pe}^2} \sin^2 \theta - \frac{\omega_{pe}^2}{\omega^2} \cos^2 \theta - \frac{\omega_{pe}^2}{\omega^2} + i \epsilon_1 = 0.$$  \hspace{1cm} (3)

Here, the first three terms give the standard cold LH waves [with dispersion relation in Eq. (2)] in the limit $\omega_{pe} / \omega_{ce} \gg 1$. The imaginary component $\epsilon_1$ drives the instability or contributes damping. A gradient of the tail population contributes the following to $\epsilon_1$:\(^{35}\)

$$\epsilon_1^D = \frac{\pi e^2}{e_0 m^* k^2} \left( \frac{k \times \nabla f}{\omega_{pe}^2} \right) d^2 \mathbf{v}_\perp.$$  \hspace{1cm} (4)

This expression differs in sign from Ref. 35 because the electron gyrofrequency is unsigned here.) To drive an instability, the gradient drive must overcome Landau damping on the electrons at the same phase velocity,

$$\epsilon_1 = -\frac{\pi e^2}{e_0 m^* k^2} \left| \frac{\partial f}{\partial v_z} \right|_{\omega k_z} d^2 \mathbf{v}_\perp.$$  \hspace{1cm} (5)

Per usual procedure, for small $\epsilon_1$, the real part of $\omega$ will derive simply from the bulk dielectric, and here will be a cold LH mode that can resonate with the fast tail electrons. Then, instability results when $\epsilon_1 < 0$, with growth rate $\gamma = -\epsilon_1 / (d\epsilon_1 / d\omega)|_{\omega_{pe}}$. The increasing dependence of $\epsilon_1^D$ on $k_{\perp} / k_{\parallel}$ pushes the unstable modes to be very perpendicular, i.e., near $\omega = \omega_{\text{LH}}$. If the fast electron population is imagined to have a gradient length scale $L$ and effective temperature $T$ near the relevant phase velocity, then the marginal stability condition is $L / \rho_e < 0.7 (k_{\perp} / k_{\parallel}) (\omega_{\text{LH}} / \omega)$, where $\rho_e$ and $\rho_i$ have been evaluated with this effective tail temperature. Therefore, gradient scales of the order of $\rho_e$ (4 cm in VTF) can trigger these modes. The energy analyzer probe has directly observed such gradient scales in the fast electron population.

As a particularly simple and violent limit of this instability, one may also consider that the heated filament extends throughout the distribution. In this case, the instability is really a LH mode driven by the temperature gradient.\(^{35}\) A derivation of the dispersion relation for temperature-gradient instability is presented in the Appendix. Using this dispersion relation, Fig. 10 presents the results of an example instability calculation based on a temperature scale length of $100\rho_e$, which is approximately 2 cm for VTF parameters. The growth rate peaks for real frequencies slightly above the LH frequency, and the most unstable wave modes have finite $k_i$, with $k_i / k_{\parallel} \sim (m_i / m_e)^{1/2}$ customary for LH modes.

While available measurements favor the energetic-
electron- or temperature-gradient-driven instabilities, it is hard to completely rule out other perpendicular drive mechanisms—density gradients or perpendicular flows—at present. Therefore, prospects for driving the associated instabilities (LHDI and MTSI) are reviewed here.

The LHDI is driven by density gradients. In the low-$\beta$, strong-guide-field regime, the LHDI can be driven if the density scale length in the current sheet approaches $\rho_s$, and this is roughly the scale of current sheets observed on VTF (Ref. 20) immediately before the onset of the reconnection event. However, as found in Fig. 9, fluctuations tend to lag the peak of the reconnection event, whereas the existence of a thin current sheet and the associated gradients precedes the reconnection event. This suggests that LHDI due simply to having a thin current sheet is not the cause of the LH waves in VTF. On the other hand, it could be that even sharper density gradients are formed during the disruption of the current sheet. In fact, computational studies of guide-field reconnection predict a strong quadrupolar density perturbation near the $x$ line in coincidence with reconnection events.

MTSI\(^{31,36}\) is driven by “nonequilibrium” perpendicular current, i.e., perpendicular currents not associated with density or temperature gradients, for instance, polarization currents. The typical instability criterion is that the perpendicular drift be larger than the ion sound speed. Such currents might arise in VTF during the onset of the reconnection events since they are observed to turn on about as fast as the ion-cyclotron time, $\omega_{ci}^{-1} \approx 6 \ \mu s$. Furthermore, ions are also potentially unmagnetized near the current sheet due to finite gyroradius, as $\rho_s \approx 3 \ \text{cm}$ is the same size scale as the current sheets.\(^{20}\) At the same time, the reconnection flows are large; the outflow speed has been observed to be up to the (upstream) Alfvén speed, $\approx 1 \times 10^4 \ \text{m/s} > c_s$. However, the electric fields associated with the outflow have not yet been measured below the $\rho_s$ scale where decoupling should occur.

Second, recent investigation has also found that the reconnection events on VTF are not completely axisymmetric but rather start at one toroidal location and appear to propagate toroidally, “unzipping” the magnetic field.\(^{37}\) Such local reconnection corresponds to a local disruption of toroidal current, but if this does not happen symmetrically around the device then substantial perpendicular currents can be driven to maintain $\nabla \cdot \mathbf{j} = 0$. Prior to reconnection onset, the electron parallel drift is estimated to be $v_{de} \approx 50c_s$; only a small fraction of this has to be diverted across the field to drive MTSI. Both of these mechanisms would be expected to correlate well with the reconnection events.

Finally, note that a number of “parallel” driven mechanisms for LH waves have also appeared in the literature. The parallel excitation mechanisms include “slide-away” LH waves\(^{38}\) or Buneman instability. Slide-away LH waves were proposed to account for LH wave observations in tokamaks; the mechanism is a positive slope in the distribution function based on the separation of the “trapped” and “circulating” populations of electrons. With enough parallel drift in the circulating population, a region of positive slope opens up in the distribution, and modes resonant with this velocity can be LH modes. However, the VTF plasma is neither hot enough to obtain the required “banana regime” nor is the observed parallel drift velocity ($v_{de} = 0.1v_{te}$) large enough to trigger these modes.\(^{39}\) Finally, Buneman instability can give rise to LH waves as a second branch due to finite magnetic field.\(^{25}\) However, this has even stronger electron current requirements than the slide-away LH waves: $v_{de} > v_{te}$. In conclusion, these parallel mechanisms are unlikely to drive the LH waves in VTF.

In summary, the wave numbers inferred from correlation techniques show that the lower-frequency waves are LH waves. A number of mechanisms have been reviewed for their ability to drive the observed LH waves. It is found that gradient instabilities, especially gradients in the fast electron population or perhaps a temperature gradient, are excellent candidates, as the required short gradient length scales have been observed in the tail of the electron distribution using a multitipped electron energy analyzer. The operation of LHDI or MTSI has also been examined but at this point is more speculative.

C. Anomalous resistivity

The nonlinear reaction of unstable waves onto the dc components of the electron and ion distributions is found from the correlated product $(\tilde{n}\tilde{E})$, where $(\cdot)$ indicates an ensemble average. Since we do not have complete measurement of $\tilde{n}$ and $\tilde{E}$, here we use quasilinear theory, which estimates the phase and amplitude relationship of these quantities to the measured $\phi$, allowing for a simple estimate of the anomalous momentum coupling due to growth of these waves. This is cast into the form of an effective parallel electric field, which is to be compared against the dc reconnection electric field. Note that there can be other quasilinear effects, i.e., quasilinear diffusion, within the electron population. However, here we restrict the discussion to the total momentum transferred from electrons to ions by the waves.

One begins with the quasilinear diffusion operator (e.g., Eq. II–27 in Ref. 40), taking the velocity moment to find the net momentum transfer to ions. Because of the strong-guide-field in VTF, the parallel moment and thus the parallel quasilinear electric field are calculated for comparison against the reconnection electric field. This yields the following sum over the wave spectrum,

$$E_{v_{de}} = \frac{2e}{m_i} \sum_k k^2 \frac{\gamma_k}{\omega_k} |\phi_k|^2,$$

Note that the direction of $k_0$ observed (Fig. 7) is in the direction of the electron drift, which is the correct direction to remove momentum from the drifting electron population.

One further approximation is needed to complete the estimate; due to the absence of a complete measurement of the $k$-spectrum of $\phi$, we instead replace the factor $k^2\gamma/\omega^3$ with its maximum value from linear theory and pull it out from the integral. Then, the Faltung theorem is used to convert $\Sigma_k |\phi_k|^2$ back to $(\phi^2)$, i.e., the measured power in fluctuations. This is a reasonable approximation when the fluctuation power peaks near the most unstable mode. This yields
\[ E_{tq,j} = \frac{2e}{m_e} \left[ \frac{\gamma}{\omega^3} k_j k^2 \right]_{\text{th,max}} \times \langle \phi^2 \rangle_{\text{expt}}, \]

where the subscripts th and expt indicate source of each term. In the last step, the theoretical term is converted to a nondimensional coefficient, \( \Gamma_{\text{th}} \), by normalizing \( k \) to \( 1/\rho_e \) and \( \omega \) and \( \gamma \) to \( \omega_{\text{LH}} \).

\[ \Gamma_{\text{th}} = \left( \frac{k \rho_e (k^2 \rho_e^2) (\gamma/\omega_{\text{LH}})}{(\omega/\omega_{\text{LH}})^3} \right)_{\text{max}}. \]

With this substitution,

\[ E_{tq,j} \approx \frac{k_B T_e}{e \rho_e} \times \Gamma_{\text{th}} \times \left( \frac{e}{k_B T_e} \right)^2 \langle \phi^2 \rangle. \]

The theory is best regarded as a simple scaling law prediction: assuming that the momentum coupling scales linearly with the fluctuation power and the instability growth rate, the scaling as \( v^3 \) is determined uniquely—the phase velocity is the only velocity in the problem for generating a formula with the correct dimensions. The factor \( \Gamma_{\text{th}} \) is naturally found to depend on the instability theory invoked, gradient length scales assumed, and growth rates thus calculated; therefore, a range of plausible values are attained.

Despite the limitations of the quasilinear theory, uncertainty in Langmuir probe calibration, and wide shot-shot variation in observed fluctuation amplitudes, it is still important to attempt a quantitative comparison. As an example, we use the temperature-gradient instability. In numerical investigations, it has been found that \( \Gamma \approx 2 \times 10^{-4} \) when \( L_T=100 \rho_e, \) \( =2 \text{ cm} \), and scales roughly linearly with \( 1/L_T \) and \( 1/L_{T,e} \), where \( L_{T,e}=250 \rho_e =5 \text{ cm} \) is the critical temperature-gradient length scale for the temperature instability.

For amplitudes, we use an estimated calibration factor for raw Langmuir probe signals to potentials fluctuations \( \sim 40 \) and use a typical raw fluctuation power of \( \sim 10^{-3} \text{ V}^2 \) (see Fig. 9), giving \( e \phi_{\text{rms}}/k_B T_e \sim 0.1 \). Meanwhile, the dimensional factor \( k_B T_e/e \rho_e \sim 8 \times 10^4 \text{ V/m} \). Combining these factors with \( \Gamma_{\text{th}} \approx 2 \times 10^{-4} \), one estimates \( E_{tq,j} \approx 0.2 \text{ V/m} \). This is about two orders of magnitude too small to account for typical reconnection electric fields of \( 15 \text{ V/m} \). The maximum fluctuation powers observed, \( \sim 10^{-2} \text{ V}^2 \), if combined with more generic \( \Gamma_{\text{th}} \), may imply interestingly large quasi-linear electric fields in a handful of discharges. This topic will be discussed further in Sec. VI.

V. Trivelpiece–Gould REGIME

A. Measurements

We have conducted similar analysis of the higher-frequency branch over the frequency range of 200 MHz \(< f < 2 \text{ GHz} \). Overall, these waves have proven to be much easier to analyze owing to improved coherence. Furthermore, the modes are not nearly as anisotropic (between \( k_y \) and \( k_z \)) as the LH waves, which decreases projection effects and makes the correlation measurements more reliable. Figure 11 presents phase measurements for probes aligned parallel to the field over the entire regime up to \( f_{\text{ci}}=2 \text{ GHz} \). As before, phases are determined from the statistical cross-correlation analysis, evaluated using data over a 10 \( \mu \text{s} \) window during the reconnection events.

In this frequency regime a clear parallel dispersion relation \( k_y (\omega) \) is apparent. Interestingly, over this range the dispersion is linear with \( k_y (\omega) \propto \omega \), i.e., \( \omega/k_y \) approximately a constant phase velocity \( (v_{ph}) \). Comparing probes at three separations, one finds consistent \( v_{ph} \) so that the phase \( \Phi \) satisfies

\[ \Phi = k_y (\omega) \Delta = \frac{2 \pi f}{v_{ph}} \Delta. \]

In Fig. 11 are plotted fit curves using the known \( \Delta \) for each pair and the common fit parameter \( v_{ph}=2.2 \times 10^7 \text{ m/s} \). Note that the direction of \( k_y \) and \( v_{ph} \) is consistent with the direction of electron drift. \( k_y \) takes on a wide range of values, with the typical value of \( 300 \text{ m}^{-1} \) at 1 GHz and an extrapolated \( 600 \text{ m}^{-1} \) at 2 GHz. This phase velocity is very fast compared to the electron thermal speed, \( v_{ph}/v_{te} \sim 10 \). Coherence is also measured over the ensemble and is shown in Fig. 11(b); it is found that the parallel correlation length is of the order of 3 cm.

Figure 12 shows that the coherence falls for these waves with increasing angle from the magnetic field. (Note that coherence also falls with frequency even for parallel correlation, but this happens since fluctuation power approaches the noise level as \( f \) approaches \( f_{\text{ci}} \).) In contrast to the LH waves, here we plot data from the probe pair with 1 cm separation because the decorrelation onset is slower. Further-
more, 30° separation is required to substantially decorrelate
the waves. Coherency versus angle is plotted in Fig. 12(b),
and from an exponential fit, one infers a perpendicular cor-
relation length of the order of 1 cm. Using the wave-bath
estimate, one estimates from the decorrelation that the per-
pendicular wavelengths will be of the order of 300 m−1;
therefore in this frequency range, \( k_\perp \sim k_\parallel \).

Figure 13 shows the time correlation of these modes and
the reconnection events, evaluated for a large ensemble of
discharges, with the probes in the fan configuration as used
above. As has been discussed above, these high-frequency
modes persist long after the reconnection events; here one
sees that substantial fluctuations still exist at 25 \( \mu s \)
after the reconnection event. This strongly implies that these modes
arise as a consequence of the reconnection events. Below we
will discuss that a natural interpretation for the excitation of
these modes is instability of a tail of high-energy electrons
generated by the reconnection event.

B. Instability mechanisms

To summarize observations, the high-frequency branch
of oscillations has a fairly uniform phase velocity, and flu-
cation power is observed up to almost exactly \( f_{ce} \). Peak
fluctuation power is typically in the range of 1/4–1/2 \( f_{ce} \). In
this section we show that these are TG modes, likely driven
unstable by bump-on-tail (Čerenkov) instability of high-
energy runaway electrons. The phase-velocity measurements
imply that the modes are driven by electrons with energies
near 1 keV.

First, as is well known, the electrostatic cold dispersion
relation for high-frequency plasma waves,

\[
1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \sin^2 \theta - \frac{\omega_{pe}^2}{\omega^2} \cos^2 \theta = 0,
\]

predicts two bands of waves. In the limit of well-separated \( \omega_{pe} \gg \omega_{ce} \), one band of waves exists between \( \omega_{pe} \) and \( \omega_{th} = (\omega_{pe}^2 + \omega_{ce}^2)^{1/2} \), and the second exists for \( \omega < \omega_{ce} \), with a gap
(of cold waves) in between. The observed waves fall nicely
in the second category since we observe cutoff of fluctuation
power at \( f_{ce} \). This category is the TG branch of the dispersion
relation, with \( \omega = \omega_{ce} \cos \theta \). This connects to the LH branch
discussed earlier when the waves are highly perpendicular
and the ions begin to play a role. At high frequency, how-
ever, \( k_\perp \sim k_\parallel \), consistent with observations here; recall that we estimated \( k_\perp \sim 300 \text{ m}^{-1} \) and measured directly \( k_\parallel \)
\( \sim 300 \text{ m}^{-1} \) at 1 GHz.

Restricting to waves with \( \omega_c \leq \omega_{ce} \ll \omega_{pe} \) and including
kinetic instability mechanisms, the relevant dispersion rela-
tion is

\[
- \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \sin^2 \theta - \frac{\omega_{pe}^2}{\omega^2} \cos^2 \theta + i \epsilon_l = 0.
\]

Here \( \epsilon_l \) is a small imaginary part of the dielectric that drives
the instability. Kinetic drive mechanisms include Čerenkov
instability (inverse Landau damping) due to a small positive
slope of the distribution function or temperature-anisotropy
instability, i.e., \( T_i \neq T_e \). The anisotropy expected in this case
is \( T_i > T_e \) owing to long electron tail that can be drawn out
by the strong reconnection electric field; here the instability
mechanism derives from the “anomalous” Doppler (AD)
resonance between waves and particles with phase velocity \((\omega+i\gamma)/k_c\).\(^{41}\) (Instability drive from spatial gradients of fast electrons,\(^{35}\) discussed in the previous section in connection with the LH modes, drives instability at much lower frequencies and will therefore not be considered here.)

As per usual procedure, for weak drive, \(\epsilon_i\) is small, and therefore \(\omega-i\gamma, \omega, \gamma\) with \(\omega\) deriving from the bulk modes, i.e., \(\omega_c=\omega_c, \cos \theta, \gamma=-\epsilon_i/(\partial \epsilon_i/\partial \omega)|_{\omega}\). Here, note that

\[
\left[\partial \epsilon_i/\partial \omega\right]^{-1} = \frac{\alpha_c^3}{2 \alpha_c^2} \cos \theta \sin^2 \theta. \tag{13}
\]

In general \(\epsilon_i\) will also have some \(\theta\) dependence but the \(\theta\)-dependence in \(\partial \epsilon_i/\partial \omega\) already pushes the most unstable modes away from \(\theta=0\), i.e., to frequencies below \(f_{ce}\), consistent with our measurements.

Of these two instability mechanisms, Čerenkov drive is the most consistent with the fairly uniform phase velocity inferred from the measurements; the speed on the positive slope of the presumed bump determines the phase speed. The imaginary part of the dielectric in this case is given by Eq. (5) but with \(df/\partial v_{||}>0\) in this case. The angle dependence of \(\gamma\) in this case simply follows from \(\partial \epsilon_i/\partial \omega\) x \(\cos \theta \sin \theta\), which is maximum at \(\theta=\pm55^\circ\), having maximum growth at \(f=0.58 f_{ce}\) and a high-frequency peak. If this is in fact the drive mechanism, then the presence of such waves implies that electrons with energies above \(m_e v_{te}^2/2 = 1\) keV have been accelerated by the reconnection events. We do not yet have corroboration of observations of electrons with energies in this range but have found fast electrons with energies up to \(\sim 250\) eV using gridded electron energy analyzers, as discussed in Sec. III B. Also, note that toroidal reconnection electric fields near 15 V/m are near the runway field and will directly accelerate electrons from rest to \(2 \times 10^7\) m/s in 10 \(\mu\)s. This also naturally fits with the observed 10–20 \(\mu\)s time delay between the reconnection events and the observation of the peak of these instabilities (see Fig. 13).

The AD instability has been studied heavily in the context of fusion research,\(^{38,41}\) where it was found to have important effects on tokamaks under Ohmic heating (especially during startup at low density) or during rf current drive.\(^{42}\) The large Ohmic electric fields or rf diffusion pulled out a long tail to the electron distribution, which was first unstable to AD instabilities. In the present case, while the AD effect surely can play a role in the instability of energetic electrons, especially when the bump-on-tail is just marginally unstable, it will not produce an instability spectrum with as high a phase velocity as observed (\(\omega/k_i \sim 10^6\) ). The marginal instability of AD drive is set by competition between the drive by high-energy particles at the fast phase velocity \((\omega+i\gamma)/k_i = \omega_c/e/k_i\) and Landau damping on the bulk back at \(\omega/k_i\). Even the longest tails have some energy dependence, weakening the instability drive with higher \(\omega_c/k_i\)—this effect pushes instability to as large a \(k_i\) as possible so that typically \(\omega/k_i\) will be just past the end of the bulk, at \(\omega/k_i \geq 3v_{te}\) or so. Examining the observations here, we have measured \(\omega/k_i = 2 \times 10^7\) m/s \(\approx 10 v_{te}\) or for energies of about 1 keV. The AD resonance is therefore near 30\(v_{te}\) for \(f \approx f_{ce}/2\), roughly \(\varepsilon=10\) keV. It is therefore difficult to imagine the AD instability producing the observed phase-velocity spectrum. Interestingly, in the context of tokamak research, it was proposed that the evolution of an initial AD instability would lead to a bump-on-tail distribution, and thereby drive additional Čerenkov instability; thus the two modes would cooperate in removing the fast electron tail.\(^{41}\) In any case, the observations here, averaged over the reconnection time scale, are dominated by the Čerenkov component.

Finally, Fig. 14 presents calculated growth rates from the Čerenkov drive on a model distribution with a bump-on-tail. The bump-on-tail speed is set so that the parallel phase velocity matches the observed \(v_{pe} = 10 v_{te}\). The beam density is assumed to \(10^{-4}\) times the bulk density, consistent with a beam current of 300 A/m². The peak growth rate is \(\gamma/\omega_c = 10^{-2}\), or about \(10^6\) s⁻¹, which gives strong growth on the 10–20 \(\mu\)s time scale of the reconnection events.

An unresolved point has to do with the large perpendicular group velocity of TG modes. The group velocity satisfies \(v_{gr} = v_{pe}\), so for modes with \(\omega/k_i = v_{ni}\), one finds \(\partial \omega/\partial k_i = v_{pe} \cos \theta \sin \theta\). Therefore, wave packets rapidly transport across the field, crossing the plasma radius faster than a characteristic growth time, which might be estimated as \(\approx 1\) \(\mu\)s to have growth on the reconnection time scale. This is evident in Fig. 14(c), where \(k_{\perp, p} = 2 \times 10^5\) is found, where \(k_{\perp, i}\) is the convective growth rate. Using \(v_p = 200\) \(\mu\)m, this implies a perpendicular growth rate of \(0.1\) m⁻¹, which requires the waves to traverse the plasma numerous times as they grow.

As discussed in the previous section, this point was considered by Papadopoulos and Palmedesso,\(^{34}\) who considered beam-driven modes driven by narrow electron beams. They found that lower-frequency, LH waves would be preferentially driven because their smaller perpendicular group ve-
velocity would keep the modes better confined to the unstable region. Of course, this does not explain the high-frequency waves typically observed in VTF.

A potential resolution is to consider in more detail the transport of the generated waves using ray tracing. These waves may be reflected from the low-density edge of the plasma and therefore confined, allowing for multiple passes of growth over regions of instability. Using the cold electrostatic dispersion, one can solve for $k_\perp$ for fixed $k_\parallel$ and $\omega$, as a function of $\omega_{pe}$ and $\omega_e$. (Simple ray tracing ideas imply that $k_\parallel$ and $\omega$ should be conserved as the waves propagate across the radial cross-section: time scales for density evolution and magnetic field evolution are very long compared to the wave period, and the parallel length scales are much larger than the perpendicular owing to toroidal symmetry and the strong-guide field.) Then,

$$
\left(1 - \frac{\omega^2_{pe}}{\omega^2} \right) k^2_\perp = -\left(1 - \frac{\omega^2_e}{\omega^2} \right) k^2_\parallel. \tag{14}
$$

From this one can show that $k_\perp$ is imaginary and the waves evanescent when $\omega^2_{pe}$ falls below $\omega^2$ (the $P=0$ cutoff). That is, the waves will be reflected back into the plasma from the low-density edge. For instance, at $f = f_{ce}/2$, the waves are reflected at the critical density $n_c = 2 \times 10^{16}$ m$^{-3}$. This evanescent layer is well known from the study of LH heating or current drive on fusion devices. In that case, it is an experimental complication because launched waves must tunnel through the evanescent layer near the antenna before being able to do useful work on the core plasma. In the present case, however, the evanescent layer may confine the waves within the plasma.

Note, the fast perpendicular transport of the Trivelpiece–Gould waves may in part explain why poloidally spaced fluctuation probes typically measure similar powers of TG fluctuations (see, e.g., Fig. 3), in contrast to the much stronger spatial variations in the LH waves.

Finally, we make a few comments on possible electromagnetic effects in this frequency range. Electromagnetic effects may be retained in the dispersion relation through a solution of the full plasma conductivity tensor. In the VTF regime, for $\omega < \omega_{ce}$ and $\omega_{pe} \gg \omega_{ce}$, the electrostatic TG dispersion generalizes to the $R$-wave dispersion,

$$
\omega = \frac{k_\parallel d_e^2}{1 + k^2 d_e^2}. \tag{15}
$$

Here $d_e$ is the electron inertial length or skin-depth, $c/\omega_{pe}$. This dispersion relation includes the TG modes in the limit $kd_e \gg 1$, and the electromagnetic whistler waves ($\omega = \omega_e, k d_e^2$) in the opposite limit. Therefore, the skin-depth is seen to be the critical wavelength at which electromagnetic effects become important. (The boundary between TG modes and whistlers is relevant in other contexts, e.g., in the parametric instability of large amplitude plasma waves. 

In order to conclusively discriminate between electromagnetic and electrostatic regimes, one requires complete measurement of $|k|$. However, while $k_\parallel$ is conclusively measured here, we really have only an estimate for $k_\perp$. Still, if one assumes the electrostatic TG dispersion relation $a$ \textit{priori}, then $k = k_\parallel (\omega_{ce}/\omega)$, which gives approximately constant $k = 600$ m$^{-1}$ over the whole band. This, in combination with $d_e$ of 5 mm at $n = 1 \times 10^{18}$ m$^{-3}$, gives $kd_e > 3$, consistent with electrostatic modes. The electromagnetic effects also limit the maximum frequency; from Eq. (15) one immediately sees that $f < f_{max} = f_{ce} \times k^2 d_e^2 (1 + k^2 d_e^2)$. However, fluctuation power is observed to extend nearly to $f_{ce}$, again implying electrostatic modes with $kd_e > 1$.

In terms of instability mechanisms, the introduction of electromagnetic effects is potentially interesting since it is found that a new branch of beam-driven instability opens up at lower frequency. We have compared the growth rates of the two branches by investigating beam-driven instability using the full plasma dispersion tensor. We have found, however, that the higher-frequency TG branch always has the larger growth rate and, moreover, that the high-frequency peak cannot be explained by the whistler branch of the instability.

Overall, the high-frequency modes are well described as TG modes (potentially including weak electromagnetic effects) driven by velocity space instability of high-energy electrons. Simple growth rate calculations predict maximum growth near $f_{ce}/2$. The observation of approximately constant $\omega/k_\parallel$ is consistent with Čerenkov drive by a positive slope in the distribution of high-energy particles. In this case, the fluctuation observations are the first diagnostic available in VTF to imply the existence of runaway electrons ($E = 1$ keV) produced by the reconnection events.

VI. DISCUSSION AND CONCLUSIONS

We have presented a study of electrostatic plasma fluctuations driven during magnetic reconnection events in the low-$\beta$, strong-guide magnetic field regime of VTF. The observed strong time dependence of reconnection rates in VTF is useful for experimentally determining which are the most important mechanisms enabling fast reconnection.

We have found strong fluctuations driven during the reconnection events. Modes were studied based on frequency spectra and with wavelength measurements using correlation analysis between multiple probe tips. We have found two broad regimes of modes: LH modes and high-frequency TG modes. In general, most of the fluctuation power is in the LH waves, except typically 20–30 $\mu$s after the reconnection event completes the LH waves have disappeared and a set of TG waves is still present and dominates the spectrum.

LH waves are identified in the lower-frequency portion of the spectrum based on frequency ($f \approx f_{LH} = (f_{ce} d_e)^{1/2}$) and wavelength measurements from multiprobe correlation. While the correlation measurements are noisy, they suffice to establish two points of agreement with LH waves: the wavevectors are highly anisotropic, with $k_\parallel \ll k_\perp$, and $\omega/k_\parallel$ is of the order of the electron thermal speed or larger. The latter point excludes ion-acoustic waves, even though those also reside in the same frequency range; it is possible that ion-acoustic waves do exist, but the spectrum is dominated by a stronger set of LH waves. The perpendicular wave-vector appears large, $k_\perp \rho_e \approx 1$, as implied by the fast decorrelation

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of probe signals separated by a small distance (1 mm) in the perpendicular direction.

Multiple instabilities can be shown to drive LH waves in the plasma. The LHDI, in particular, has long been considered an important candidate for producing anomalous resistivity leading to enhanced reconnection. Its operation here has not been excluded but is more speculative at present. Instead, we propose that the LH waves in VTF are driven by the filamented population of fast electrons. Probe-probe correlation measurements demonstrate that the observed LH modes propagate in the electron parallel drift direction with superthermal phase velocity, suggesting a connection to energized electrons. Furthermore, measurements with electron energy analyzers find that the reconnection events energize electrons, and measurements with a multicolonlector energy analyzer show that this population is not uniform even over the small (1 cm) scale of the probe. Complementing the theory of Papadopoulos and Palmadesso, who showed that narrow beamlets preferentially drive LH waves as opposed to higher-frequency TG modes, here we have also shown that the spatial gradients in the fast electron population can also drive a LH instability; this instability appears to be able to operate given the observed gradients in fast electrons, i.e., one does not have to invoke beamlike components of the fast electron population to get instability.

The second regime is a broad collection of high-frequency modes, with frequencies up to the electron cyclotron frequency. These modes agree well with expectations for TG modes, electrostatic plasma fluctuations with dispersion relation $\omega = \omega_{ce} \cos \theta$ in the parameter regime $\omega_{pe} \gg \omega_{ce}$. First, the spectrum cuts off to the noise floor at $\omega_{ce}$, consistent with the dispersion relation. Second, based on correlation measurements, it is found that the typical wavevectors satisfy $k d_e > 1$, implying that the modes are primarily electrostatic. (Here $d_e$ is the electron skin-depth, $c/\omega_{pe}$, roughly 5 mm in VTF.)

Correlation measurements have also measured the parallel phase speed of these modes, which is found to be fast ($\nu_{ph} \sim 10 \nu_{te}$) and roughly uniform over the spectrum. This fast, uniform phase velocity suggests Čerenkov instability (inverse Landau damping) due to a bump-on-tail. While these high-frequency modes could also in principle be driven from the AD effect, AD instability is inconsistent with the fast, uniform-phase velocity. Instead, AD instability should drive modes with phase velocities closer to the bulk, near $\omega / k_j \sim \nu_{te}$, rather than the observed $\omega / k_j \sim \nu_{te}$. The existence of modes at this phase velocity suggests that the reconnection events create a population of highly energetic runaway electrons, with energies of $m_e(\omega / k_j)^2/2 \sim 1 \text{ keV}$. These fluctuation measurements are the first diagnostic on VTF to find such a fast electron population.

The anomalous resistivity due to these modes has also been estimated. Here, the LH waves were discussed due to stronger fluctuations in this range. Because the nonlinear term $\langle \vec{n} \vec{E} \rangle$ was not measured, quasilinear theory was used to estimate this quantity from the measured $\langle \delta n \rangle$. The quasilinear theory yields a coefficient of proportionality between the two; here the coefficient was estimated using the theory for the LH instability driven by a temperature gradient. Using this coefficient, the anomalous resistivity from the LH waves was estimated to be too small, by about a factor of 50–100, for typical wave powers to account for the typical inductive portion of the electric field $(1/R \times d\nu / dt)$ during the reconnection events.

Overall it is hard to draw firm conclusions from this estimate based on the limitations of the theory, uncertainties in calibration of the Langmuir probes, and wide shot-to-shot and probe-to-probe variations in measured power. Furthermore, detailed experimental measurements of exactly where in the reconnection region the frozen-in law is broken were not available for these experiments. Typically, $\nu \times B$ flows will balance the reconnection electric field over much of the poloidal cross-section, except over a finite area near the x line where nonideal effects must be invoked. One can envision that this region could be localized toroidally as well. Therefore, it is possible that the fluctuation measurements have systematically “missed” the volume in the experiment where the frozen-in law is broken. Regarding this point, note, however, that the fluctuation powers reported here are consistent with observations over a larger number of experimental discharges, obtained using a few different iterations of the fast Langmuir probe, and sampling the experimental accessibility parameter space of toroidal magnetic field strength, discharge timing, plasma densities, and probe location compared to the x line.

The small estimated anomalous resistivity and wide variations in observed fluctuation power both in space and time make it difficult to conclude that the observed modes supply anomalous resistivity critical to the reconnection dynamics on VTF. Nonetheless, based on the present experimental understanding of the location of and extent to which the frozen-in law is broken, an incomplete model connecting fluctuation power to resistivity, and standard difficulties calibrating Langmuir probes, it is not possible to definitively rule these waves out.

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APPENDIX: LH WAVES DRIVEN BY THE ELECTRON TEMPERATURE GRADIENT

This appendix summarizes the theory of LH instabilities driven by the gradient of the parallel component of electron temperature. We review the instability mechanisms and calculate stability boundaries and growth rates for parameters relevant to VTF. While LH waves based on density gradients (the “LHDI”) or strong perpendicular flow (“MTSI”) have been extensively studied, modes due to temperature gradients appear to have received much less attention. A local theory is presented here; a nonlocal theory of the mode excitation in a strong gradient region would make for interesting future work.
This mode derives from the warm, electrostatic plasma dispersion relation including gradients.\textsuperscript{35} This is written schematically as
\[ 0 = 1 + \epsilon^{(e)} + \epsilon^{(i)}, \quad (A1) \]
where \( \epsilon^{(e)} \) and \( \epsilon^{(i)} \) are the electron and ion contributions to the dielectric.

For calculating the dielectric, the ions are treated as unmagnetized because the growth rates are larger than the ion gyrofrequency and \( kp_i \gg 1 \), so straight-line orbits are used. (This is found to be the correct treatment as long as the growth rates are larger than \( \omega_{ci} \).\textsuperscript{35}) The electrons are treated by full-orbit integrals, allowing \( k_z \rho_z \sim 1 \), with gradients included by the “local” or slab approximation, where it is assumed that perpendicular wavelengths are much smaller than gradient scale lengths, e.g., \( L_{Te} \approx T_i/(dT_i/dx) \). The latter is therefore regarded as approximately constant over the region where the mode is localized.

The gradients are assumed to lie in the \( \hat{x} \)-direction, and the drifts (including diamagnetic drifts) lie in the \( \hat{y} \)-direction. The magnetic field is in the \( \hat{z} \)-direction. Modes resonant with drifting species therefore have finite \( k_z \). Because growth rates maximize when \( k_z \sim k \), the two will be used largely interchangeably in this discussion. In practice, a finite \( k_z \) will be required so that the wave packet can be localized within the gradient regions. However, theory typically finds that \( k_z \sim 1/\rho_i \) at maximum growth, much larger than the \( k \) required by this wave packet criterion, as typical gradient scales are near \( \rho_i \) in these theories. Finally, for the theory here, \( k \) will be taken as positive, and therefore \( \omega \) is signed, with negative \( \omega \) indicating a mode propagating in the negative direction. In this convention, the sign of \( \omega \) will match that of the drift frequency (e.g., \( \omega_{ci} \)) if the mode propagates in the diamagnetic direction of the associated species.

As mentioned above, the ions are considered to be unmagnetized and are treated by straight-line orbits. For instability driven by the electron temperature gradient, the electron and therefore ion densities are uniform. Therefore, the ions are treated simply at rest, and
\[ \epsilon^{(i)} = \frac{1}{k^2 \lambda_{De}^2} \left[ 1 + \frac{\omega}{k\nu_i} \right] \left( \frac{\omega}{k\nu_i} \right) \Gamma_0(z). \quad (A2) \]
Here \( Z \) is the well-known plasma dispersion function,\textsuperscript{46} and \( \lambda_{D_i} \) is the Debye length evaluated with the ion temperature.

Next, the electron contribution is calculated with the standard orbit integrals, including gradients calculated within the local wave approximation.\textsuperscript{35} This yields
\[ \epsilon^{(e)} = \frac{1}{k^2 \lambda_{De}^2} \left[ 1 + \frac{\omega}{k\nu_{Te}} \right] \left( \frac{\omega}{k\nu_{Te}} \right) \Gamma_0(z). \quad (A3) \]
where \( \mathcal{L} \) is an operator which contains the gradient effects,
\[ \mathcal{L} = 1 + \frac{k_z}{m_{i} \omega_{i} \epsilon_{e}} \left[ \frac{1}{n} \frac{\partial n}{\partial x} + \frac{\partial T}{\partial x} \frac{\partial}{\partial T} \right]. \quad (A4) \]
Here the expression differs in a sign from Ref. 35; there the cyclotron frequencies are kept as signed algebraic quantities. In the discussion here, \( \omega_{i} \) is the unsigned electron cyclotron frequency. \( \Gamma_0(z) = I_0(z)e^{-z} \), where \( z = k_z^2 T_e^2/(m_e \omega_{ce}^2) \), and \( I_0 \) is the modified Bessel function of the first kind. This is the well-known Bessel function term that contains the finite gyroradius effects for the electrons. Finally, compared with the full expression here, we have assumed \( \omega \ll \omega_{ce} \) and kept only the \( n=0 \) term.

Finally, keeping the gradient of parallel temperature, the electron dielectric is\textsuperscript{35}
\[ \epsilon^{(e)} = \frac{1}{k^2 \lambda_{De}^2} \times \left[ 1 + \left( \xi_z Z - \frac{\omega_{i} T}{\omega} \left( \xi_e + \left( \xi_z - \frac{1}{2} \xi T \right) \right) \Gamma_0 \right] \right]. \quad (A5) \]
Here \( \omega_{i} T = -k_z T_i/eBL_{Te} \) is the corresponding drift frequency for the electron temperature gradient, whose scale length is \( L_{Te} \). Also, \( Z(\xi_e) \) is abbreviated as simply \( Z \). The \( Z \)-function recursion relationship \( Z' = -2(1+\xi Z) \) was also used to simplify the resulting expression. The expression above assumes that \( T_{||} \approx T_{\perp} \).

Armed with the dispersion relation [Eq. (A5) plus Eq. (A2)], we can proceed to calculate growth rates. Figure 10, presented in the main text, shows a calculation of instability due to temperature gradient. The dispersion relation is solved for a range of \( k_{||} \) and \( k_{\perp} \), and Fig. 10(a) plots contours of the growth rate \( \gamma \), where it is positive, over \( (k_{||}, k_{\perp}) \) space (assuming \( k_{\perp} = k_i \)). For this plot we have used \( L_{Te} = 100 \rho_{i} \approx 2 \) cm and found maximum growth rates at \( k_{||} \rho_{i} = 0.8 \) and \( k_{\perp} \rho_{i} = 3 \times 10^{-2} \). Thus, \( k_{||} \rho_{i} = 4 \times 10^{-3} \approx (m_{e}/m_i)^{1/2} \), which is exactly in the LH regime. At each \( (k_{||}, k_{\perp}) \), \( \omega \) is found as well, and Fig. 10(b) plots the maximum growth rate associated with each frequency; peak growth rates are found for real \( \omega \) near the LH frequency. For completeness, Fig. 10(c) plots the \( k_{||} \) and \( k_{\perp} \) associated with the (\( \omega, \gamma \)) pairs from Fig. 10(b). These trace out the heavy black curve superimposed on the contours in Fig. 10(a).

Figure 15 presents some simple parametric scans over the temperature scale length \( L_{Te} \) (a) and electron-ion temperature ratio \( T_i/T_e \) (b). The temperature scale length is crucial, and the peak growth rate increases sharply with decreasing \( L_{Te} \). In fact, gradient scales \( L_{Te} = \rho_i \approx 200 \rho_{i} \) will be required for strong growth on the 10 \( \mu s \) time scale of the reconnection events (i.e., \( \gamma \sim 0.1 \omega_{Te} \sim 5 \times 10^6 \) s\(^{-1}\)). The ion temperature plays a more minor role in the growth rate. One effect is that, at \( T_i = 0 \), a broad band of higher-frequency modes can be excited in addition to the peak instability near \( \omega_{Te} \). These are actually perpendicular ion-acoustic waves;\textsuperscript{35} they are found to be stabilized, however, when the ions acquire a finite temperature.

Finally, for insight into the overall analytic structure of the instability, one can take the cold plasma limit of the dispersion relation and find the cubic equation (similar to that derived in Ref. 35),
\[ 0 \approx 1 - \frac{\omega_{ce}^2 \cos^2 \theta}{\omega^2} \left( 1 - \frac{\omega_{i} T}{\omega} \right) - \frac{\omega_{i}^2}{\omega^2}. \quad (A6) \]
(Here we have also taken the limits \( \omega_{pe}^2/\omega_{ce}^2 \rightarrow \infty \) and \( \sin^2 \theta \rightarrow 1 \) but left \( \cos^2 \theta \) intact.) First, if \( \omega_{i} T = 0 \), then the cold LH...
dispersion relation $\omega^2 = \omega_{\text{LH}}^2 + \omega_c^2 \cos^2 \theta$ is recovered. Next, by solving the cubic equation, one can find that $[\omega_{\text{LH}}] > \omega_{\text{LH}}$ for instability. Kinetic theory is required to set the limit on $k_\perp < 1/\rho_e$; this additional fact limits $\omega_{\text{LH}}$ and determines the temperature length scale required for instability. Finally, Eq. (A6) shows that unstable modes have the sign of $\omega$ match that of $\omega_{\text{LH}}$, confirming that the instability propagates in the electron diamagnetic direction.