Top quark mass measurement using the template method at CDF

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We present a measurement of the top quark mass in the lepton + jets and dilepton channels of $t\bar{t}$ decays using the data sample corresponding to an integrated luminosity of 5.6 fb$^{-1}$ of $p\bar{p}$ collisions at Tevatron with $\sqrt{s}=1.96\text{ TeV}$, collected with the CDF II detector. We construct templates of two reconstructed top quark masses from different jets-to-quarks combinations and the invariant mass of two jets from the $W$ decays in the lepton + jets channel, and a reconstructed top quark mass and $m_T^Z$, a variable related to the transverse mass in events with two missing particles, in the dilepton channel. The simultaneous fit of the templates from signal and background events in the lepton + jets and dilepton channels to the data yields a measured top quark mass of $M_{\text{top}} = 172.1 \pm 1.1(\text{stat}) \pm 0.9(\text{syst}) \text{ GeV}/c^2$. 


The top quark ($t$) is by far the heaviest known elementary particle [1]. The top quark contributes significantly to electroweak radiative corrections relating the top quark mass ($M_{\text{top}}$) and the $W$ boson mass to the mass of the predicted Higgs boson [2,3]. Precision measurements of $M_{\text{top}}$ provide therefore important constraints on the Higgs boson mass. Since the discovery of the top quark in 1995 [4] at the Fermilab Tevatron $p\bar{p}$ Collider, both the CDF and D0 experiments have been improving the precision of the $M_{\text{top}}$ measurement [5]. However it is important to measure $M_{\text{top}}$ using different techniques and independent data samples in different decay channels. Significant differences in the measurements of $M_{\text{top}}$ in different decay channels could indicate contributions from new physics beyond the SM [6].

This letter reports a measurement of the top quark mass using the template method [7–9]. We use samples of $t\bar{t}$ candidates in the lepton + jets and dilepton channels, corresponding to an integrated luminosity of 5.6 fb$^{-1}$ of proton-antiproton collisions at $\sqrt{s}=1.96\text{ TeV}$, collected by the CDF II detector [10]. This is a general-purpose detector designed to study $p\bar{p}$ collisions at the Fermilab Tevatron. A charged-particle tracking system, consisting of a silicon microstrip tracker and a drift chamber, is immersed in a 1.4 T magnetic field. Electromagnetic and hadronic calorimeters surround the tracking system and measure particle energies. Drift chambers and scintillators, located outside the calorimeters, detect muon candidates.

Assuming unitarity of the three-generation CKM matrix, the top quark decays almost exclusively into a $W$ boson and a $b$ quark [1]. The case where one $W$ decays leptonically into an electron or a muon plus a neutrino and the other hadronically into a pair of jets defines the lepton + jets decay channel. The dilepton channel is defined as the case where both $W$’s decay leptonically into an electron or a muon plus a neutrino.

Lepton + jets events are selected by requiring one isolated [11] electron (muon) with $E_T > 20 \text{ GeV}$ ($p_T > 20 \text{ GeV}/c$) and pseudorapidity $|\eta| < 1.1$ [12]. We also require high missing transverse energy [13], $E_T > 20 \text{ GeV}$, and at least four jets. Jets are reconstructed with a cone algorithm [14] with radius $R=\sqrt{\Delta \eta^2 + \Delta \phi^2}=0.4$. Jets originating from $b$ quarks are identified (tagged) using a secondary vertex tagging algorithm [15]. We request at least one jet to be tagged as a $b$ jet. We divide the sample of candidate lepton + jets events into subsamples of one $b$-tagged jet (1-tag) and two or more $b$-tagged jets (2-tag). In events with more than two $b$-tagged jets, we consider the two highest $E_T$ jets as $b$ quark candidates and treat the other $b$-tagged jets as non-$b$-tagged jets. In the 1-tag sample, we require exactly four jets with transverse energy $E_T > 20 \text{ GeV}$ and $|\eta| < 2.0$. In the 2-tag sample, three jets are required to have $E_T > 20 \text{ GeV}$ and $|\eta| < 2.0$, and at least one more jet is required to have $E_T > 12 \text{ GeV}$ and $|\eta| < 2.4$. We apply an additional cut on the scalar sum of transverse energies in the event, $H_T = E_{T\text{lepton}} + E_T + \sum_{\text{jets}} E_{T\text{jet}}$, requiring $H_T > 250 \text{ GeV}$ for all events to further reject backgrounds. $E_{T\mu\text{muon}} = p_{T\mu\text{muon}}$ is assumed in the $H_T$ calculation.

The primary sources of background in the lepton + jets channel are $W$ + jets and QCD multijet production. We also consider small contributions from $Z$ + jets, diboson, and single-top production. To estimate the contribution of each process, we use a combination of data and Monte Carlo (MC)-based techniques described in Ref. [16]. For the $Z$ + jets, diboson, single top, and $t\bar{t}$ events we normalize MC simulation events using their respective theoretical cross sections [17–19]. QCD multijet background is estimated using the data referring to techniques described in Ref. [20]. The shape of $W$ + jets background is obtained from MC while the number of $W$ + jets events is determined from the data by subtracting all the other backgrounds and $t\bar{t}$.

Three observables are used from each lepton + jets event: two reconstructed top quark masses ($m_{\text{top}}^{\text{reco}}$ and $m_{\text{top}}^{\text{reco(2)}}$) and the invariant mass of the two jets from the
hadronically decaying $W$ boson ($m_{jj}$). We have complete reconstruction of the $t\bar{t}$ kinematics in the lepton + jets channel [7,8] with constraints from the precise $W$ boson mass and requiring the $t$ and $\bar{t}$ masses to be the same. With the assumption that the leading four jets in the detector come from the $t\bar{t}$ decay products, there are six and two possible assignments of jets to quarks for 1-tag and 2-tag, respectively. A minimization is performed for each assignment using a $\chi^2$ comparison to the $t\bar{t}$ hypothesis with $m_{\text{reco}}$ taken from the assignment that yields the lowest $\chi^2$. To increase the statistical power of the measurement, we employ an additional observable $m_{t_j}^{\text{reco}(2)}$ from the assignment that yields the second lowest $\chi^2$. Events with the lowest $\chi^2 > 9.0$ are removed from the sample to reject poorly reconstructed events. The dijet mass $m_{jj}$ is calculated as the invariant mass of two non $b$-tagged jets which provides the closest value to the world average $W$ boson mass of 80.40 GeV/$c^2$ [1].

To select dilepton candidate events, we require two oppositely charged leptons with $E_T > 20$ GeV (for electrons) or $p_T > 20$ GeV/$c$ (for muons). One lepton is required to be isolated in the central region ($|\eta| < 1.1$) of the detector, but the other can be a nonisolated lepton in the central region or an isolated electron in the forward region ($1.1 < |\eta| < 2.0$). We also require $E_T > 25$ GeV, and at least two jets with $E_T > 15$ GeV and $|\eta| < 2.5$. To further reject backgrounds, we require $H_T > 200$ GeV. In measuring the top quark mass, we divide the dilepton sample into events with $b$-tagged jets (tagged) and without $b$-tagged jets (nontagged).

Drell-Yan, diboson, and $W$ + jets (fake lepton) events are the primary sources of background in the dilepton channel. We estimate the rate of the Drell-Yan and diboson events with calculations based on MC simulations. For the Drell-Yan $Z$ + jets process, we normalize the MC sample by matching the number of $Z$ events predicted and observed in the $Z$ mass region between 76 GeV/$c^2$ and 106 GeV/$c^2$. We use data to estimate the rate of $W$ + jets (fake lepton) events where an event has one real lepton and one of the jets misidentified as the other lepton. The detailed procedure of background estimation in the dilepton channel is described in Ref. [21]. For each event, we calculate a reconstructed top quark mass $m_{t_{\text{NWA}}}$ using the neutrino weighting algorithm [22], and we calculate a quantity $m_{t_{2}}$ [23]. Here, $m_{t_{2}}$ is a variable related to the transverse mass of the mother particles (top quark in the $t\bar{t}$ system) in events with two missing particles from pair production of the mother particles. We first use this variable for the top quark mass measurement in the dilepton channel [9]. We require these observables to be consistent with the top quark signal by demanding $100 \text{ GeV}/c^2 < m_{t_{\text{NWA}}} < 350 \text{ GeV}/c^2$ and $30 \text{ GeV}/c^2 < m_{t_{2}} < 200 \text{ GeV}/c^2$. The estimated number of background events and the observed number of events after event selection are listed in Table II for the dilepton decay channel.

We estimate the probability density functions (PDFs) of signal and background using kernel density estimation (KDE) [24]. In the lepton + jets channel, we use the three dimensional KDE that accounts for the correlation between the three observables. In the dilepton channel, instead, we use the two-dimensional KDE. The dijet mass $m_{jj}$ of the two jets assigned to the $W$ in the lepton + jets channel is used for in situ calibration of jet energy scale (JES) [7,8]. The PDFs for the observables are estimated at discrete values of $m_{\text{top}}$ from 130 GeV/$c^2$ to 220 GeV/$c^2$, with increments from 0.5 GeV/$c^2$ in the region immediately above and below 172.5 GeV/$c^2$ to 5 GeV/$c^2$ near the extreme mass values, and at discrete values of $\Delta_{\text{JES}}$ from

| TABLE I. Expected and observed numbers of signal and background events assuming $t\bar{t}$ production cross section $\sigma_{t\bar{t}} = 7.4$ pb and $M_{\text{top}} = 172.5$ GeV/$c^2$ in the lepton + jets channel. |
|-------------|-------------|
| $W$ + jets  | 53.4 ± 17.5 | 8.5 ± 3.0 |
| QCD multijet| 13.1 ± 10.6 | 1.8 ± 1.5 |
| $Z$ + jets  | 4.7 ± 1.0   | 0.5 ± 0.1 |
| Diboson     | 6.3 ± 0.8   | 0.8 ± 0.1 |
| Single top  | 4.9 ± 0.4   | 2.0 ± 0.2 |
| Background  | 105 ± 21    | 14.2 ± 3.5|
| $t\bar{t}$ signal | 590 ± 74 | 293 ± 45 |
| Expected    | 694 ± 77    | 307 ± 45  |
| Observed    | 695         | 286       |

| TABLE II. Expected and observed number of signal and background events assuming $t\bar{t}$ production cross section $\sigma_{t\bar{t}} = 7.4$ pb and $M_{\text{top}} = 172.5$ GeV/$c^2$ in the dilepton channel. |
|-------------|-------------|
|            | nontagged   | tagged      |
| Diboson     | 19.2 ± 3.3  | 0.7 ± 0.2   |
| Drell-Yan   | 31.5 ± 3.9  | 3.7 ± 0.2   |
| $W$ + jets  | 30.8 ± 9.4  | 4.6 ± 1.3   |
| Background  | 81.6 ± 10.4 | 8.9 ± 1.4   |
| $t\bar{t}$ signal | 124 ± 16 | 151 ± 19 |
| Expected    | 205 ± 19    | 160 ± 19    |
| Observed    | 237         | 155         |
...from the input top mass) and the width of the pull (the ratio of the residual to the uncertainty reported by MINUIT) for simulated experiments are corrected to be unity and zero, respectively. The corrections are \( m_{\text{corr}} = 1.04 \times m_{\text{meas}} - 6.8 \text{ GeV/c}^2 \), \( m_{\text{corr}} = 1.03 \times m_{\text{meas}} - 5.5 \text{ GeV/c}^2 \), and \( m_{\text{corr}} = 1.03 \times m_{\text{meas}} - 5.9 \text{ GeV/c}^2 \) for combined fit, lepton + jets, and dilepton channel, respectively, where \( m_{\text{meas}} \) is the raw value from likelihood fit and \( m_{\text{corr}} \) is the corrected value of the measurement. We increase the measured uncertainty by 4% for combined fit and lepton + jets channel and 3% for dilepton channel to correct the width of the pull.

We examine various sources of systematic uncertainties that could affect the measurement by comparing the results of pseudoexperiments in which we vary relevant parameters within their systematic uncertainties. The dominant sources of systematic uncertainty are the residual JES [8,25] and signal modeling. We vary JES parameters within their systematic uncertainties. The dominant uncertainties in both signal and background MC generated event sand interpret the shifts in the results of their uncertainties in both signal and background MC generated event sand interpret the shifts in the results of the pseudoexperiments as uncertainties. For the dilepton channel, which has no in situ calibration, the JES is the single dominant uncertainty. The uncertainty arising from the choice of MC generator (signal modeling) is estimated by comparing the results of pseudoexperiments generated with PYTHIA [29] and HERWIG [30]. The b-JES systematic uncertainty arising from our modeling of b fragmentation, b hadron branching fractions, and calorimeter response captures the additional uncertainty not taken into account in the (residual) JES. We estimate the systematic uncertainty due to modeling of initial-state gluon radiation and final-state gluon radiation by extrapolating uncertainties in...
the $p_T$ of Drell-Yan events to the $t\bar{t}$ mass region [7]. We vary the parameters of parton distribution functions and gluon fusion fraction within their uncertainties to account systematic effects. We estimate systematic uncertainties due to the lepton energy and momentum scales by propagating shifts in electron energy and muon momentum scales within their uncertainties. Background shape systematic uncertainties account for the variation of the background composition. We estimate the multiple hadron interaction systematic uncertainty to account the effect from the difference in the average number of interactions between MC samples and the data. The color reconnection systematic uncertainty [31] is evaluated by MC samples generated with and without color reconnection effects using different tunes [32] of PYTHIA. All systematic uncertainties are summarized in Table III. The total systematic uncertainties, adding individual components in quadrature, are $0.9\, \text{GeV}/c^2$ in the combined fit, $0.9\, \text{GeV}/c^2$ in the lepton + jets channel, and $3.1\, \text{GeV}/c^2$ in the dilepton channel.

We perform the likelihood fits to the data using the observables discussed in this letter and apply the corrections obtained using the simulated experiments. We obtain for the lepton + jets channel, a top quark mass

$$M_{\text{top}} = 172.2 \pm 1.2\,(\text{stat}) \pm 0.9\,(\text{syst})\,\text{GeV}/c^2$$

$$= 172.2 \pm 1.5\,\text{GeV}/c^2,$$

while for the dilepton channel,

$$M_{\text{top}} = 170.3 \pm 2.0\,(\text{stat}) \pm 3.1\,(\text{syst})\,\text{GeV}/c^2$$

$$= 170.3 \pm 3.7\,\text{GeV}/c^2.$$
In conclusion, we have performed a measurement of the top quark mass using the template method simultaneously in the lepton + jets and dilepton channels. The result, $M_{\text{top}} = 172.1 \pm 1.4 \text{GeV}/c^2$, is consistent with the most recent world average of $M_{\text{top}} = 173.3 \pm 1.1 \text{GeV}/c^2$ [5]. In the lepton + jets channel, we use the same data set as the best single $M_{\text{top}}$ measurement [33], and have a consistent result with slightly larger uncertainty. In the dilepton channel, we achieve the single most precise measurement of $M_{\text{top}}$ in this channel to date and the result is in good agreement with the measurement in the lepton + jets channel.

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[2] ALEPH, CDF, D0, DELPHI, L3, OPAL, SLD, the LEP Electroweak Working Group, the Tevatron Electroweak Working Group, the SLD Electroweak, and Heavy Flavor Working Groups, arXiv:1012.2367v2.
[11] A lepton is isolated when $(p_T^\text{total} - p_T^\text{lepton})/p_T^\text{lepton} < 0.1$, where $p_T^\text{total}$ is the total transverse momentum (energy) and $p_T^\text{lepton}$ is the lepton transverse momentum (energy) for muon (electron) in a cone of radius $\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = 0.4$ [12] with axis along the direction of the lepton.
[12] We use a right-handed spherical coordinate system with the origin at the center of the detector with the $z$-axis along the proton beam and the $y$-axis pointing up. $\theta$ and $\phi$ are the polar and azimuthal angles, respectively. The pseudorapidity is defined by $\eta = -\ln(\tan(\theta/2))$. The transverse momentum and energy of a detected particle or jet are defined by $p_T = p \sin(\theta)$ and $E_T = E \sin(\theta)$, respectively, where $p$ and $E$ are the momentum and energy of the particle. For the reconstructed top quark decay products used in the $m_{T^2}$ calculation, the transverse energy is defined by $E_T = \sqrt{m^2 + p_T^2}$, where $m$ is the mass of the product.
[13] The missing transverse energy, an imbalance of energy in the transverse plane of the detector, is defined by $E_T = |\sum_{\text{towers}} E_T \hat{n}_T|$, where $\hat{n}_T$ is the unit vector normal to the beam and pointing to a given calorimeter tower and $E_T$ is the transverse energy measured in that tower.