Measurement of the mass and width of the $D_{s1}(2536)^{+}$ meson

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<td>As Published</td>
<td><a href="http://dx.doi.org/10.1103/PhysRevD.83.072003">http://dx.doi.org/10.1103/PhysRevD.83.072003</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>American Physical Society</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Sun Feb 03 21:20:41 EST 2019</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/67494">http://hdl.handle.net/1721.1/67494</a></td>
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Measurement of the mass and width of the $D_{s1}(2536)^+$ meson

The decay width and mass of the $D_{s1}(2536)^+$ meson are measured via the decay channel $D_{s1}^+ \rightarrow D^{*+}K_S^0$ using 385 fb$^{-1}$ of data recorded with the BABAR detector in the vicinity of the $Y(4S)$ resonance at the PEP-II asymmetric-energy electron-positron collider. The result for the decay width is 
\[ \Gamma(D_{s1}^+) = 0.92 \pm 0.03 \text{(stat.)} \pm 0.04 \text{(syst.)} \text{ MeV.} \]
For the mass, a value of $m(D_{s1}^+) = 2535.08 \pm 0.01 \text{(stat.)} \pm 0.15 \text{(syst.)} \text{ MeV}/c^2$ is obtained. The mass difference between the $D_{s1}^+$ and the $D^{*+}$ is measured to be $m(D_{s1}^+) - m(D^{*+}) = 524.83 \pm 0.01 \text{(stat.)} \pm 0.04 \text{(syst.)} \text{ MeV}/c^2$, representing a significant improvement compared to the current world average. The unnatural spin-parity assignment for the $D_{s1}^+$ meson is confirmed.

DOI: 10.1103/PhysRevD.83.072003
PACS numbers: 14.40.Lb, 13.25.Ft, 13.66.Bc
I. INTRODUCTION

The theoretical description of $D^{+}_{s}$ mesons is problematic because, unlike $D$ mesons, the masses and widths of the $D^{*0}_{s}(2317)^{+}$ and $D^{+}_{s}(2460)^{+}$ states [1–6] are not in agreement with potential model calculations based on HQET [7]. Theoretical explanations for the discrepancy invoke $D(\pi K)$ molecules [8], chiral partners [9,10], unitarized chiral models [11,12], tetraquarks [13,14], and lattice calculations [15,16], but a satisfactory description is still lacking (see [17,18] for more details). Improved measurements of the $D^{+}_{s}$ meson parameters can lead to a better understanding of these states.

In this analysis a precise measurement of the $D^{+}_{s}(2536)^{+}$ mass and decay width is performed based on a high statistics data sample [19]. The $D^{+}_{s}(2536)^{+}$ meson, referred to as the $D^{+}_{s}$ below, was first seen in $c\bar{c}$-continuum reactions [20], and more recently in $B$ decays. The current world average mass value published by the Particle Data Group is based on measurements with large statistical and systematic uncertainties: $2535.29 \pm 0.20$ MeV/$c^2$ [21]; the mass difference between the $D^{+}_{s}$ and the $D^{*+}$ meson has been measured to be $525.04 \pm 0.22$ MeV/$c^2$ [21]. An upper limit on the decay width ($\Gamma < 2.3$ MeV at 90% confidence level), and a measurement of the spin-parity of the $D^{+}_{s}$ meson ($J^{P} = 1^{+}$), have been obtained, but based on low-statistics data samples only [21–23]. The mixing between the $D^{+}_{s}$ meson and the other $J^{P} = 1^{+}$ state $D^{+}_{s}(2460)^{+}$ was investigated in Ref. [24].

This analysis is based on a data sample corresponding to an integrated luminosity of 349 fb$^{-1}$ recorded at the $Y(4S)$ resonance and 36 fb$^{-1}$ recorded 40 MeV below that resonance with the BABAR detector at the asymmetric-energy $e^{+}e^{-}$ collider PEP-II at the SLAC National Accelerator Laboratory. In this analysis, $D^{+}_{s}$ mesons are reconstructed from $c\bar{c}$ continuum events in the $D^{*+}K^{0}_{S}$ channel; those originating from $B$ decays are rejected.

The BABAR detector is described briefly in Sec. II. The principal criteria used in the reconstruction of the $D^{*+}K^{0}_{S}$ mass spectrum and the acceptance of $D^{+}_{s}$-candidates are discussed in Sec. III. The relevant Monte Carlo (MC) simulations are described in Sec. IV, while the detector resolution parametrization is considered in Sec. V. Measurements of the mass and total width for the $D^{+}_{s}$ state are obtained from a fit to the $D^{*+}K^{0}_{S}$ invariant mass distribution as discussed in Sec. VI. Decay angle distributions are studied in Sec. VII, where the implications for the spin-parity of the $D^{+}_{s}$ state are also discussed. Sources of systematic uncertainty are described in Sec. VIII, and the results of the analysis are summarized in Sec. IX and X.

II. THE BABAR DETECTOR

The BABAR detector is described in detail elsewhere [25]. Charged particles are detected, and their momenta measured, with a combination of five layers of double-sided silicon microstrip detectors (SVT) and a 40-layer cylindrical drift chamber (DCH), both coaxial with the cryostat of a superconducting solenoidal magnet that produces a magnetic field of 1.5 T. Charged particle identification is achieved by measurements of the energy-loss $dE/dx$ in the tracking devices and with an internally reflecting, ring-imaging Cherenkov detector. The energy of photons and electrons is measured with a CsI(Tl) electromagnetic calorimeter, covering 90% of the $4\pi$ solid angle in the $Y(4S)$ rest frame. The instrumented flux return of the magnetic field is used to identify muons and $K^{0}_{S}$.

III. SELECTION AND RECONSTRUCTION OF EVENTS

The $D^{+}_{s}$ is reconstructed via its decay mode $D^{*+}K^{0}_{S}$, with $K^{0}_{S} \rightarrow \pi^{+}\pi^{-}$ and $D^{*+} \rightarrow D^{+}\pi^{-}$. The $D^{0}$ is reconstructed through two decay modes, $K^{-}\pi^{+}$ and $K^{-}\pi^{+}\pi^{+}\pi^{-}$, which will be labeled $K\pi\pi$ and $K\pi\pi\pi$, respectively, in the following. To improve the mass resolution, the mass difference $\Delta m(D^{+}_{s}) = m(D^{+}_{s}) - m(D^{*+}) - m(K^{0}_{S})$ is examined.

Events are selected by requiring at least five charged tracks, at least one of which is identified as a charged kaon. Also, at least one neutral kaon candidate is required. Each track must approach the nominal $e^{+}e^{-}$ interaction point to within 1.5 cm in the transverse direction, and to within 10 cm in the longitudinal (beam) direction. Kaon candidates are identified using the normalized kaon, pion and proton likelihood values ($L_K$, $L_\pi$ and $L_p$) obtained from the particle identification system, by requiring $L_K/(L_K + L_\pi) > 0.50$ and $L_K/(L_K + L_p) > 0.018$. Furthermore, the track must be inconsistent with the electron hypothesis or have a momentum less than 0.4 GeV/c. Tracks that fulfill $L_K/(L_K + L_\pi) < 0.98$ and $L_p/(L_p + L_\pi) < 0.98$ are selected as pions.

Candidates for the $D^{0}$ decay are formed by selecting all $K^{-}\pi^{+}\pi^{-}$ pairs ($K^{-}\pi^{+}\pi^{+}\pi^{-}$ combinations in the second mode) that have an invariant mass within $\pm 100$ MeV/$c^2$ of the nominal mass [21]. Candidates for the $D^{*+}$ decay are formed by adding a $\pi^{+}$ to the $D^{0}$, such that the mass difference between $D^{*+}$ and $D^{0}$ is less than 170 MeV/$c^2$. A $K^{0}_{S}$ candidate consists of a $\pi^{+}\pi^{-}$ pair with invariant mass within $\pm 25$ MeV/$c^2$ of the nominal mass [21]. A kinematic fit is applied to the complete decay chain, constraining the $D^{+}_{s}$ candidate vertex to be consistent with the $e^{+}e^{-}$ interaction region. Mass constraints are not applied to intermediate states. Those $D^{+}_{s}$ candidates with a $\chi^2$ fit probability greater than 0.1% are retained. To suppress combinatorial background and events from $B$ decays, we require the momentum $p^{*}(D^{+}_{s})$ of the $D^{+}_{s}$ in the $Y(4S)$ center-of-mass (CM) frame to exceed 2.7 GeV/c.
The $K\pi$ and $K\pi\pi\pi$ mass spectra for accepted $D^0$ candidates, shown in Figs. 1(a) and 1(d), are fitted with a signal function consisting of a sum of two Gaussians with a common mean value, and a linear background function. The width of the signal regions for $D^0$, $D^{*+}$ and $K_S^0$ candidates is defined as twice the full width at half maximum (FWHM) of the signal line shapes. A signal window of $\pm 18 (14)$ MeV/$c^2$ for the $K4\pi$ ($K6\pi$) mode around the mean mass of 1863.5 (1863.5) MeV/$c^2$ obtained from the fit is used to select $D^0$ candidates. For these candidates, the $D^0\pi^+ - D^0$ mass difference distributions are shown in Figs. 1(b) and 1(e). These are fitted with a Double-Gaussian signal function and a linear background function consisting of a polynomial times an exponential function. A $D^{*+}$ signal region of $\pm 1$ MeV/$c^2$ around the fitted mean value of 145.4 MeV/$c^2$ is chosen for both decay modes. To further reduce the background, the angle between the flight direction of the $K_S^0$ candidate and the line connecting the $e^+e^-$ interaction point and the $K_S^0$ decay vertex is required to be less than 0.15 radians. For candidates passing these selection criteria, the $K_S^0$ candidate invariant mass distributions (Figs. 1(c) and 1(f)) are fitted with the sum of a signal function, consisting of the sum of two Gaussians, and a linear background function. A signal window of $\pm 6$ MeV/$c^2$ around the fitted mean mass of 497.2 MeV/$c^2$ is selected for both decay modes.

In the case of an event with multiple candidates, the candidate with the best fit probability is chosen. The $\Delta m(D_{s1}^+)$ candidate spectra after all selection criteria are shown in Figs. 2(a) and 2(b). The fits to these spectra use a Double-Gaussian signal function and a linear background function. Note that for this preliminary fit the intrinsic width and the resolution are not taken into account. The FWHM values obtained are $(2.2 \pm 0.1)$ MeV and $(2.0 \pm 0.1)$ MeV, respectively, with corresponding signal yields of about 3500 and 4000 entries.

**IV. MONTE CARLO SIMULATION AND COMPARISON WITH DATA**

Monte Carlo events are generated for $D_{s1}^+ \rightarrow D^{*+}K_S^0$, $D^{*+} \rightarrow D^0\pi^+$, with $D^0 \rightarrow K^-\pi^+$ and $D^0 \rightarrow K^-\pi^+\pi^-\pi^-$, by EVTGEN [26]. The detector response is simulated using the GEANT4 [27] package. For each $D^0$ decay mode, and for each of the corresponding $D_{s1}$ decays, 776000 events are generated. The $D_{s1}^+$ line shape is generated using a nonrelativistic Breit-Wigner function having central value $m(D_{s1}^+)$gen $= 2535.35$ MeV/$c^2$ and intrinsic width $\Gamma(D_{s1}^+)$gen $= 1$ MeV (this sample is labeled $\Gamma_1$ in the following). The range of generated $D_{s1}^+$ masses is restricted to values between $m(D_{s1}^+)$gen $- 10$ MeV/$c^2$ and $m(D_{s1}^+)$gen $+ 15$ MeV/$c^2$. The masses of the daughter particles are taken from Ref. [21].
In order to test the mass resolution model, a second set of MC samples with 381000 events for each $D^0$ decay mode is generated using a Breit-Wigner width of $\Gamma(D^\ast_{s1})_{\text{gen}} = 2$ MeV ($\Gamma_2$ sample). In addition to these signal MC samples, separate $D^0$ and $K^0_s$ samples are created from data and generic $c\bar{c}$ MC simulations without requiring a $D^{*+}$ or $D^0_s$. They are used mainly for resolution studies.

The MC and data are in good agreement for the transverse momentum distributions of pions, kaons, $D$ and $D^*$ mesons, and for the number of SVT coordinates of pions and kaons. The agreement is worse for the number of DCH coordinates, where the data show systematically fewer coordinates than the MC, giving rise to a resolution that is about 10% smaller in the MC than in data. This is illustrated in Fig. 3, which shows the $p^*(K^0_s)$ and $p^*(D^0)$ dependence of the ratio between the FWHM of the resolution functions in $c\bar{c}$ MC and data, where $p^*$ is the momentum in the CM frame. This effect will be discussed further in Sec. VIII. There is also disagreement between the number of $D^0_s$ signal entries in MC and data as a function of $p^*(D^0_s)$ (Fig. 4). This effect will be addressed in Sec. V and VIII.

V. RESOLUTION MODEL

The resolution model is derived from the $D^0_s$ signal MC by studying the difference $\Delta m_{\text{res}}$ between the reconstructed and generated $D^+_s$ mass values. The Multi-Gaussian ansatz

$$G(\Delta m_{\text{res}}) = \int_{\sigma_0}^{r\sigma_0} \frac{1}{r} e^{-\left((\Delta m_{\text{res}}-\Delta m_{\text{gen}})^2/(2r^2)\right)} d\sigma$$

is found to accurately model the mass resolution spectra. This represents a superposition of Gaussian distributions with the same mean value $\Delta m_{\text{res}}$ but variable width $\sigma$, starting from minimum width $\sigma_0$ and increasing to maximum width $r\sigma_0$. The FWHM of the distribution is numerically calculable once $\sigma_0$ and $r$ are known. The mass resolution for the different particles depends on the CM momentum $p^*(D^0_s)$. Therefore, the parameter $\sigma_0$ of Eq. (1) is obtained as a function of $p^*(D^0_s)$.

Figs. 5(a) and 5(b) show $\Delta m_{\text{res}}$ distributions for the full $p^*(D^0_s)$ range. From these plots the value of the parameter $r$ is determined to be $4.78 \pm 0.04$ and $5.20 \pm 0.05$ for the $K4\pi$ and $K6\pi$ modes, respectively. Events are divided into 30 $p^*(D^0_s)$ intervals from 2.7 GeV/c to 4.7 GeV/c and the fit repeated for each interval, resulting in $p^*(D^0_s)$-dependent $\sigma_0$ values (Figs. 6(a) and 6(b)). The corresponding $p^*(D^0_s)$-dependent FWHM of the resolution functions is shown in Figs. 7(a) and 7(b).

In order to validate this resolution model, the $p^*(D^0_s)$-dependent resolution function with the corresponding parameters $\sigma_0$ and $r$ is convolved with a
nonrelativistic Breit-Wigner function and fitted to the \( \Delta m(D_{s1}^+) \) signal MC distribution (MC sample \( \Gamma_1 \)). The results are shown in Figs. 8(a) and 8(b), and the reconstructed values for mean \( \Delta m(D_{s1}^+) \) and width \( \Gamma(D_{s1}^+) \) are listed in Table I. The corresponding generated values for both decay modes are \( \Delta m(D_{s1}^+)_{\text{gen}} = 27.744 \text{ MeV}/c^2 \) for the mean and \( \Gamma(D_{s1}^+)_{\text{gen}} = 1.000 \text{ MeV} \) for the width. The small deviations between generated and reconstructed values are discussed in Sec. VIII.

VI. FIT TO THE \( D^+K_{3}^0 \) MASS SPECTRUM

For the final fit to the \( D^+K_{3}^0 \) mass spectra, as represented by the \( \Delta m(D_{s1}^+) \) distributions of Figs. 2 and 9, the signal function consists of a relativistic Breit-Wigner line shape numerically convolved with the \( p^*(D_{s1}^+) \)-dependent resolution function (Eq. (1)). A linear function is used to describe the background.

The relativistic Breit-Wigner function used takes the form

\[
\frac{(p_{1,m})^{2L+1}}{p_{1,m_0}} \frac{(m_0/m)\frac{m F_L(p_{1,m})^2}{(m_0^2 - m^2 + \Gamma_m^2 m_0^2)}}{\left(m_0 - m^{2} + \Gamma_m^{2} m_0^{2}\right)}, \tag{2}
\]

where \( m_0 \) is an abbreviation for \( \Delta m(D_{s1}^+) \) and \( m \) stands for \( \Delta m(D_{s1}^+) \). The variable \( p_{1,m} \) is the momentum of the \( D^+ \) in the rest frame of the \( D_{s1}^+ \) resonance candidate, which has mass \( m \), and \( p_{1,m_0} \) is the value for \( m = m_0 \). The respective

FIG. 4 (color online). \( p^*(D_{s1}^+) \)-dependence of the \( D_{s1}^+ \) signal yield for data (open squares) and reconstructed MC (solid points) for the (a) \( K4\pi \) and (b) \( K6\pi \) decay modes.

FIG. 5 (color online). Fit of the resolution function (Eq. (1)) to \( \Delta m_{\text{res}} \) with the \( r \) and \( \sigma_0 \) parameters free to vary for the (a) \( K4\pi \) and (b) \( K6\pi \) decay modes.

FIG. 6. \( p^*(D_{s1}^+) \) dependence of the resolution function parameter \( \sigma_0 \), represented by a linear parametrization (\( r \) fixed) for the (a) \( K4\pi \) and (b) \( K6\pi \) decay modes.
the momentum given by is defined as in Ref. [28]. The mass-dependent width is where the Blatt-Weisskopf barrier factors $F_L(p_{1,m})$ for orbital angular momentum $L$ between the $D^{*+}$ and $K_S^0$ are

$$F_0(p_{1,m}) = 1,$$

$$F_1(p_{1,m}) = \frac{\sqrt{1 + (R p_{1,m})^2}}{\sqrt{1 + (R p_{1,m})^2}},$$

$$F_2(p_{1,m}) = \frac{\sqrt{9 + 3(R p_{1,m})^2 + (R p_{1,m})^4}}{\sqrt{9 + 3(R p_{1,m})^2 + (R p_{1,m})^4}},$$

where

$$R = 1.5 \, (\text{GeV/c})^{-1}$$

is defined as in Ref. [28]. The mass-dependent width is given by

$$\Gamma_m = \Gamma(D_{s1}^{*+}) \left( B_1 \frac{p_{1,m}}{p_{1,m_0}} \right)^{2L+1} \left( \frac{m_0}{m} \right) F_L(p_{1,m})^2$$

$$+ B_2 \left( \frac{p_{2,m}}{p_{2,m_0}} \right)^{2L+1} \left( \frac{m_0}{m} \right) F_L(p_{2,m})^2$$

with $\Gamma(D_{s1}^{*+})$ the total intrinsic width of the $D_{s1}^{*+}$ resonance. This relation takes into account the $D_{s1}^{*+} \rightarrow D^{*+}K^0$ and the $D_{s1}^{*+} \rightarrow D^{*0}K^+$ decay modes, with the corresponding branching fractions $B_1$ and $B_2$, respectively.

FIG. 7. $p^*(D_{s1}^{*+})$ dependence of the FWHM of the resolution function ($r$ fixed) for the (a) $K4\pi$ and (b) $K6\pi$ decay modes.

Blatt-Weisskopf barrier factors $F_L(p_{1,m})$ for orbital angular momentum $L$ between the $D^{*+}$ and $K_S^0$ are

$$F_0(p_{1,m}) = 1,$$

$$F_1(p_{1,m}) = \frac{\sqrt{1 + (R p_{1,m})^2}}{\sqrt{1 + (R p_{1,m})^2}},$$

$$F_2(p_{1,m}) = \frac{\sqrt{9 + 3(R p_{1,m})^2 + (R p_{1,m})^4}}{\sqrt{9 + 3(R p_{1,m})^2 + (R p_{1,m})^4}},$$

where

$$R = 1.5 \, (\text{GeV/c})^{-1}$$

is defined as in Ref. [28]. The mass-dependent width is given by

$$\Gamma_m = \Gamma(D_{s1}^{*+}) \left( B_1 \frac{p_{1,m}}{p_{1,m_0}} \right)^{2L+1} \left( \frac{m_0}{m} \right) F_L(p_{1,m})^2$$

$$+ B_2 \left( \frac{p_{2,m}}{p_{2,m_0}} \right)^{2L+1} \left( \frac{m_0}{m} \right) F_L(p_{2,m})^2$$

with $\Gamma(D_{s1}^{*+})$ the total intrinsic width of the $D_{s1}^{*+}$ resonance. This relation takes into account the $D_{s1}^{*+} \rightarrow D^{*+}K^0$ and the $D_{s1}^{*+} \rightarrow D^{*0}K^+$ decay modes, with the corresponding branching fractions $B_1$ and $B_2$, respectively.

FIG. 8 (color online). Fit of a nonrelativistic Breit-Wigner convolved with the resolution function to the $D_{s1}^{*+}$ candidate mass difference spectra in the $\Gamma_1$ MC sample for the (a) $K4\pi$ and (b) $K6\pi$ decay modes.

$$B_i = \frac{p_{1,m_0}^{2L+1}}{p_{1,m_0}^{2L+1} + p_{2,m_0}^{2L+1}}.$$

Since the $D_{s1}^{*+}$ mass lies close to threshold for both decay modes, the mass values of the decay particles make a significant difference. The momenta $p_{2,m}$ and $p_{2,m_0}$ correspond to $p_{1,m}$ and $p_{1,m_0}$, respectively, but are calculated for the $D^{*0}K^+$ decay mode.

It is assumed that the $D_{s1}^{*+}$ has spin-parity $J^{P} = 1^+$ and from parity conservation that the orbital angular momentum $L$ is either 0 or 2. The $S$ wave usually dominates in $1^+$ decays, so $L = 0$ is chosen for the main fit and an additional $L = 2$ contribution is used to estimate a systematic uncertainty. Further discussion on the $J$ and $L$ values is presented in Sec. VII.

<table>
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<tr>
<th>$\Delta m(D_{s1}^{*+})_0/\text{MeV}/c^2$</th>
<th>$\Gamma(D_{s1}^{*+})/\text{MeV}$</th>
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<tr>
<td>(K4\pi)</td>
<td>27.737 ± 0.003</td>
</tr>
<tr>
<td>(K6\pi)</td>
<td>27.734 ± 0.003</td>
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TABLE I. Reconstructed values for $\Delta m(D_{s1}^{*+})_0$ and $\Gamma(D_{s1}^{*+})$ (fit to MC sample $\Gamma_1$). The resolution model used is derived from MC sample $\Gamma_1$.
VII. ANGULAR DISTRIBUTION

The assigned spin-parity $J^P = 1^+$ of the $D_{s1}^{+}$ is based on studies with small data samples (less than 200 reconstructed events) [22,23]. There, fits of an angular distribution corresponding to unnatural spin-parity $(1^+, 2^-, \ldots)$ yielded the highest confidence level. In this analysis clean signals with a total number of about 8000 reconstructed $D_{s1}^{+}$-candidates are available, making a detailed study possible.

$D^{++}$ decay angle. Since in this analysis the origin of the $D_{s1}^{+}$ is not known, the decay angle $\theta'$ between the $D^0$ momentum vector in the $D^{++}$ CM system and the $D^{++}$ momentum vector in the $D_{s1}^{+}$ CM system (Fig. 10(a)) is used for the $J^P$ analysis. The resulting angular distribution $dN(D_{s1}^{+})/d\cos\theta'$ is influenced by the spin of the $D_{s1}^{+}$. The expected distributions for different $D_{s1}^{+}$ spin-parity values are calculated using the helicity formalism [29–31] and are listed in Table II.

The data are corrected for the detection efficiency and divided into 20 bins of $\cos\theta'$. The signal entries for the $\cos\theta'$ bins are obtained from separate fits to the data with the mass and decay width of the $D_{s1}^{+}$ fixed to the values reported in Sec. VI. The $dN(D_{s1}^{+})/d\cos\theta'$ distribution shown in Fig. 11 is the combined result from the $K4\pi$ and $K6\pi$ samples.

Comparison with the theoretical distributions shows a clear preference for the unnatural spin-parity values $J^P = 1^+, 2^-, 3^+ \ldots$, confirming the earlier results [22,23]. The signal function for these $J^P$ values is

$$I(\theta') = a(\sin^2\theta' + \beta\cos^2\theta'),$$  

where $\beta = |A_{00}|^2/|A_{10}|^2$ and $a$ is a constant. The helicity amplitudes $|A_{00}|$ and $|A_{10}|$ correspond to the $D^{++}$ helicities 0 and $\pm 1$, respectively.

The lowest value $J^P = 1^+$ is the most probable one: assuming $1^+$ implies $l = 1$ (orbital momentum between the light and heavy quark), while the higher $J$ values demand $l \geq 2$; such mesons are expected to be highly

The fit to the $\Delta m(D_{s1}^{+}) = m(D_{s1}^{+}) - m(D^+) - m(K^0_S)$ mass difference spectrum in data (Fig. 9) yields mean mass differences

$$\Delta m(D_{s1}^{+})_0 = 27.231 \pm 0.020 \text{ MeV}/c^2 \quad (K4\pi),$$  

$$\Delta m(D_{s1}^{+})_0 = 27.205 \pm 0.018 \text{ MeV}/c^2 \quad (K6\pi),$$

and total width values

$$\Gamma(D_{s1}^{+}) = 1.000 \pm 0.049 \text{ MeV} \quad (K4\pi),$$  

$$\Gamma(D_{s1}^{+}) = 0.941 \pm 0.045 \text{ MeV} \quad (K6\pi).$$

The fitted values for the two $D^0$ decay modes agree within the statistical errors. The signal yield is $3704 \pm 71$ for $K4\pi$ and $4334 \pm 78$ for $K6\pi$. 

![Fig. 9](color online). Fit of a relativistic Breit-Wigner convolved with the resolution function to the $D_{s1}^{+}$ candidate mass difference spectra in data, for the (a) $K4\pi$ and (b) $K6\pi$ modes. The dotted line indicates the background line shape. The upper parts of the figures show the normalized fit residuals.

![Fig. 10](a) Decay angle $\theta'$ of the $D^{++}$. b) Decay angle $\theta$ of the $D_{s1}^{+}$.}
TABLE II. List of spin-parity values \( J^P \) for the \( D_{s1}^+ \) and the corresponding decay angle distributions for the \( D_{s1}^+ \). Under the assumption of a strong decay, \( 0^+ \) is forbidden. The last three columns show the \( \chi^2/NDF \) of the fits to the \( \cos \theta' \)-distribution for efficiency-corrected data, with \( NDF \) being the number of degrees of freedom.

<table>
<thead>
<tr>
<th>( J^P )</th>
<th>( dN(D_{s1}^+)/d \cos \theta' )</th>
<th>( \chi^2/NDF(K4\pi) )</th>
<th>( \chi^2/NDF(K6\pi) )</th>
<th>( \chi^2/NDF ) (combined data)</th>
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<tr>
<td>0^-</td>
<td>acos^2 ( \theta' )</td>
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<td>. . .</td>
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<tr>
<td>1^- , 2^- , 3^- , . . .</td>
<td>asin^2 ( \theta' )</td>
<td>2142.7/19</td>
<td>2440.8/19</td>
<td>4578.0/19</td>
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<tr>
<td>1^+ , 2^- , 3^- , . . . (S wave only)</td>
<td>const</td>
<td>103.2/19</td>
<td>108.8/19</td>
<td>190.9/19</td>
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<tr>
<td>1^+ , 2^- , 3^- , . . . (S , D wave)</td>
<td>a(sin^2 ( \theta' ) + ( \beta )cos^2 ( \theta' ))</td>
<td>392.1/19</td>
<td>425.1/19</td>
<td>802.5/19</td>
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\[ \chi^2/NDF = NDF \left( \beta = 0.23 \pm 0.03 \right) \]

Results from a fit of both a constant and a distribution proportional to \( 1 + \gamma \cos^2 \theta \) (based on Eq. (10)) are given in Table III. Using the value of \( t \) from the \( \cos \theta \) fit, the result for \( |A_{10}|^2/|A_{00}|^2 \) from the \( \cos \theta' \) fit, and the coefficients from Eq. (10), we determine \( \rho_{00} = 0.48 \pm 0.03 \) for the combined \( K4\pi \) and \( K6\pi \) samples, and \( 0.44 \pm 0.04 \) and \( 0.53 \pm 0.04 \) for the individual samples, consistent with the Belle result \( \rho_{00} = 0.490 \pm 0.012 \) [24].

Several effects that might affect the results of the angular analysis have been studied.

**Test for nonflat efficiency.** The formalism used for the calculation of \( I(\theta') \) assumes a flat acceptance in \( \cos \theta \). In this study, the efficiency decreases a few percent for values of \( \cos \theta > 0 \). In order to assess the impact of this effect, all \( D_{s1}^+ \) candidates with \( \cos \theta > 0 \) are removed from the data sample. The results for \( \beta \) from fits to the reduced \( \cos \theta' \) spectra are consistent with the nominal results, ruling out an observable effect due to nonflat efficiency.

**Test for possible interference.** Possible interference with unreconstructed recoil particle(s) \( X \) in the decay chain \( e^+e^- \rightarrow D_{s1}^+X \) is considered. The effect of interference is expected to depend on the flight direction of the \( D_{s1}^+ \). Therefore the data are divided into four subsamples based on their \( \cos \theta_d \) value, where \( \theta_d \) is the flight angle of the \( D_{s1}^+ \) relative to the beam axis (calculated in the \( e^+e^- \) CM system). For each of these reduced data samples, the fit to the \( \cos \theta' \) distribution is repeated. The values obtained

![Efficiency-corrected signal yield as function of \( \cos \theta' \) in data. The following models are fitted to the distribution: \( a(\sin^2 \theta' + \beta \cos^2 \theta') \) (solid line); a constant (dash-dotted line); \( acos^2 \theta' \) (dashed line); \( asin^2 \theta' \) (dotted line).](image1)

![Efficiency-corrected signal yield as function of \( \cos \theta \) in data. The following models are fitted to the distribution: constant (dotted line); \( a(1 + \gamma \cos^2 \theta) \) (solid line).](image2)
for the parameter $\beta$ are fully consistent within errors with each other and with the nominal value (full data sample), ruling out a significant interference effect. The same consistency between results is found in fits to $\cos \theta$.

**VIII. SYSTEMATIC STUDIES**

The investigated sources of systematic uncertainty can be separated into three main categories: uncertainties arising from the resolution model, fit procedure, and reconstruction. The uncertainties are defined by taking the differences $\Delta_{\Delta m}$ and $\Delta \Gamma$ between the standard result for the mass difference $\Delta m(D_{s1}^+)$ and width $\Gamma(D_{s1}^+)$ given in Sec. VI and the result obtained with the correspondent modification. A summary of the results is listed in Table IV. If not otherwise stated, the momentum-dependent resolution model and the relativistic Breit-Wigner signal function combined with a first order polynomial for background parametrization from the standard fit are used. Deviations smaller than 0.5 keV/$c^2$ for $\Delta_{\Delta m}$ and smaller than 0.5 keV for $\Delta \Gamma$ are considered as negligible.

**A. Resolution model uncertainties**

*General comparison between MC and Data.* The $D^0$ and $K^0_\ell$ test samples (see Sec. IV) demonstrate that the mass resolution is underestimated by 10% in MC (Fig. 3), yielding an overestimated decay width from the fits to data. The effect of this is quantitatively studied by increasing the width of the resolution function by 10%. The repeated fits yield no significant deviations for the mass difference, but a 51 (45) keV smaller decay width. The nominal decay width values obtained from the fits in Sec. VI are thus corrected by these values, yielding values of $\Gamma(D_{s1}^+)$ = 0.949 (0.896) MeV for the $K4\pi$ ($K6\pi$) mode.

To estimate the corresponding systematic uncertainty, the resolution function modification is varied within $(10 \pm 4)$% to take a possible $p^0$ dependence into account (this value is derived from Fig. 3(a), which shows the largest variation in $p^0$). There are no effects on $\Delta m(D_{s1}^+)$, and a deviation of $\pm 3.0$ keV for $\Gamma(D_{s1}^+)$ is observed in both decay modes, compared to the corrected results from above. As a conservative estimate, the larger deviation is used as a two-sided uncertainty, providing the largest systematic error for the decay width.

*Further validation of the resolution model.* To further validate the procedure used to obtain the resolution model, the results from fits to the $\Gamma_1$ and $\Gamma_2$ MC samples are compared. The derived resolution function parameters are in good agreement between the two samples. The widths of the reconstructed $\Delta m(D_{s1}^+)$ distributions

<table>
<thead>
<tr>
<th>Systematic uncertainty</th>
<th>$\Delta_{\Delta m}$/keV/$c^2$</th>
<th>$\Delta \Gamma$/keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution +10%</td>
<td>$&lt;0.5$</td>
<td>$&lt;0.5$</td>
</tr>
<tr>
<td>MC validation</td>
<td>$\pm 7$</td>
<td>$\pm 10$</td>
</tr>
<tr>
<td>Alternative resolution models</td>
<td>$&lt;0.5$</td>
<td>$&lt;0.5$</td>
</tr>
<tr>
<td>Multi-Gaussian resolution: $r \pm \delta r$</td>
<td>$&lt;0.5$</td>
<td>$&lt;0.5$</td>
</tr>
<tr>
<td>Multi-Gaussian resolution: $\sigma_0$</td>
<td>$&lt;0.5$</td>
<td>$&lt;0.5$</td>
</tr>
<tr>
<td>Breit-Wigner signal line shape: Value of $L$</td>
<td>$\pm 9$</td>
<td>$\pm 8$</td>
</tr>
<tr>
<td>Mass window for $\Delta m(D_{s1}^+)$</td>
<td>$&lt;0.5$</td>
<td>$&lt;0.5$</td>
</tr>
<tr>
<td>Background parametrization</td>
<td>$&lt;0.5$</td>
<td>$&lt;0.5$</td>
</tr>
<tr>
<td>Tracking region material density</td>
<td>$\pm 21$</td>
<td>$\pm 13$</td>
</tr>
<tr>
<td>SVT Alignment</td>
<td>$\pm 6$</td>
<td>$\pm 7$</td>
</tr>
<tr>
<td>Magnetic field strength</td>
<td>$\pm 13$</td>
<td>$\pm 19$</td>
</tr>
<tr>
<td>Length scale</td>
<td>$\pm 4$</td>
<td>$\pm 6$</td>
</tr>
<tr>
<td>Drift chamber hits</td>
<td>$\pm 11$</td>
<td>$\pm 15$</td>
</tr>
<tr>
<td>$\phi$-dependency</td>
<td>$\pm 13$</td>
<td>$\pm 14$</td>
</tr>
</tbody>
</table>
determined from fitting the \( \Gamma_2 \) samples, \( \Gamma(D_{21}^+) = 2.004 \pm 0.016 \text{ MeV} \) for the \( K4\pi \) mode and \( 2.018 \pm 0.022 \text{ MeV} \) for the \( K6\pi \) mode, are in good agreement with the generated values. Similarly, when the resolution function from the \( \Gamma_2 \) sample is used to determine the width for the \( \Gamma_1 \) sample, values of \( \Gamma(D_{11}^+) = 1.003 \pm 0.005 \text{ MeV} \) and \( 0.999 \pm 0.006 \text{ MeV} \), respectively, are obtained, in agreement with the generated values.

As a conservative estimate, the small deviations found during the validation procedure for the resolution model using the \( \Gamma_1 \) sample in Sec. V are used as systematic uncertainties: \( \Delta \sigma_m = -(7\pm10) \text{ keV}/c^2 \); \( \Delta \Gamma = +1(-9) \text{ keV} \) for \( K4\pi (K6\pi) \).

Alternative resolution models. Using the resolution model obtained from the \( \Gamma_2 \) MC sample, a fit to data yields uncertainties \( \Delta \sigma_m < 0.5(\pm0.5) \text{ keV}/c^2 \) and \( \Delta \Gamma = +1(+12) \text{ keV} \) for \( K4\pi (K6\pi) \).

Instead of the momentum-dependent resolution model of the standard analysis, an alternative model has been tested, based on the comparison of MC and data distributions that show disagreement, such as the \( p^+(D_{11}^+) \) dependence of the \( D_{11}^+ \) yield. By dividing the MC and data spectra from Fig. 4, a correction function is derived. MC data are modified with this function such that the two distributions in Fig. 4 coincide. From these corrected MC, a new resolution model is derived. The results for \( \Delta m(D_{11}^+) \) and \( \Gamma(D_{11}^+) \) in data agree within the error with the momentum-dependent treatment (systematic uncertainties \( \Delta \sigma_m < 0.5(\pm0.5) \text{ keV}/c^2 \), \( \Delta \Gamma = -2(+1) \text{ keV} \) for \( K4\pi (K6\pi) \)).

The larger deviations listed above are reported as the systematic uncertainties associated with the use of alternative resolution models.

Parameters of the \( p^+(D_{11}^+) \)-dependent resolution model. The parameter \( r \) of the \( p^+(D_{11}^+) \)-dependent resolution model is modified within its error leading to negligible deviations in \( \Delta m(D_{11}^+) \) and \( \pm 6(\pm7) \text{ keV} \) in \( \Gamma(D_{11}^+) \) for \( K4\pi (K6\pi) \). A different parametrization of the \( \sigma_0(p^+(D_{11}^+)) \)-dependence (second order polynomial) results in a negligible deviation for \( \Delta m(D_{11}^+) \) and \( -3(-2) \text{ keV} \) for \( \Gamma(D_{11}^+) \).

B. Fit procedure uncertainties

Breit-Wigner line shape. In the standard fit, a pure \( S \)-wave decay of the \( D_{11}^+ \) to \( D^+K_0^* \) is assumed, using a Breit-Wigner line shape corresponding to \( L = 0 \). To estimate a systematic uncertainty, a combination of an \( S \)-wave and a \( D \)-wave Breit-Wigner is used instead. Relative contributions of 72% and 28% are used, based on a decay angle analysis of the \( D_{11}^+ \) by the Belle Collaboration [24]. Using the modified signal line shape, uncertainties of \( -9(\pm8) \text{ keV}/c^2 \) in \( \Delta m(D_{11}^+) \) and \( -2(-3) \text{ keV} \) in \( \Gamma(D_{11}^+) \) are derived, compared with the standard results.

As an additional check, the value of \( R \) (Eq. (6)) is set to \( 2.0 (\text{GeV}/c)^{-1} \). No effect on \( \Delta m(D_{11}^+) \) and \( \Gamma(D_{11}^+) \) is observed.

The effect of neglecting \( D^0K^+ \) decays (Sec. VI) is studied by setting \( B_1 = 1 \) and \( B_2 = 0 \). The resulting uncertainties are negligible for both \( \Delta m(D_{11}^+) \) and \( \Gamma(D_{11}^+) \).

Numerical precision of convolution. The integration range and number of steps in the numerical convolution of the signal line shape and resolution function (Sec. VI) are varied, resulting in a negligible deviation both for the mass and the width.

Mass window. The mass window for \( \Delta m(D_{11}^+) \) is enlarged, resulting in no significant change for \( \Delta m(D_{11}^+) \) and a difference for the width of \( \Delta \Gamma = +9(+3) \text{ keV} \).

Background parametrization. For background parametrization, a power law function proportional to \( m^\alpha \) is used instead of a linear function, leaving \( \Delta m(D_{11}^+) \) unaffected but yielding \( \Delta \Gamma = -5(-7) \text{ keV} \) for \( K4\pi (K6\pi) \).

C. Reconstruction uncertainties

Tracking region material. Uncertainties in the \( D_{11}^+ \) mass may arise from uncertainties in the energy-loss correction in charged particle tracking. Studies of \( \Lambda \) and \( K_0^* \) decays suggest that the correction might be underestimated [33].

Two possibilities are considered, one with the SVT material density increased by 20% and the other with the tracking region material density (SVT, DCH) increased by 10%, as was investigated in detail in Refs. [4,33]. The deviations indicate that the fit results for the mass might be underestimated. The larger values from the two studies \( \Delta \sigma_m = +21(+13) \text{ keV}/c^2 \) and \( \Delta \Gamma = +14(-15) \text{ keV} \) for \( K4\pi (K6\pi) \) are chosen as a two-sided systematic uncertainty.

SVT alignment. Slight possible deviations in the alignment of SVT components may affect the measurement of angles between tracks and thus the mass measurement. This is studied by applying small distortions to the SVT alignment in simulated data, comprising general changes between different run periods and radial shifts. Results are \( \Delta \sigma_m = -6(-7) \text{ keV}/c^2 \) and \( \Delta \Gamma = +2(14) \text{ keV} \) for \( K4\pi (K6\pi) \).

Magnetic field. The magnetic field inside the tracking volume has several components: the main solenoidal field, fields from permanent magnets and an additional magnetization of the latter due to the main field. To understand the effect of the field on the track reconstruction, the solenoid field strength is varied by \( \pm 0.02% \) and the magnetization of the permanent magnets by \( \pm 20% \) [4,33]. For the mass difference, the larger deviations arising from the change in rescaled solenoid field and magnetization are added in quadrature and the sum is assigned as a systematic uncertainty associated with the magnetic field; the same is done for the decay width. The results are \( \Delta \sigma_m = -13(\pm19) \text{ keV}/c^2 \) and \( \Delta \Gamma = +19(+11) \text{ keV} \) for \( K4\pi (K6\pi) \).
Distance scale. A further source of uncertainty for the momentum determination comes from the distance scale. The positions of the signal wires in the drift chamber are known with a precision of 40 μm, corresponding to a relative precision of 0.01%. As an estimate of the uncertainty of the momentum due to the distance scale, a systematic error half the size of the uncertainty obtained from the 0.02% variation of the solenoid field is assigned. For the mass difference this yields a shift of ±4 (±6) keV/c² for K4π (K6π); the width is shifted by ±8 (±4) keV for K4π (K6π).

Drift Chamber hits. In the standard D_{s1}^+ selection no lower limit is set for the number of drift chamber hits. Requiring at least 20 hits per track, thereby excluding the low momentum pions from D^{*+} decays, modifies ∆m(D_{s1}^+) by −11 (−15) keV/c² and Γ(D_{s1}^+) by −7 (−7) keV for K4π (K6π).

Angular dependence. For the reconstructed K₀^0 and D⁰ masses from the test data samples (see Sec. IV), a sinelike dependence on the azimuthal angle ϕ is observed. This effect was also observed in a previous BABAR analysis and might be related to the internal alignment of the DCH [33]. For a detailed study, the same ϕ-dependence is introduced into the signal MC samples by modifying the kaon and pion track momenta accordingly. Because of symmetry, the effect disappears when all ϕ angles are taken into account, but as a conservative estimate the amplitude of the sinelike shift on the reconstructed D_{s1}^+ mass in MC (13 (14) keV/c² for K4π (K6π)) is taken as a systematic error for ∆m(D_{s1}^+).

IX. RESULTS

For the combination of the measurements, a Best Linear Unbiased Estimate (BLUE, [34]) technique is used, where correlations between the systematic uncertainties are taken into account. Adding the nominal D⁺ and K₀^0 masses, 2010.25 MeV/c² and 497.614 MeV/c² (with their respective errors of 0.140 MeV/c² and 0.024 MeV/c² [21]), the final value for the D_{s1}^+ mass is

\[
m(D_{s1}^+) = 2535.08 \pm 0.15 \text{ MeV/c}^2.
\]

Using a slightly different method for the combination of the individual results [4], a value of

\[
m(D_{s1}^+) = 2535.08 \pm 0.01 \pm 0.15 \text{ MeV/c}^2
\]

is obtained, where the first error denotes the statistical and the second the systematic uncertainty. The latter is dominated by the uncertainty of the D^{*+} mass. The mass difference between the D_{s1}^+ and the D^{*+} is

\[
m(D_{s1}^+) - m(D^{*+}) = 524.83 \pm 0.04 \text{ MeV/c}^2,
\]

using the BLUE technique, and for the alternative combination method

\[
m(D_{s1}^+) - m(D^{*+}) = 524.83 \pm 0.01 \pm 0.04 \text{ MeV/c}^2,
\]

which has a significantly smaller systematic uncertainty than the m(D_{s1}^+) result.

For the total decay width of the D_{s1}^+, combining the results from the two measurements in the same way as for the mass yields

\[
Γ(D_{s1}^+) = 0.92 \pm 0.05 \text{ MeV},
\]

using the BLUE technique, and for the alternative combination method

\[
Γ(D_{s1}^+) = 0.92 \pm 0.03 \pm 0.04 \text{ MeV}.
\]

The corrections of −51 (−45) keV for the K4π (K6π) decay mode, based on the systematic resolution studies (Sec. VIII A), are applied prior to the combination process.

X. SUMMARY

In this paper, precision measurements of the mass and decay width of the charmed-strange meson D_{s1}(2536)^+ via the decay D_{s1}(2536)^+ → D^{*+}K₀^0 are presented. Two decay modes are analyzed, with the D⁰ that originates from the D^{*+} decaying either through K^−π^+ or K^-π^+π^+π^-.

The results include the first significant measurement of the total decay width of the D_{s1}^+. This width is determined to be

\[
Γ(D_{s1}^+) = 0.92 \pm 0.03 \pm 0.04 \text{ MeV},
\]

compared to the 90% confidence level upper limit of 2.3 MeV given in Ref. [21]. The mass of the D_{s1}(2536)^+ is measured to be

\[
m(D_{s1}^+) = 2535.08 \pm 0.01 \pm 0.15 \text{ MeV/c}^2,
\]

and the D_{s1}^+ − D^{*+} mass difference to be

\[
m(D_{s1}^+) - m(D^{*+}) = 524.83 \pm 0.01 \pm 0.04 \text{ MeV/c}^2.
\]

The result for the D_{s1}^+ − D^{*+} mass difference represents a significant improvement compared to the current world average of 525.04 ± 0.22 MeV/c² [21].

Based on a decay angle analysis, the J^P = 1^+ assignment for the D_{s1}^+ meson is confirmed.

ACKNOWLEDGMENTS

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), the Commissariat à l’Energie Atomique and Institut National de Physique.
Nucle´aire et de Physique des Particules (France), the Bundesministerium fu¨r Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Education and Science of the Russian Federation, Ministerio de Educaci´on y Ciencia (Spain), and the Science and Technology Facilities Council (United Kingdom). Individuals have received support from the Marie-Curie IEF program (European Union) and the A. P. Sloan Foundation.

[19] The use of charge conjugated reactions is implied throughout the text.