Integrated modeling for design of lightweight, active mirrors

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Integrated modeling for design of lightweight, active mirrors

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Integrated modeling for design of lightweight, active mirrors

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Abstract. Lightweight, active, silicon carbide mirrors have the potential to enable larger primary aperture, space-based optical systems, hence improving the resolution and sensitivity of such systems. However, due to the lack of design heritage, the best mirror designs are not yet known. Therefore, an integrated model of the lightweight mirrors is created in order to explore the design space. The model determines the achievable radius of curvature change within wavefront error limits, the peak launch stress, and the mass of a mirror segment. However, designing a mirror to meet any of these individual objectives results in a system that performs poorly in terms of the other objectives. Therefore, a full trade space analysis is run to determine the portions of the design space that best balance the trade-offs between metrics. These results are used to determine designs that perform well with respect to specific missions and can be used for future mirror designs. © 2011 Society of Photo-Optical Instrumentation Engineers (SPIE). [DOI: 10.1117/1.3592520]

Subject terms: integrated model; lightweight mirrors; space telescope; active optics.

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1 Introduction

The next generation of space-based imaging systems will push the limits of current technology and design methodologies, while achieving performance that has previously been impossible. Whether the goals are Earth imaging systems with better ground resolution and located in higher orbits or astronomical telescopes looking back into time, the desired improvement in optical resolution and sensitivity can be obtained through the use of larger primary apertures. However, larger apertures bring about a number of design challenges including mass, volume, and flexibility. Mirrors larger than about three meters in diameter encounter packaging constraints due to the size and volume limits of the launch vehicle shroud. Also, mass-to-orbit is limited and extremely expensive, requiring the areal density or mass per unit area of the mirrors to decrease as the diameter increases in order to maintain an acceptable launch mass. Furthermore, the large size and the lower mass combine to significantly increase the flexibility of the mirrors, lowering flexible mode frequencies and making them more susceptible to static and dynamic distortion, so maintaining optical tolerances across the mirror surface becomes increasingly difficult. While these challenges are immense, they can be dealt with through the use of lightweight, actuated, segmented primary mirrors. Instead of the traditional monolithic design, the primary aperture is made up of multiple smaller mirror segments that are easier to manufacture and can deploy from a stowed configuration that will fit within a launch vehicle. Furthermore, the mirrors can be rib-stiffened and made of silicon carbide (SiC) to achieve low mass while maintaining adequate stiffness, and the ribs can contain embedded actuators to control the shape of the mirror to optical tolerances. The size of the achievable aperture, and hence the potential imaging resolution, is very promising.1–3

As with many promising new technology developments, lightweight active mirror segments solve one problem (aperture size), but introduce a new set of challenges that must be addressed. One issue that arises is the ability to design the mirror to accommodate multiple environments and disturbance sources in an efficient way. For example, launch survival is a key challenge in the mirror design.4 Launch is an extremely harsh environment and silicon carbide is brittle and could break at low areal densities when exposed to the vibrations and acoustics from launch. Yet it is imperative that these fragile optical components arrive on-orbit undamaged. Once on-orbit, the mirror design must also meet tight optical performance requirements in the face of static and dynamic disturbances during operation. However, designing a mirror to best survive launch would yield a mirror that is substantially different than one that performs well with respect to on-orbit metrics. Therefore, the mirror structure and control system design must be carefully analyzed and optimized in an integrated fashion in order to advance the state of the art in active mirror design.

This paper focuses on the design of lightweight mirrors, specifically with respect to the launch and on-orbit correction, resulting in an integrated design methodology that can be used for technology optimization and advancement. First, the integrated modeling methodology is discussed, followed by the specific launch and on-orbit mirror models. Next, a trade space analysis is presented, along with the best mirror designs, in terms of the metrics of interest. Finally, conclusions are presented illustrating the advantages of using integrated modeling for lightweight active mirror design.

2 Integrated Modeling

Traditionally, the design of space-based opto-mechanical systems occurs by choosing a point design very early in the design life cycle.5 This design is typically chosen based on heritage, engineering judgment, basic design principles, or

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very simple trade studies. While this methodology can work well for designs that are based on heritage systems, when there are major design changes or technology breakthroughs, the engineering experience and past data are lacking, and the probability of initially picking a point design that meets all of the requirements drastically decreases. This is particularly apparent in technology development programs where there is very little knowledge about the performance of the system.

Lightweight active SiC mirrors are complex systems, and the best designs are as of yet unknown. Therefore, an integrated modeling methodology is used to explore the trade space of designs. The integrated modeling considered here uses auto-generating models based on parameterized inputs, considers multiple disciplines, and is adaptable and upgradeable due to a modular modeling environment. This enables trade space exploration, where many different designs, defined by different values for the input parameters, can be analyzed with the integrated model to determine the desired performance outputs. These different designs can be compared in terms of the performance metrics to determine which designs or families of designs are best for the particular metric of interest, or to determine the trade-offs between performance metrics. Also, by using the single mirror model, the interactions between disciplines can be captured and the design can proceed without iterating between different disciplinary models to converge on a design.

The integrated modeling methodology is particularly useful for the design of lightweight active mirrors. The mirrors represent a significant deviation from traditional monolithic glass telescope mirror designs. The high degree of actuation, and specifically surface-parallel actuation, is a substantial change. The lightweight segmented mirrors necessitate the actuation to maintain the shape of the optical surface. Surface-parallel actuation is used because it does not require a massive reaction structure on which to react. Additionally, prototyping is expensive and time-consuming, so determining the best design from a model can save time and money in addition to mapping out the design space. These factors make lightweight actuated SiC mirrors a good candidate for integrated modeling.

3 Mirror Model

Section 1 discussed achieving large aperture systems through segmented apertures composed of multiple smaller mirror segments. The mirror segments can be combined in a number of ways to obtain primary mirrors with varying overall diameters and numbers of mirror segments. As an example, Fig. 1 shows primary mirrors made up of differing numbers of mirror segments. The exact number and layout of segments will be mission specific. In this paper, only a single mirror segment is modeled. A single segment model limits the size and the complexity of the model, as well as generalizes the result to be applicable regardless of the exact segment layout and support structure. The elimination of the other mirror segments and spacecraft results in a worst case analysis for the launch vibration analysis (Sec. 5). However, the rms wavefront error and the radius of curvature change due to actuation, which are the on-orbit correction metrics utilized in this study (Sec. 4), can be defined with respect to the single mirror segment, and are thus unaffected by the single segment assumption. Therefore, for the parameters of interest, the best segment designs will also translate to the best full mirror designs. In a full mirror consisting of multiple segments, the alignment between segments must also be accounted for, though this is outside of the scope of this study.

Therefore, the mirror model here focuses on the design of a single silicon carbide, rib-stiffened mirror segment containing embedded surface-parallel piezoelectric actuators. Figure 2 shows the back surface of the mirror model, including the rib structure and actuators. The large triangles show the rib structure, and the bars on the ribs are the embedded actuators. The mirror segment is mounted on three kinematic bipod mounts, which are connected to a rigid back structure (not shown).

As discussed in Sec. 2, the mirror model is generated using parameterized inputs. The key parameters considered in this study are as follows:

- Segment size: point-to-point diameter of the hexagonal mirror segment.
- F#: focal length divided by the diameter; defines the curvature of the mirror. Computed assuming a six-segment mirror system (shown on the left side of Fig. 1).
- Areal density: mass per reflecting area of the silicon carbide.
- Number of rib rings: number of concentric rings of primary ribs in the segment. Ribs are filled in as defined...
by the number of rings (see Fig. 3). This also defines the number of actuators as one per cell side.

- Rib aspect ratio: height divided by thickness of the ribs.
- Face sheet mass fraction: percentage of SiC mass in the face sheet. The remainder of the SiC mass is in the ribs.
- Actuator length: length of the actuator, including tabs and spacers. Represents the distance between the moment on either side of the actuator, rather than the length of the piezoelectric material.
- Bipod reinforcement: factor to taper ribs near reinforced bipod connection points.
- Damping ratio: modal damping ratio

In this paper, a number of those inputs are held constant to compare similar systems. All mirrors are hexagonal measuring 1.2 m from point-to-point. The curvature of the mirror is defined assuming a six-segment, parabolic mirror (shown on the left in Fig. 1) with an F# of 2. This results in a radius of curvature of 12.5 m. Additionally, the damping ratio is fixed at 0.01. The baseline values and ranges for the remaining parameters can be seen in Table 1. The mirror and actuator material properties are also parameterized, though they are set to be those of silicon carbide and PMN, respectively. The baseline material properties are defined in Table 2.

The model of the mirror segment utilizes multiple disciplines, including structural finite element modeling, optical performance modeling, control system analysis, vibroacoustic disturbance analysis, and on-orbit disturbance and actuation analysis. The model is physics-based and includes enough detail to obtain the performance of the system without heritage-based empirical models.

The mirror model is based in MATLAB. The modeling process is illustrated in Fig. 4. MATLAB is used to define the input parameters and create the mirror structural model, including geometry, mounting configuration, and material properties. This is formatted as a finite element model that can be used with the MSC.NASTRAN finite element solver. Two different types of analyses are considered: a quasi-static analysis for on-orbit correction and a dynamic analysis for launch, which are discussed in Secs. 4 and 5, respectively. The quasi-static analysis uses static loading to determine the nodal displacements due to actuation. The dynamic analysis uses a normal modes analysis to determine the frequencies and mode shapes of the mirror, which are later transformed into stresses in the MATLAB state-space model. The NASTRAN results are brought back into MATLAB, where control systems are added and the analysis routines are run, including the optical performance and disturbance analysis, to determine the desired performance metrics. The control systems and performance metrics vary by the type of analysis and are discussed in detail in Secs. 4 and 5.

### 4 On-orbit Correction

On-orbit optical performance is the principal function of the primary mirror. The mirror must reflect light and optically perform well. On-orbit performance can be described in a variety of ways. Here, only metrics that are directly influenced by the mirror parameters of interest will be considered. Therefore, issues such as spacecraft jitter, that will affect the optical performance regardless of the primary mirror design, are ignored. The main on-orbit performance metrics of interest are correctability and high spatial frequency wavefront error. Correctability refers to the size of the achievable radius of curvature change with the embedded actuators, while wavefront error refers to the distortion of the corrected mirror surface with respect to the ideal shape.

Low order shape correctability is desirable for a number of reasons, including thermal distortion, manufacturing imperfections, and optical prescription changes. Due to the relatively high coefficient of thermal expansion (CTE) and high thermal conductivity of silicon carbide, thermal variations

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**Table 1** Baseline mirror properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Baseline value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment size</td>
<td>m</td>
<td>1.2</td>
<td>fixed</td>
</tr>
<tr>
<td>F#</td>
<td></td>
<td>2.0</td>
<td>fixed</td>
</tr>
<tr>
<td>SiC areal density</td>
<td>kg/m²</td>
<td>8</td>
<td>2.5–15</td>
</tr>
<tr>
<td>Number of rib rings</td>
<td>–</td>
<td>4</td>
<td>3–6</td>
</tr>
<tr>
<td>Rib aspect ratio</td>
<td>–</td>
<td>25</td>
<td>10–80</td>
</tr>
<tr>
<td>Face sheet mass fraction</td>
<td>–</td>
<td>0.72</td>
<td>0.3–0.9</td>
</tr>
<tr>
<td>Actuator length</td>
<td>cm</td>
<td>2.5</td>
<td>1–10</td>
</tr>
<tr>
<td>Bipod reinforcement</td>
<td>–</td>
<td>2</td>
<td>1–4</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>–</td>
<td>0.01</td>
<td>fixed</td>
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</tbody>
</table>

**Table 2** Baseline mirror material properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>SiC</th>
<th>Actuator</th>
</tr>
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<tr>
<td>Young's modulus</td>
<td>GPa</td>
<td>375</td>
<td>93</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>–</td>
<td>0.17</td>
<td>0.3</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>3200</td>
<td>7650</td>
</tr>
<tr>
<td>Yield stress</td>
<td>MPa</td>
<td>600</td>
<td>140</td>
</tr>
<tr>
<td>Coefficient of thermal expansion</td>
<td>m/deg</td>
<td>$4.5 \times 10^{-6}$</td>
<td>$3.6 \times 10^{-10}$</td>
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result in low order deformations in the mirror.\textsuperscript{6} Therefore, either tight thermal control or the ability to correct for the thermally induced shape changes are necessary. Also, issues in the manufacturing process result in variations in the radii of curvature between segments. In order to use multiple mirror segments as a single primary aperture, the curvatures of the segments must be matched using a low order correction. Third, there is a desire to have the ability to change the optical prescription of the mirror while on-orbit or from mission to mission so that the same mirror design can be used for multiple systems. A large range of correctability improves all of these issues and allows the mirror to be more functional. However, there are a limited number of actuators with finite stroke lengths, so there is a limit to the achievable shape change.

The low order shape change is accomplished through the use of discrete embedded piezoelectric actuators. The discrete nature of the actuators, acting to induce a continuous shape, results in a high spatial frequency dimpling on the surface, as seen in Fig. 5. This dimpling is above the spatial frequency of the actuators and is thus uncorrectable. The dimpling causes distortions of the mirror surface, which are quantified as wavefront error. As mentioned in Sec. 3, there are additional contributors to overall wavefront error, including segment mismatch, which are not considered here.

The disturbances of interest act on a long time scale and can be considered quasi-static. In other words, the control inputs change, but at long intervals that do not require active control systems. This results in much simpler control algorithms and no stability concerns.

4.1 Quasi-static On-orbit Model

The quasi-static on-orbit model uses the finite element mirror model to determine the actuator influence functions, which describe the effect of each actuator on the mirror surface. The actuator influence functions are created using the MSC NASTAN finite element solver. Each actuator is given a unit actuation command, and the surface normal ($z$) displacements of all nodes on the surface of the mirror are computed. The influence function for each actuator is then stored in a column of the influence function matrix, $\Sigma$. The symmetry of the mirror segment is exploited in order to minimize the computational burden of computing the actuator influence functions. More details on the actuator influence function and computation can be found in Gray.\textsuperscript{7} An example influence function can be seen in Fig. 6, where the actuator location is shown in white, and the contour represents the influence function across the mirror. The shape and size of the influence function depends on both the location within the mirror and the mirror parameters.

In reality, the actuation is induced through a voltage command. The piezoelectric equation in the along-axis direction is given in\textsuperscript{8–10}

\[
S = d_{33} \frac{V}{L} + s_{33} E T,
\]

where $S$ is the mechanical strain, $d_{33}$ is the piezoelectric constant, $V$ is the voltage, $L$ is the length of the piezo, $s_{33}$ is the compliance at short circuit, $T$ is the vector of material stress, and $3$ corresponds to the along-axis direction.
Applying a voltage to the piezo induces both stress and strain in the system. Therefore, the actuator cannot be considered as a displacement actuator, which induces only strain, or a force actuator, which induces only stress. NASTRAN does not directly support piezoelectric elements and actuation. Instead, the actuation is achieved in NASTRAN through the use of temperature changes. The actuators are thermally isolated from the structure in the finite element model through the use of rigid elements. By applying a temperature change to the actuator, the coefficient of thermal expansion causes it to change length. However, the SiC structure resists the deformation from the actuator, causing a surface error of a given length to result in an optical path length error of twice the surface error.

The WFE limit is assumed to be $\lambda/20$, where $\lambda$ is the wavelength of light. Assuming visible wave lengths, $\lambda \approx 600$ nm, leading to a 30 nm wavefront error limit, and hence a 15 nm surface dimpling error limit. Therefore, the correctability is then the maximum change in RoC while maintaining the induced surface dimpling error less than 15 nm rms.

### 4.3 On-orbit Results

Results for the on-orbit correction are presented for similar mirror segments (i.e., ones with the same diameter and F#, as discussed in Sec. 3). Note that the output values will differ for various diameters and F#s, but the trends will be the same. Figure 7 shows the correctability as a function of the number of rib rings, which are the number of concentric hexagonal rings of ribs in the segment (Fig. 3), while holding all other parameters constant at their baseline values (Table 1). Note that increasing the number of rib rings increases the number of actuators, which are defined by the rib structure. The correctability significantly increases with more rib rings; more actuators are better able to approximate the continuous surface actuation, leading to larger RoC changes before the WFE limit is met.

Figure 8 shows the correctability as a function of actuator length. The correctability increases with longer actuators. This is again because longer actuators provide longer moments and can better approximate the desired continuous shape.

Finally, Fig. 9 shows the correctability as a function of the SiC areal density. The correctability increases with larger areal densities because increasing the areal density helps to...
broaden the influence functions, so that the actuators create a more global shape change, and dimpling is minimized. However, as areal density continues to increase, the actuators have less control authority and will saturate before the wavefront error limit is reached. Single-axis trades such as these could be created for a variety of other metrics, including rib structure (aspect ratio and height), face sheet structure, diameter, etc., to map out the design space with respect to correctability. The resulting best designs for correctability have many long actuators, thick face sheets, and medium to high areal densities. However, it is also important to take into account the interaction between parameters as well as other mirror design issues such as launch survival.

5 Launch

In addition to performing well in terms of on-orbit correction, the mirrors must also survive launch and arrive on-orbit undamaged. As will be shown, many mirror designs are quite close to launch stress limits. Therefore, it is important to verify that the mirror designs will survive launch and to understand the launch stress implications of various design decisions.

Traditional launch load modeling is performed either with a very simple quasi-static model, or with a full coupled loads analysis on the spacecraft-launch vehicle system. However, given the proximity of the launch stresses to limitations, it is desirable to have the ability to analyze launch in the early design phases. Therefore, a dynamic, state-space model of the mirror subjected to vibroacoustic launch loads is developed. While it is a deviation from traditional launch analysis methods, this method allows trade space exploration early in the design process, as well as the possible addition of launch load alleviation.

5.1 Dynamic Launch Model

Launch is a highly dynamic process and thus necessitates a dynamic model. The model used here is a state-space model derived from finite element normal modes analysis. The inputs to the model are random vibration and acoustic spectra from the launch vehicle and the outputs of the model are stresses in the elements, resulting in a stress distribution across the mirror. The total input disturbance is 6.2 g for the random vibrations and 145.2 dB for the acoustics. The input spectra utilized are representative of typical launch vehicle disturbance spectra. The vibration spectra are based on the space shuttle acceptance spectra, and the acoustic spectrum is a qualification spectrum that is comparable to the Delta IV spectrum. The resulting peak stresses in both the SiC and actuators is compared to stress limits to determine the probability of launch survival. The stresses are determined using a frequency domain steady-state disturbance analysis where the launch spectra are applied to the mirror model. More details about the model and launch disturbances can be found in Cohan.

The stress outputs from this model are validated to the extent possible. They are compared to available data from multiple similar systems and match within 10% in all cases. The parametric nature of the model allows it to be validated against multiple sets of test data since the model can easily be made to represent the test setup.

5.2 Launch Metrics

The primary metric from the launch model is Von-Mises stress. The model output stress value is a 1-σ value. In order to obtain the desired level of conservatism, the 1-σ model output is compared to two limit levels, representing two launch survival certainty levels: 3-σ, or 99.7% certainty, and 6-σ, or 99,999,999,998% certainty. If the model value is below the given certainty level, the mirror will survive launch with at least that level of confidence.

In addition to minimizing the peak stress, one would also like to minimize the mass of the mirror, as launch mass is limited and extremely expensive. Therefore, the second launch metric is mirror mass.

5.3 Launch Results

If one considers only launch stress, single-axis trades can be performed where, as in Sec. 4.3, parameters are individually varied to assess the effect of that particular parameter on the performance output. All parameters other than the parameter of interest are held constant at their baseline values. Here,
While these single-axis trade results can give important information as to the effect of individual design parameters, they lack information about the complete design problem. First, they do not illustrate the interdependence of the various design parameters. Many parameters are coupled, so looking at the effect of a single parameter variation, while holding all other parameters constant, misses many important parameter interactions. Additionally, it is clear that the desire to maximize on-orbit correction, minimize peak launch stress, and minimize mass lead to vastly different results. Therefore, the analyses and metrics must be simultaneously considered, using the full integrated model to determine an overall good design.

6 Trade Space Analysis

Due to the aforementioned issues of coupled parameters and conflicting performance metrics, a full trade space of designs is examined in order to find the best families of mirror designs. Latin hypercube sampling is used to define 4000 distinct mirror designs. Latin hypercube sampling is a design of experiments sampling technique where the parameter ranges are divided into bins and the samples are chosen such that there is exactly one sample in each row or column of the \( n \)-dimensional hypercube, where \( n \) is the number of the parameters. The bins are based on uniform probabilities across the parameter ranges (shown in Table 1). This method ensures uniform coverage of the parameter space. However, it does not ensure full coverage of the output trade space, and it is possible to miss portions of the design space. However, the large number of samples relative to the number of parameters helps to more fully cover the design space, while maintaining computational efficiency.

In this trade space, six mirror design parameters are varied: number of rib rings, SiC areal density, actuator length, rib aspect ratio, percentage of mass in the face sheet, and mounting location taper ratio (bipod reinforcement). Each of the 4000 designs is determined by specifying a unique set of the six input parameters. The integrated model (Fig. 4) is used to determine the correctability, mass, and peak stress of each design so that they can be compared. The correctability is defined using the maximum RoC change while maintaining the WFE within the \( \lambda/20 \) limit, as discussed in Sec. 4.2. Additionally, the launch stress is limited; any designs that exceed the peak launch stress at the specified certainty level are eliminated from the trade space. This leaves two independent metrics with which to compare each design: mass and correctability. Note that the results will vary based on the desired launch survival certainty level, so results are presented showing multiple levels.

6.1 Integrated Model Results

By limiting the peak launch stress and wavefront error, the designs can be compared using mass and correctability. One would like to minimize mass while maximizing correctability. The first set of results uses the more conservative, 6-\( \sigma \) launch survival certainty level. Since it is imperative that the mirror survive launch, and there is significant uncertainty in the acoustic disturbance levels, the more conservative limit is assumed. These trade space results can be seen in Fig. 12. Each point represents a distinct mirror design, as specified by a set of input parameters. The best designs are in the upper left hand corner, maximizing correctability while minimizing...
mass. Notice that there is no clear best design. Rather, there is a trade-off of designs, demarcated by the nondominated designs. The nondominated designs are those designs for which there exist no designs that are better in all performance metrics. These nondominated designs approximate the Pareto front, or the front where one must sacrifice performance in terms of one metric in order to gain performance in terms of another metric. In order to better understand the Pareto designs and the trade-offs involved, the results can be plotted by the different parameters, as will be done in this section. Additionally, Sec. 6.3 discusses the parameters that make up each of the Pareto optimal designs in Fig. 12.

In order to distinguish the trends in the data, Fig. 13 shows the same trade space as in Fig. 12, but with each of the designs distinguished by the number of ribs in the mirror segment. While utilizing more ribs improves correctability, as was shown in Sec. 4.3, it also necessitates higher mass, primarily in order to meet the stress limits.

Figure 14 shows the trade space differentiated by the areal density. As expected, the mass is very dependent on the areal density. However, also notice that as areal density decreases, fewer designs are feasible because of the large launch stresses.

6.2 Effect of Launch on Mirror Design

Section 6.1 presented trade space results for the case with the more conservative, 6-σ launch certainty limit. However, the launch survival limits have a significant effect on the results since many designs exceed the stress limits. If one relaxes the stress limits to the less conservative, 3-σ level, more designs are feasible and the resulting trade space can be seen in Figs. 15 and 16, which show the designs colored by the number of rib rings and areal density, respectively. In Fig. 15, notice that more designs with many rib rings are feasible and many of them have lower masses than in Fig. 13. Also, Fig. 16 shows that more low areal density designs meet the stress limits and are feasible.

The difference between the two stress limits can be best illustrated by looking at only the set of nondominated designs for each case, shown in Fig. 17. The entire Pareto front is shifted to the upper left when the stress limits are relaxed,
indicating that it is possible to improve overall performance of the Pareto front by relaxing the stress limits. The launch stress has a significant effect on the performance of the mirror and should therefore be considered early in the design process.

Additionally, one could add launch load alleviation techniques to decrease the launch stress in the mirrors. These techniques include vibration isolation, and passive and active damping techniques making use of the existing embedded actuators. Four types of launch load alleviation are considered. First, whole-spacecraft isolation is used to reduce vibration levels that are transferred from the launch vehicle to the spacecraft. This is modeled using a low pass filter with a variable corner frequency that levels off at high frequency. The second method is to passively add damping to the mirror using the embedded actuators by utilizing shunt circuits in conjunction with the piezos. The third alleviation method is to use to existing embedded actuators in an active control system to add additional damping to the system. The active damping is accomplished using a positive position feedback filter. These launch load alleviations are discussed in detail in Ref. 4 and all serve to decrease the launch stress in the system, but at the expense of added complexity. By including launch load alleviation techniques, the stress can be reduced and the resulting nondominated designs can be seen in Fig. 18. In the case with alleviation, the more conservative 6-σ uncertainty level can be retained, while still improving the performance of the entire Pareto front and enabling better mirror designs.

6.3 Pareto Optimal Designs

As was seen in the trade space plots in Secs. 6.1 and 6.2, there is a clear trade off between mass and correctability. The optimal designs for on-orbit correctability have very high masses, while the lowest mass designs have low correctabilities. The conflicting performance metrics motivate the use of the trade space, rather than simply designing for one metric. By simultaneously considering all metrics, one can find a Pareto optimal design that best balances the metrics, based on the requirements for the specific mission.

The Pareto optimal designs for the 6-σ certainty limits (Fig. 12) can be seen in Table 3. As the masses increase and the correctabilities improve, the Pareto designs transition from having few ribs to having many ribs. The designs in the middle of the Pareto front have alternating numbers of ribs, and one can obtain similar performance from designs with differing numbers of ribs. Areal density is another metric to move along the Pareto front; increasing areal density increases both correctability and mass. Together, the rib aspect ratio and face sheet mass fraction determine the height and thickness of the ribs and the thickness of the face sheet, which must be optimized at each point in the design space. This makes it difficult to determine a clear trend with respect to the two variables, especially with the strict stress limits and no alleviation. The strict stress limits require there to be a significant amount of mass in the ribs, thus keeping the face sheet mass fraction at a moderate value. With less stringent stress limits, the face sheet mass fraction will be low in the low mass designs, maximizing the mass in the ribs, and it will be higher in the very correctable designs, resulting in broader influence functions. Similarly, the rib aspect ratio tends to be high in the lower mass designs, as the taller ribs are necessary to meet the stress requirements.
Table 3 Mass versus correctability nondominated designs with 6-σ stress limit.

<table>
<thead>
<tr>
<th># of Rib rings</th>
<th>Rib aspect ratio</th>
<th>Face sheet mass fraction</th>
<th>Actuator length [cm]</th>
<th>SiC areal density [kg/m²]</th>
<th>Mass [m]</th>
<th>Correctability [mm]</th>
</tr>
</thead>
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In examining the Pareto front designs, it is desirable to understand how to move along the Pareto front, particularly whether movement along the Pareto front is dictated by a single variable made up of a linear combination of design parameters or a more complex relationship, necessitating the full integrated modeling effort. In order to determine the complexity of the relationships, a singular value decomposition (SVD) is utilized.

SVD involves factoring a matrix into three matrices as follows:

\[ M = U \Sigma V^T, \]  \hspace{1cm} (4)

where \( M \) is the original \([m \times n]\) matrix, \( \Sigma \) is the \([m \times n]\) diagonal matrix with the singular values on the diagonal, \( U \) is an \([m \times m]\) unitary matrix, and \( V \) is an \([n \times n]\) unitary matrix. The columns of \( V \) form an orthonormal basis for the inputs of \( M \), and the singular values can be thought of as scalar gains. Furthermore, the number of nonzero singular values, which is equal to the rank of \( M \), determines the number of linearly independent columns of \( M \).

Consider the set of parameters from Table 3 that make up the Pareto front designs. There are 23 Pareto designs and six design parameters, resulting in a \([23 \times 6]\) matrix. One can perform an SVD on the normalized matrix of Pareto parameters, which can then be used to determine linear independence of those parameters. The singular value matrix is:

\[
\Sigma = \begin{bmatrix}
7.86 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.44 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.99 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.65 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.54 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.27 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] \hspace{1cm} (5)
Notice that there are six nonzero singular values, indicating that moving along the Pareto front does not involve a linear combination of parameters. Though none of the singular values are negligible, the first one is significantly greater than the others and will be used to determine a linear combination of parameters that best approximates movement along the Pareto front.

As previously mentioned, the columns of the $V$ matrix make up an orthonormal basis for the design parameters. Therefore, the first column of $V(v_1)$, which corresponds to the largest singular value, can be used as a basis vector to move along the direction of the maximum singular value. One can start at any point on the Pareto front and use the direction of $v_1$ to move along the front. A single variable, corresponding to the direction of $v_1$ and simultaneously changing all design parameters, is varied in even increments with respect to the initial point. Therefore, the designs chosen are dependent on the starting location.

Figures 19 and 20 show the Pareto front along with the newly calculated designs for two cases. These two cases differ in their initial Pareto points, indicated by the circle. Figure 19 begins at a low mass design, while Fig. 20 begins at a high correctability design. The new designs determined from the SVD analysis that are feasible are marked with a +, while the new designs that violate the constraints are marked by an ×. It is clear that moving along the direction corresponding to the largest singular value does roughly follow the Pareto front locally. However, as one moves farther from the initial point, the designs veer away from the Pareto front and are suboptimal, and many of the designs are infeasible due to constraint violations.

Therefore, while it may be desirable to determine a single linear combination of parameters that allows one to move along with Pareto front, thus minimizing the amount of necessary computation, the complexity involved makes it unlikely. The linear combination corresponding to the direction of the maximum singular value would be extremely difficult to determine without an integrated model and trade space. However, even if this were possible due to past experience or engineering judgment, utilizing a single parameter to approximate the Pareto front will result in suboptimal or infeasible designs, thus necessitating the integrated model and full trade space exploration to determine the actual Pareto front. One could also attempt to utilize a polynomial combination of variables to determine a single parameter to move along the Pareto front. However, determining that combination of parameters would necessitate trade space exploration due to the complex nature of the mirror design.

6.4 Trade Space Summary
As is visible from the Pareto plots, there is no single best mirror design. Rather, there is a set of best designs, and the portion of the Pareto front from which the mirror design is chosen will be based on the specific mission. Therefore, it is important to understand which designs are on the Pareto front and which parameters allow the design to move along the front. Figure 21 summarizes the design knobs that allow one to move along the Pareto front. The utopia point refers to the best portion of the design space where the designs have both low mass and high correctability.

The lowest mass designs have the following properties: few rib rings and actuators, tall thin ribs, low areal density, short actuators, and no launch load alleviation. The designs in the top, right corner of Fig. 21 with the largest correctabilities have the following characteristics: many rib rings and actuators, thick face sheets, high areal density, long actuators, and launch load alleviation.
Once a specific mission objective is specified, then a design can be chosen from the Pareto front using these guidelines. However, it is important to ensure that the correct balance of parameters is determined so that the design is on the Pareto front and is not dominated. The wrong combinations of parameters can result in a design that performs poorly with respect to both mass and correctability. For example, a mirror with a high areal density and few actuators performs poorly with respect to both metrics. Similarly, a design with many rib rings but a low face sheet mass fraction has very localized influence functions, so the high spatial frequency error dominates and the design is far from the Pareto front. Therefore, the trade space results should be used to find a design with good performance, since single-axis trades examining the effects of individual parameters will not capture the coupling between the parameters and metrics and result in a sub-optimal design.

7 Conclusions

Achieving better performance in space-based optical systems, in terms of resolution and sensitivity, can be obtained through the use of larger primary apertures. However, large primary apertures are challenging for a number of reasons, including launch mass, launch volume, and flexibility. Segmented apertures composed of lightweight, rib-stiffened, surface-parallel actuated, silicon carbide mirror segments can solve many of the problems encountered in large aperture telescope systems. However, as they are a deviation from traditional design, there is very little knowledge on how to best design the lightweight mirrors. Therefore, an integrated model is created to better understand the design space of lightweight, active mirrors, specifically with respect to launch and on-orbit correctability.

The integrated model discussed in this paper consists of two main parts: a quasi-static model of on-orbit correction and wavefront error and a dynamic launch model. Changing the low spatial frequency shape of the mirror solves a number of issues, including radius of curvature mismatch between mirror segments, thermal variations, and optical prescription changes. However, the finite length and spacing of the actuators limits the achievable shape change and causes a high spatial frequency dimpling error. This analysis maximizes the correctability, while keeping the wavefront error within prescribed limits. The best designs for on-orbit correction have many ribs and actuators, long actuators, thick face sheets, and high areal densities. The dynamic launch model limits the peak stresses in the mirror to ensure survival. The mirrors that minimize launch stress have high areal densities, few rib rings, and tall thin ribs. However, mass-to-orbit is extremely expensive, so it is also desirable to minimize mass, and therefore areal density. The conflicting parameter trends among the different performance metrics reiterate the necessity for using an integrated model and simultaneously analyzing all metrics.

A large trade space is run using the integrated model, and the designs considered are compared in terms of mass and correctability, while meeting the prescribed launch survival certainty level and wavefront error limit. From the trade space, one can find areas of the design space that perform best for specific missions. The best designs in terms of on-orbit performance have many ribs, long actuators, high areal densities, and include launch load alleviation, while the lowest mass designs have few thin ribs, short actuators, low areal density, and no alleviation. One can move along the Pareto front to find good designs by varying the design parameters in a specific way, though this must be carefully done to ensure nondominance. If one varies the parameters in a careless manner, then the design will end up away from the Pareto front, where one could obtain better performance in both metrics. Furthermore, single-axis trades would not have allowed one to find the best designs, as the parameters must be concurrently varied to move along the Pareto front. Therefore, the integrated modeling has provided insights into the design of lightweight active mirrors that would not otherwise have been discovered.

References

David W. Miller is a professor, the director of the Space Systems Laboratory, and the head of the Systems Sector in the Department of Aeronautics and Astronautics at MIT. Professor Miller has played a strong engineering role in the development of space-based telescopes. His research includes dynamics and controls for satellite formation flight, vibration suppression and isolation, and thin face-sheet adaptive optics. Professor Miller has also developed an extensive set of dynamics and controls technology laboratories on the Shuttle (MODE STS-40, 48, 62; MACE STS-67) and ISS (MACE Increments 1 & 2 and SPHERES Increments 13 & 14). He is currently developing reconfigurable spacecraft concepts that permit repair, inspection, assembly, upgrade, and multi-mission functionality through rendezvous and docking of modular satellites utilizing universal standardized interfaces. He has also developed a technique to control satellite formations, without the need for propellant, using high temperature superconducting electromagnets.