The Perils of Behavior-Based Personalization

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The Perils of Behavior-Based Personalization

Abstract

“Behavior-based personalization” has gained popularity in recent years, whereby businesses offer personalized products based on consumers’ purchase histories. This paper highlights two perils of behavior-based personalization in competitive markets. First, although purchase histories reveal consumer preferences, competitive exploitation of such information damages differentiation, similar to the classic finding that behavior-based price discrimination intensifies price competition. With endogenous product design, there is yet a second peril. It emerges when forward-looking firms try to avoid the first peril by suppressing the information value of purchase histories. Ideally, if a market leader serves all consumers on day one, purchase histories contain no information about consumer preferences. However, knowing that their rivals are willing to accommodate a market leader, firms are more likely to offer a mainstream design at day one, which jeopardizes differentiation. Based on this understanding, I investigate how the perils of behavior-based personalization change under alternative market conditions, such as firms’ better knowledge about their own customers, consumer loyalty and inertia, consumer self-selection, and the need for classic designs.

Keywords: behavior-based personalization; behavior-based price discrimination; revealed preference; segmentation; targeting; competition; customer relationship management; endogenous information generation
1 Introduction

“Behavior-based personalization” is gaining popularity in recent years, whereby businesses offer personalized products based on consumers’ purchase histories. Firms that have adopted this strategy include Amazon, Barnes and Noble, Digg, eBay, iTunes, Netflix, and YouTube, to name a few. The main argument for behavior-based personalization is the notion of revealed preferences; since consumers’ past choices reflect their tastes, firms can serve consumer needs better by modifying product offerings based on choice histories (Arora et al. 2008).

However, I argue that behavior-based personalization can hurt the profits of competing firms. Suppose a consumer subscribes to Netflix. Blockbuster can infer that this consumer likely appreciates the convenience of online DVD rentals. By offering her an online rental option, Blockbuster serves this consumer better. But this move only diminishes the differentiation between Netflix and Blockbuster to both firms’ detriment. I label this effect the “first peril of behavior-based personalization,” which echoes the classic result that behavior-based price discrimination intensifies price competition (Villas-Boas 1999; Fudenberg and Tirole 2000).

With endogenous product design, there is yet another peril. It arises as forward-looking firms try to strategically avoid the first peril. One obvious way is to have a market leader serve all consumers at day one. For example, let us suppose Netflix adopts a more “mainstream” service mode convenient for everyone, and captures the entire market. Subscription to Netflix then contains no information about a consumer’s relative preference for online rentals, thus preventing both firms from offering personalized services. In other words, the mere incentive to avoid the first peril would compel Blockbuster to accommodate Netflix as a market leader. However, the same argument holds true had Blockbuster become the market leader in the first place. Indeed, knowing that their rivals are willing to welcome a market leader, both firms will offer a more mainstream service on day one which jeopardizes differentiation. I label this effect the “second peril of behavior-based personalization.”

I formalize the above intuition with a two-period duopoly model. Consumers have heterogeneous tastes and are uniformly distributed on a Hotelling line. Firms cannot collude, or
commit to ignoring consumer purchase histories, in making product and pricing decisions. I start with a simple way to conceptualize consumers’ purchase histories: each firm offers one standard product in the first period and recognizes its customers. In the second period, a prisoner’s dilemma outcome arises as each firm chooses to offer different personalized designs to its customers and the rival firm’s patrons even though simultaneous personalization by both firms reduces differentiation. This effect leads to the first peril of behavior-based personalization, where both firms are worse off in period two than if personalization were impossible. In addition, the more symmetric first-period market shares, the more informative purchase histories are in the “entropy” sense (Shannon and Weaver 1949), and the more severe the first peril.

For a detailed view of the market force leading to the second peril, suppose firms were sharing the market evenly in period one. Now, firm $A$ unilaterally moves its product closer to the market center. Firm $B$ will normally respond by lowering prices to recoup some of its lost market share. However, firm $B$ also realizes that its current loss in market share helps attenuate the first peril in the future, and will thus respond with a smaller price cut than if it were myopic. Anticipating firm $B$’s mild response in prices, firm $A$ will be more aggressive with its product strategy and will offer a design closer to the market center. However, the same reasoning applies to firm $B$ as well. The net result is reduced differentiation and lower profits in period one, although in this symmetric equilibrium firms still fail to avoid the first peril.

The emergence of the second peril contrasts findings from the behavior-based price discrimination literature. In particular, when analyzing the same market setting, Fudenberg and Tirole (2000) find that price discrimination based on purchase histories hurts profits in period two, similar to the first peril, but benefits firms in period one by softening price competition. The benefit comes from a demand-side effect, that forward-looking consumers are less price-sensitive in period one, anticipating period-two price discrimination. Notably, forward-looking firms’ incentive to avoid the first peril has no impact on period-one competition. This is because a change in period-one market shares from an even split has zero first-order effect on period-two profits—each firm’s marginal gain from one clientele is offset by the marginal loss from the other. Firms thus have no incentive to vary period-one prices to influence market shares.
This last result no longer holds with endogenous product design. In a symmetric equilibrium, a change in period-one market shares still has zero first-order effect on period-two profits. However, forward-looking firms’ incentive to avoid the first peril affects how they respond to their rivals’ period-one product design in the pricing sub-game. As a result, the first-order conditions of firms’ period-one product choices shift with their degree of patience, which gives rise to the second peril. The second peril can outweigh the effect of decreased period-one demand elasticity, so that period-one profits can be even lower than if behavior-based personalization were impossible, in contrast to Fudenberg and Tirole (2000).

I extend the main model to explore factors that affect the perils of behavior-based personalization. I find that the attenuation of one peril often exacerbates the other. For example, one might conjecture that consumer preference information will be more beneficial to a firm if the rival cannot access this information: a firm may observe its customers’ exact preferences besides purchase histories; alternatively, a new generation of consumers may enter the market in period two such that the rival cannot perfectly identify the firm’s previous customers. In both cases, knowing more about their own customers does improve firms’ profits in period two, thus mitigating the first peril, but hurts profits in period one as firms compete more aggressively for customers. The same result holds if firms can exploit consumer loyalty and inertia. Behavior-based personalization becomes more profitable in period two when customers are reluctant to switch. However, firms in period one will again compete more intensively for market shares.

Nevertheless, there do exist market conditions under which firms can reduce the perils of behavior-based personalization. If consumers are able to self-select among all personalized designs, firms will abandon personalization altogether to avoid intra-firm cannibalization. Alternatively, if firms are committed to providing a “classic design” for their old customers, it will help mitigate both perils. Finally, asymmetric patience between firms attenuates the first peril, whereas a larger segment of forward-looking consumers reduces the second peril.

I continue in §2 with a literature review. §3 introduces the model setup. §4 presents the main analysis, and §5 extends the main model in several ways. §6 discusses a set of testable empirical implications. §7 concludes the paper. All proofs are collected in the Online Appendix.
2 Related Literatures

This paper is related to the theoretical literature on “behavior-based price discrimination,” meaning price discrimination based on customers’ purchase histories. A well-known result is that conditioning prices on purchase histories can damage profits. Villas-Boas (1999) and Fudenberg and Tirole (2000) show that since purchases reveal preferences, firms will poach their rivals’ customers with lower prices than if purchase histories were unobservable. Villas-Boas (2004) finds that behavior-based price discrimination hurts a monopolist seller’s profits because strategic consumers foresee future price cuts offered to low-valuation consumers. Acquisti and Varian (2005) consider a seller’s ability to commit to a pricing policy, and find that conditioning prices on purchase histories is generally unprofitable. Pazgal and Soberman (2008) explore the possibility of adding value to past customers. Although firms can lock in their customers in this way, they compete more aggressively for customers on day one.

This literature also uncovers a number of reasons why behavior-based discrimination might benefit firms, including different enhanced services to high- versus low-valuation past customers (Acquisti and Varian 2005), the coexistence of loyal versus price-sensitive consumers (Chen and Zhang 2009), stochastic consumer preferences and heterogeneous purchase quantities (Shin and Sudhir 2009). Notably, Fudenberg and Tirole (2000) find that the anticipation of behavior-based price discrimination makes consumers less price sensitive, which helps soften early-stage price competition. Table 1 presents a detailed summary of these studies on behavior-based price discrimination, comparing their main findings and underlying market forces.

This paper extends the behavior-based price discrimination literature by endogenizing firms’ product design decisions. Practically, the product design perspective is important today for at least two reasons. First, the revolution in flexible manufacturing has significantly lowered the costs of personalization (Dewan, Jing and Seidmann 1993). Second, price discrimination

\footnote{See Fudenberg and Villas-Boas (2007) for a comprehensive review of this literature. There is also an empirical literature that investigates the efficacy of conditioning prices on purchase histories (see for example Rossi, McCulloch, and Allenby 1996; Besanko, Dubé, and Gupta 2003.)}

\footnote{For comparability, in presenting the main findings of Fudenberg and Tirole (2000), I focus on the setting closest to mine, which features a two-period market with uniformly distributed tastes and short-term contracts.}
<table>
<thead>
<tr>
<th>Study</th>
<th>Main findings</th>
<th>Mechanism</th>
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<tbody>
<tr>
<td>Villas-Boas (1999)</td>
<td>Hurts firms when firms and consumers are patient</td>
<td>In an infinite-horizon duopolist market with overlapping generations of consumers, firms price below static levels to poach rivals’ customers; patient consumers are more indifferent to which product to buy first and more sensitive to current prices, which intensifies price competition</td>
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<tr>
<td>Fudenberg and Tirole (2000)</td>
<td>Hurts firms in period two; benefits firms in period one</td>
<td>In period two, duopolist firms price below static levels to poach their rival’s customers, who have revealed their relative preference for the rival; in period one, firms price above static levels because consumers, who anticipate second-period poaching, are less price sensitive</td>
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<tr>
<td>Villas-Boas (2004)</td>
<td>Hurts firms</td>
<td>In an infinite-horizon monopolist market with overlapping generations of consumers, prices cycle in equilibrium; the firm is worse off than without customer recognition because consumers foresee future prices cuts</td>
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<td>Acquisti and Varian (2005)</td>
<td>Generally hurts firms; can benefit firms if they provide enhanced services to returning consumers</td>
<td>If consumers who hold higher valuation of the product also hold higher valuation of the enhanced services provided to returning consumers, the firm can profitably target high-valuation consumers with a high price conditional on initial purchase</td>
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<tr>
<td>Pazgal and Soberman (2008)</td>
<td>Generally hurts firms who add the same values to past customers</td>
<td>Since duopolist firms can lock in their past customers by adding value in period two, they compete more aggressively for customers in period one</td>
</tr>
<tr>
<td>Chen and Zhang (2009)</td>
<td>Can benefit firms in both periods if the market consists of loyal vs. price-sensitive consumers</td>
<td>In period two, duopolist firms profit from being able to target loyal vs. price-sensitive consumers; in period one, firms want to charge a higher price than the rival to identify the loyal consumers, thus softening competition</td>
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<tr>
<td>Shin and Sudhir (2009)</td>
<td>Can benefit firms in both periods with stochastic preferences and heterogeneous purchase quantities</td>
<td>In period two, duopolist firms compete less aggressively if some consumers automatically prefer their product due to stochastic preferences, or if low prices disproportionately attract low-value customers; in period one, consumers anticipate second-period poaching and are less price sensitive, which softens price competition</td>
</tr>
<tr>
<td>This study</td>
<td>Hurts firms in period two (the first peril); can hurt firms in period one (due to the second peril) with endogenous product design</td>
<td>In period two, duopolist firms offer less differentiated designs and lower prices than static levels to poach rivals’ customers, who have revealed their relative preference for the rival; in period one, prices may drop below static levels as firms become less differentiated, knowing that the rival is more likely to accommodate mainstream designs</td>
</tr>
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Notes: The static benchmark refers to the case in which consumer purchase histories are unavailable.
may raise regulatory concerns and cause customer antagonism (Anderson and Simester 2010), forcing many businesses to obfuscate price disparities by varying product features. Therefore, product design has frequently been an endogenous variable even for the short run. Theoretically, endogenizing product design reveals the second peril of behavior-based personalization; firms can be even worse off in the early stage of competition than if behavior-based discrimination were impossible, a result opposite to the findings of Fudenberg and Tirole (2000).

In modeling firms’ product decisions, I draw on the literature of spatial competition (e.g., Hotelling 1929; Wernerfelt 1986; Moorthy 1988; Thisse and Vives 1988; Desai 2001). In particular, this paper is related to the research on product customization. Dewan, Jing and Seidmann (2003) analyze the strategic implications when firms can offer a continuous spectrum of perfectly customized products. Syam, Ruan, and Hess (2005) show that competing firms will customize only one of two product attributes to soften price competition. Syam and Kumar (2006) find that offering customized products in addition to standard products can expand demand and improve profits. In the customization literature, firms are often assumed to exogenously know at least the distribution of consumer preferences. This paper, on the other hand, emphasizes consumer purchase history as the endogenous information basis of product design.4

This paper is also connected with the literature on targetability, which refers to a firm’s ability to predict customer characteristics such as brand loyalty (Chen, Narasimhan, and Zhang 2001). Another related literature is customer addressability, where database technologies allow firms to know the tastes of a fraction of consumers and offer customized prices accordingly (Chen and Iyer 2002). This study contributes to these literatures by highlighting firms’ endogenous production of customer preference information by influencing purchase decisions, and sophisticated consumers’ strategic release of preferences information.

4 Arora et al. (2008) distinguish between customization and personalization: customization refers to a consumer’s own specification of product features to purchase, while personalization refers to a firm’s tailored product offerings to an individual consumer based on its data about that consumer. I follow this terminology and use the word “personalization” for the strategy I analyze.
3 Model Setup

Two horizontally differentiated firms, denoted $A$ and $B$, compete in a nondurable-goods market. Both firms incur the same marginal cost of production, which is normalized to zero. In each period consumers have unit demands. I assume that the intrinsic value of the product $v$ is sufficiently high so that all customers are served in equilibrium. Assuming full market coverage allows us to focus on the competition effects.

A unit mass of consumers are uniformly distributed along the Hotelling interval $[0, 1]$. A consumer’s location $x$ represents her ideal product attribute. A consumer incurs a disutility of taste mismatch, or a “transportation cost,” by consuming a product away from her ideal point. I assume that a consumer’s transportation costs are quadratic in her distance to the product. Specifically, if customer $x$ buys a product located at $a$, she incurs transportation costs of $t(x - a)^2$, where $t$ measures the degree of consumer taste heterogeneity. The quadratic assumption is appealing for Hotelling models with endogenous location choices. It ensures that for any product locations a pure-strategy price equilibrium exists. This property may not hold for other functional forms. For example, with linear transportation costs, a pure-strategy price equilibrium does not exist if the two firms are located relatively close to each other (see d’Aspremont, Gabszewicz, and Thisse 1979 for a detailed discussion).

Key to the analysis is the dynamic revelation of consumer taste information, and firms’ and consumers’ influences over this revelation process. I model the dynamics with a two-period game. In period one, firms $A$ and $B$ simultaneously choose the location, or “design,” of their products on the Hotelling line, denoted as $a$ and $b$ respectively. After observing both location choices, firms then simultaneously determine the prices $p_a$ and $p_b$ respectively. It is common in the spatial competition literature to assume that firms’ price decisions occur subsequent to product decisions, since prices are typically faster to change than product design. Consumers choose between products $a$ and $b$ after observing their locations and prices. Each firm recognizes consumers’ choices. However, firms do not observe each consumer’s exact taste (i.e., her exact

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5To identify the market force of interest in a clean and tractable way, I assume that each firm offers one product in the first period. I will discuss this assumption in Section 5.1.1.
location on the Hotelling interval). In other words, I make a minimum-information assumption about consumer purchase histories; a consumer’s product choice reveals her relative preference between the two firms but not the precise strength of her preference.\footnote{Often adopted in the behavior-based price discrimination literature, this minimum-information assumption best describes industries where firms do no have data on consumer preferences other than a snapshot of their previous choices. Theoretically, this assumption highlights the endogenous nature of consumer preference information, that it can only be revealed through actual purchases. In Section 5.1.1 I ask what happens if a firm observes the exact preferences of its own customers.}

In period two, each firm can offer either a \textit{standard product design} to all consumers, or \textit{personalized product designs} to its own customers and the rival’s customers respectively.\footnote{Given the model setup, each firm offers at most two personalized designs as an equilibrium outcome (Lemma 2). In practice, however, if firms have finer-grained preference information, personalization may imply a wider array of products further tailored to individual consumer preferences.} If both firms choose standard product design, second-period competition reduces to a static location-price game. If a firm chooses to personalize its product design, I use the subscript “o” to refer to the design for its own customers, and “n” to denote the design for its new customers. For the main analysis, I allow each firm’s second-period product designs to be different from its first-period design, which is plausible if flexible manufacturing enables firms to update product designs frequently at negligible costs. After observing each others’ location choices, firms simultaneously set prices. I also assume that firms can target a consumer segment with a specific design. That is, if both firms have selected personalization, then firm $A$’s old customers will choose between designs $a_o$ and $b_n$, priced at $p_{a_o}$ and $p_{b_n}$ respectively; firm $B$’s old customers will choose between $a_n$ and $b_o$, priced at $p_{a_n}$ and $p_{b_o}$. To focus on the information role of purchase histories, I assume that consumers incur zero switching costs.

Firms and consumers maximize their total discounted payoffs over the two periods. Let $\beta$ and $\delta$ denote firm and consumer discount factors respectively. I treat $\beta$ and $\delta$ as free parameters to trace the different impact of firm versus consumer patience on market outcomes.

4 Main Analysis

I derive the equilibrium of the two-period game through backward induction, using perfect Bayesian equilibrium as the solution concept (see also Fudenberg and Tirole 2000). In each
period, I first establish consumers’ choices given firm decisions, and then derive firms’ optimal decisions anticipating consumer responses.

4.1 The Second Period

4.1.1 Consumers’ Second-Period Choices

In the second period, each consumer is offered two products given her first-period choice and firms’ personalization strategies. For example, if both firms adopt standard product design, consumers choose between these two products as in a static game. If both firms offer personalized designs, firm $A$’s old customer $x$ can either stay with firm $A$ and purchase product $a_o$, or switch to firm $B$ and buy $b_n$. Consumer $x$ will prefer product $a_o$ if and only if $p_{a_o} + t(x - a_o)^2 < p_{b_n} + t(x - b_n)^2$. A similar decision rule applies to firm $B$’s old customers. Therefore, if both firms offer personalized designs, they will compete over two markets in the second period, each market composed of one firm’s customer base.

4.1.2 Firms’ Second-Period Decisions

Suppose each firm had a positive market share in period one. Consumers’ period-one choices thus reveal their relative preferences, providing firms with a new segmentation variable. The question then is whether firms in period two will indeed base their product and pricing decisions on this segmentation variable. I state the answer in the following lemma (see the Appendix for proof).

**Lemma 1** Having segmented consumers based on their first-period product choices, in the second period both firms (weakly) prefer personalized product designs to standard product design, and (weakly) prefer to offer different prices to different segments.

Lemma 1 reflects the well-established result that firms without commitment power would unilaterally prefer the extra degree of freedom from a discriminatory policy (Thisse and Vives 1988; Shaffer and Zhang 1995). However, it is worth noting that the unilateral profitability of a discriminatory policy does not come from a larger strategy space *per se* (i.e., being able to offer
multiple designs as opposed to one), but from market segmentation as its information basis. Indeed, without segmentation information, each firm will offer only one product even if it has the option to offer multiple products. Lemma 2 states this important result formally (see the Appendix for proof).

Lemma 2 If no segmentation information is available over a uniformly distributed mass of consumers, in a static equilibrium both firms adopt standard product design even if offering additional products is costless.

The intuition behind Lemma 2 is as follows. When a firm offers multiple products without segmentation information, it cannot target a specific product at a specific consumer. Instead, it has to allow consumers to self-select among all products in the market, a practice called “product proliferation” (Arora et al. 2008). Proliferation creates cannibalization within a firm’s own product line, and the decrease in profit margins more than offsets any gain in market share.\(^8\)

Lemma 1 and Lemma 2 together imply that, supposing firms have shared the first-period market, in the second period there will be four personalized products: products \(a_o\) and \(b_o\) for firm A’s old customers, and products \(b_n\) and \(a_n\) for firm B’s old customers. Assume that firm A locates to the left of firm B in any static location-price game without loss of generality. I show in the Appendix that there exists a consumer \(\hat{x}\) who is indifferent between the two firms in period one, such that the segment of consumers over \([0, \hat{x})\) are firm A’s old customers, and the segment \(\hat{x}, 1]\) forms firm B’s clientele. Therefore, period-two competition over either segment is a standard duopolist location-price game. The following lemma summarizes the equilibrium of this class of games (see the Appendix for proof):

Lemma 3 (Equilibrium of static location-price games): Consider an arbitrary segment \([Z, Z + L] \subset \mathbb{R}\) of uniformly distributed consumers of mass \(m\). Two firms first simultaneously

\(^8\)From firms’ perspective, uniform taste distribution represents the least demand information in the entropy sense, as every taste is equally possible (Shannon and Weaver 1949). Therefore, offering multiple personalized products, even if production is free, is unattractive because each product lacks a solid target market. In practice, with demand knowledge sufficiently better than a uniform distribution, a firm may offer a proliferation of multiple products. For example, Wernerfelt (1986) analyzes a market where consumers are concentrated at the two ends of the Hotelling interval, and finds that firms may each offer two products. In this sense, Lemma 2 best describes new markets where firms initially have little demand information.
choose locations, and then simultaneously set prices observing both location choices. Consumers’ transportation costs are quadratic with coefficient $t$. The unconstrained equilibrium locations are $Z - L/4$ and $Z + 5L/4$; firms charge the same price of $3tL^2/2$, each serving a demand of $m/2$ and earning a profit of $3tL^2m/4$.

For the rest of the analysis I shall leave product locations unconstrained, which means firms can locate outside the Hotelling interval (see Tyagi 2000 for the same assumption). By doing so I admit the possibility that firms may locate “off the market” to serve the market. For example, many shopping malls choose remote locations and all their customers have to travel. More generally, products may contain features that all consumers find undesirable; firms may even introduce “nuisance attributes” for differentiation purposes (Gerstner, Hess, and Chu 1993). Nevertheless, the key intuition of the paper remains unchanged if firms must locate within the Hotelling interval (see Section A-6 of the Appendix).

It follows from Lemma 3 that when $x = 0$ or 1, the second-period game degenerates to a static location-price game with the two standard designs located at $-1/4$ and $5/4$ in equilibrium. Correspondingly, each firm charges an equilibrium static price of $3t/2$ and earns an equilibrium static profit of $3t/4$. In subsequent analyses I will frequently compare market outcomes to this static benchmark.

When $x \in (0, 1)$, the equilibrium second-period product locations are:

$$
a_n^*(\hat{x}) = \frac{5}{4} \hat{x} - \frac{1}{4}, \quad a_o^*(\hat{x}) = \frac{1}{4} \hat{x}, \quad b_n^*(\hat{x}) = \frac{5}{4} \hat{x}, \quad b_o^*(\hat{x}) = -\frac{1}{4} \hat{x} + \frac{5}{4}. \quad (1)
$$

Figure 1 illustrates the equilibrium product locations in the second period given that firms shared the first-period market. The smaller a segment, the more tailored the personalized products are for consumers in that segment. This result is intuitive as a smaller segment represents finer knowledge of consumer tastes. Meanwhile, through personalization each firm offers its old customers a design better tailored to their tastes, compared to the case of standard design (since $-\frac{1}{4} < a_o^*(\hat{x}) < 0$ and $1 < b_o^*(\hat{x}) < 5/4$ for any $\hat{x} \in (0, 1)$). Among the four new personalized designs, firm $A$’s products are located to the left of firm $B$’s products. That is,
personalization maintains firms’ core image although part of its purpose is to poach the rival’s customers. Consumers with strong preference for one firm over the other stay with the same firm in period two, while the relatively indifferent consumers switch to the rival firm.

**Figure 1**: First-Period Consumer Choices and Second-Period Product Equilibrium

<table>
<thead>
<tr>
<th>Period one</th>
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<tbody>
<tr>
<td>Buy a</td>
</tr>
<tr>
<td>Buy b</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Period two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay and buy a</td>
</tr>
<tr>
<td>a_o 0</td>
</tr>
<tr>
<td>Switch and buy a</td>
</tr>
<tr>
<td>a_n x</td>
</tr>
<tr>
<td>Stay and buy b</td>
</tr>
</tbody>
</table>

It also follows from Lemma 3 that for $\hat{x} \in (0, 1)$ the second-period equilibrium prices are $p^*_a(\hat{x}) = p^*_b(\hat{x}) = 3t\hat{x}^2/2$ and $p^*_a(\hat{x}) = p^*_b(\hat{x}) = 3t(1 - \hat{x})^2/2$. These prices are all lower than the equilibrium price of $3t/2$ in a static location-price game without personalization. In addition, the equilibrium prices of personalized designs all increase with the size of the target segment, since serving a more diverse set of tastes increases differentiation. Combining these results, firms’ second-period profits are:

$$\pi^*_{A2}(\hat{x}) = \pi^*_{B2}(\hat{x}) = \frac{3}{4}t[\hat{x}^3 + (1 - \hat{x})^3], \quad \hat{x} \in [0, 1].$$

The analysis so far reveals a difference between behavior-based price discrimination and personalization. In the former case, a firm is able to charge a higher price yet occupy a larger market share among its old customers than its rival firm (Villas-Boas 1999; Fudenberg and Tirole 2000). This incumbent advantage comes from the fact that a firm’s clientele naturally consists of consumers whose tastes are more aligned with the firm. However, this advantage dissipates when cost-effective personalization becomes available. On the one hand, the rival firm can offer a product better designed for the incumbent firm’s old customers. On the other hand, the incumbent firm will offer a design less suited to its old customers in order to differentiate.
That is, due to the personalization capacity, incumbent firms can no longer commit to offering the same product designs that attracted their clientele in the first place. As a result, firms earn equal profits over each other’s customer base, as Equation (2) suggests.

It is worth noting that consumer preference information hurts both firms in the second period. The only payoff-relevant variable for the second period is $\hat{x}$, the location of the first-period indifferent consumer. From the information theory perspective, $\hat{x}$ also measures the informativeness of purchase histories. Given uniform taste distribution over $[0, 1]$, the closer $\hat{x}$ is to $1/2$, the lower the Shannon entropy, and the more information it contains (Shannon and Weaver 1949). In the extreme case of $\hat{x}$ being 0 or 1, purchase histories contain no information beyond the prior distribution of consumer tastes. However, both firms’ second-period profits are the lowest when $\hat{x} = 1/2$ and the highest when $\hat{x}$ equals 0 or 1. This is because if purchase history information is available, by Lemma 1 both firms will exploit it as the basis for personalization, but simultaneous personalization “localizes” competition to both firms’ detriment. Therefore, a prisoner’s dilemma outcome emerges in period two, where firms are worse off than if personalization were infeasible (i.e., the static benchmark). I call this effect the “first peril of behavior-based personalization,” and summarize this result in the following proposition.

**Proposition 1 (The first peril of behavior-based personalization):** In the second period, both firms will offer personalized designs based on consumer purchase histories. By doing so, both firms are worse off than if personalization were infeasible.

To escape the first peril, firms would ideally want to avoid consumer preference information altogether. At the extreme, they can suppress preference information by having one firm sell to all consumers in period one, such that firms cannot differentiate consumers based on their past choices. In less extreme scenarios, firms can weaken the informativeness of purchase history by splitting the first-period market more asymmetrically. However, strategic consumers may change their first-period choices to influence the amount of preference information they reveal, and thus the product offers they receive in period two. In the following subsection, I analyze how these opposing incentives affect the first-period market outcome.
4.2 The First Period

4.2.1 Consumers' First-Period Choices

In the first period, consumers choose between products $a$ and $b$ knowing that this decision also determines their choice set in period two.\(^9\) If consumer $x$ buys $a$, in period two she will choose between products $a_o$ and $b_n$. The discounted total cost she pays to buy $a$ is therefore

$$p_a + t(x - a)^2 + \delta \min[p_{a_o} + t(x - a_o)^2, \ p_{b_n} + t(x - b_n)^2].$$

(3)

Similarly, if consumer $x$ buys $b$ in the first period, she incurs a discounted total cost of

$$p_b + t(x - b)^2 + \delta \min[p_{a_n} + t(x - a_n)^2, \ p_{b_o} + t(x - b_o)^2].$$

(4)

By definition, these two costs are equal when $x = \hat{x}$. Meanwhile, since switching always happens in the second period, if the indifferent consumer $\hat{x}$ chooses $a$ she will switch and buy $b_n$ in period two, and if she chooses $b$ she will subsequently purchase $a_n$. Therefore, $\hat{x}$ solves $p_a + t(x - a)^2 + \delta [p_{b_n} + t(x - b_n)^2] = p_b + t(x - b)^2 + \delta [p_{a_n} + t(x - a_n)^2]$. Plugging in the equilibrium values of $a_n$, $b_n$, $p_{a_n}$ and $p_{b_n}$, which in turn are functions of $\hat{x}$, I derive the first-period indifferent consumer as

$$\hat{x} = \frac{a + b}{2} - \frac{p_b - p_a}{25\delta t/8 + 2t(b - a)} + \frac{\delta (1 - a - b)}{2\delta + 32(b - a)/25}.$$  

(5)

Equation (5) reveals the underlying mechanism by which consumers’ strategic reactions influence the market outcome. The first term on the right-hand side, $(a + b)/2$, captures the direct demand-expansion effect of firms positioning towards the market center. The second term is the price effect on first-period demand, which is attenuated by not only product differentiation $(b - a)$ and taste heterogeneity $t$, but also by consumer patience $\delta$. To see why, suppose firms share the market evenly in period one. A unilateral price cut helps a firm attract a larger clientele, but consumers in a larger clientele will receive less tailored products at higher prices.

---

\(^9\)Since there is a continuum of consumers, each consumer’s first-period decision alone does not alter firms’ strategies in the second period.
in period two. Therefore, the more forward-looking these marginal consumers are, the less sensitive they are to current prices. The third term is more subtle. It has the opposite sign of \((a + b)/2 - 1/2\) whenever \(\delta > 0\), but is smaller in magnitude (unless \(a = b\)). In other words, consumer patience weakens the demand-expansion effect; if product \(a\) is closer to the center of the market than product \(b\) (i.e., \(a + b > 1\)), the third term partially offsets this location advantage. To understand this result, suppose consumers are myopic and the corresponding indifferent consumer is \(\hat{x}_{\text{myopic}} > 1/2\). By choosing product \(b\) over \(a\), this consumer will be strictly better off in the second period due to more localized competition over the smaller segment \([\hat{x}_{\text{myopic}}, 1]\). Had this consumer been patient, she would have strictly preferred product \(b\) in anticipation of receiving better offers in the second period. As a result, \(\hat{x}_{\text{patient}} < \hat{x}_{\text{myopic}}\) when \(\hat{x}_{\text{myopic}} > 1/2\). By the same logic, \(\hat{x}_{\text{patient}} > \hat{x}_{\text{myopic}}\) if \(\hat{x}_{\text{myopic}} < 1/2\). That is, patients consumers are less responsive to aggressive designs that are closer to the market center. In fact, absent the price effect, the indifferent consumer will be closer to the center of the market when consumers are patient than when they are myopic.

In summary, consumer patience makes it more likely that firms split the first-period market symmetrically, an outcome representing the most preference information and the least second-period firm profits. Therefore, firms and consumers have conflicting interests regarding the production of consumer taste information. Below I investigate firms’ first-period decisions and the market equilibrium.

4.2.2 Firms’ First-Period Decisions and the Market Equilibrium

In period one, firms first simultaneously choose locations \(a\) and \(b\), and then simultaneously set prices \(p_a\) and \(p_b\) observing both location choices. In doing so, firms maximize their discounted total profits

\[
\pi_A = \pi_{A1} + \beta \pi_{A2}(\hat{x}) = p_a \hat{x} + \beta \pi_{A2}^*(\hat{x}), \quad \pi_B = \pi_{B1} + \beta \pi_{B2}^*(\hat{x}) = p_b (1 - \hat{x}) + \beta \pi_{B2}^*(\hat{x})
\]

(6)

where \(\pi_{A2}^*(\hat{x})\) and \(\pi_{B2}^*(\hat{x})\) are given by Equation (2) and \(\hat{x}\) given by Equation (5). These profit functions suggest another source of conflicts in the market place. Not only do firms
and consumers have opposite preferences regarding personalization, each firm itself also faces an intertemporal tradeoff. Should firms only care about second-period profits, they would have one firm sell to all consumers in period one. However, if firms were completely myopic, they would play a static location-price game in period one and split the market evenly. The equilibrium outcome is therefore jointly shaped by firm patience and consumer patience. I first present the results and then discuss the intuition (see the Appendix for proof).

**Proposition 2 (The second peril of behavior-based personalization):** There exists a symmetric equilibrium in which firms split the market evenly in period one. In this equilibrium, forward-looking firms’ incentive to avoid the first peril of behavior-based personalization further erodes profits: first-period differentiation and prices both decrease with firm patience. Meanwhile, first-period differentiation and prices both increase with consumer patience.

The condition for the symmetric equilibrium to arise holds for the majority of the parameter space, including the case where firms and consumers are equally forward-looking (see Figure A-2 in the Appendix). The rest of this section will focus on describing this symmetric equilibrium. Figure 2 presents how consumer patience and firm patience affect equilibrium first-period product differentiation and prices.

First-period differentiation, as measured by $b^* - a^*$, increases as consumers become more forward-looking. As we have seen, consumer patience makes $\hat{x}$ sticky to the market center (the third term of Equation (5)). The more forward-looking consumers are, the less likely it is for a firm to become the market leader by offering a design close to the market center, thereby decreasing firms’ incentive to do so. Similarly, first-period prices increase with consumer patience since forward-looking consumers are less price sensitive (the second term in Equation (5)). This price effect is well-known in the behavior-based price discrimination literature (Villas-Boas 1999; Fudenberg and Tirole 2000). However, with endogenous product decision the effect is amplified as consumer patience increases first-period differentiation, which further softens price competition.
Notes: Each graph presents a set of curves for consumer patience $\delta \in \{0, .2, .4, .6, .8, 1\}$. The arrow indicates the direction along which $\delta$ increases. Each curve is plotted over $\beta \in [0, \bar{\beta}(\delta)]$, where $\bar{\beta}(\delta)$ is the maximal value of $\beta$ for the symmetric equilibrium to exist given $\delta$.

Notably, first-period prices decrease with firm patience, contrary to findings from the behavior-based price discrimination literature. In particular, Fudenberg and Tirole (2000), when analyzing a similar two-period duopolist model with uniformly distributed demand, show that firm patience does not affect first-period prices. In their model, behavior-based discrimination only affects first-period prices by lowering consumers’ price sensitivity. Even though firms are forward-looking, the first-order effect of shifting the indifferent consumer equals zero around the market center—a firm’s marginal gains in profit over one segment are exactly canceled out by losses over the other in period two. This is an intriguing result because it implies that forward-looking firms’ incentive to avoid the unprofitable use of purchase history information has no impact on the first-period market equilibrium.

I investigate why this classic result changes with endogenous product locations. Given any $a$ and $b$, the interior sub-game price equilibrium is specified by Equation (A-3) of the Appendix. Note that whenever first-period products are symmetrically locate ($a+b = 1$), these equilibrium
prices become

\[ p_a^*(a, b) = p_b^*(a, b) = t(b - a) + \frac{25}{16} \delta t. \]  

(7)

With exogenously fixed \( a \) and \( b \), as in most behavior-based price discrimination models, Equation (7) replicates the result that first-period prices increase with consumer patience \( \delta \) but are invariant to firm patience \( \beta \). Therefore, the only way by which firm patience affects first-period prices is through its impact on first-period differentiation. As products become less differentiated, price competition escalates. The key question then is why does first-period differentiation decrease with firm patience.

The reason is subtle. Consider firm A’s location decision as an example. By the chain rule, the derivative of firm A’s discounted total profit (Equation (6)) with respect to location \( a \) is

\[ \frac{\partial \pi_A}{\partial a} = \frac{\partial \pi_{A1}}{\partial a} + \beta \frac{d \pi_{A2}^*(\hat{x})}{d \hat{x}} \frac{\partial \hat{x}}{\partial a}. \]

(8)

However, we also know from Equation (2) that the marginal effect of first-period market shares on second-period profits is zero in a symmetric equilibrium:

\[ \frac{d \pi_{A2}^*(\hat{x})}{d \hat{x}} \bigg|_{\hat{x} = \frac{1}{2}} = 0. \]

(9)

The removal of this last term from Equation (8) implies that product location has no impact on second-period profits. One might then conclude that firm patience does not affect first-period product locations. However, there is a less obvious effect. Suppose the two firms were symmetrically located, but firm A unilaterally relocates \( a \) closer to the market center. This perturbation shifts the indifferent consumer to the right. Firm B will then react by cutting prices to regain some market share. However, with sufficient foresight, firm B should realize that its current loss of market share improves its second-period profits, and will hence price less aggressively in response. Mathematically, it can be verified that in the sub-game price equilibrium \( \frac{\partial^2 p_b}{\partial a \partial \beta} > 0. \) Anticipating less resistance in prices from a forward-looking rival,
firm A is then more inclined to locate closer to the market center. Indeed, it can be shown that

\[
\frac{\partial^2 \pi_A}{\partial a \partial \beta} = \frac{\partial^2 \pi_{A1}}{\partial a \partial \beta} > 0. \tag{10}
\]

The same reasoning applies to firm B, who will also move closer to the market center with greater firm patience. In other words, firm patience reduces first-period differentiation not because firms are able to influence second-period profits via first-period locations; it is because, as firms try to improve second-period profits, firm patience shifts the first-order conditions of their period-one location choices, making unilateral aggressive designs (those close to the market center) more profitable in the first period.

In contrast, for the exogenous product design setting of Fudenberg and Tirole (2000), the first-order condition of firm A’s period-one price decision is \( \partial \pi_A / \partial p_a = \partial \pi_{A1} / \partial p_a \) in a symmetric equilibrium due to the same effect of Equation (9). However, \( \partial \pi_{A1} / \partial p_a = p_a \hat{x} / \partial p_a + \hat{x} \), where \( \hat{x} \) as specified in Equation (5) does not depend on firm patience \( \beta \). Therefore, \( \partial^2 \pi_A / \partial p_a \partial \beta = \partial^2 \pi_{A1} / \partial p_a \partial \beta = 0 \). Intuitively, firms’ incentive to avoid the first peril does not improve period-two profits in a symmetric equilibrium; without a separate product design stage, neither does such incentive change period-one profits. As a result, in equilibrium firm patience has no effect on first-period competition.

It will be useful to compare equilibrium first-period differentiation and prices to the benchmark values from a static location-price game (as given by Lemma 3). As Figure 2 shows, first-period differentiation is lower than the static value of \( 3t/2 \) except when firms are myopic (\( \beta = 0 \)). First-period prices can be either higher or lower than the static level of \( 3t/2 \). This is because two countervailing forces affect first-period prices: while consumer patience lowers price sensitivity and increases first-period prices directly, firm patience reduces differentiation and decreases prices indirectly. The overall comparison with the static level depends on which effect dominates. Notably, behavior-based discrimination can intensify competition in the first period beyond the static benchmark, which differs from the finding of Fudenberg and Tirole (2000) that first-period prices are higher than the static level. I summarize these results in the following corollary.
Corollary 1  In the first period equilibrium, products are less differentiated than in a static location-price game unless firms are myopic; prices can be even lower than the static level.

To complete the analysis, I derive the second-period equilibrium that follows from the symmetric first-period equilibrium. The first-period indifferent consumer is $\hat{x}^* = 1/2$, which produces the most consumer preference information and leads to the most unprofitable personalization. There are four personalized designs in the second period:

$$a_o^* = -\frac{1}{8}, \quad a_n^* = \frac{3}{8}, \quad b_n^* = \frac{5}{8}, \quad b_o^* = \frac{9}{8}.$$ 

The second-period prices for all four products equal $3t/8$. Consumers are all better off in the second period compared with the static case: every consumer receives a better design at a lower price. Each firm’s second-period total profit is $3t/16$, lower than the static-game value of $3t/4$, which reflects the first peril. Moreover, all consumers are better off in period two than in period one, but firms are worse off due to lower prices (see the Appendix for proof). I state this last result below.

Corollary 2  Firms are worse off and consumers are better off in period two than in period one due to behavior-based personalization.

In summary, the main analysis identifies the double perils of behavior-based personalization: firms cannot help but use consumer purchase information to personalize products which hurts second-period profits; but trying to suppress consumer purchase information hurts firms in the first period.

5 Extensions

In this section, I investigate how alternative market conditions affect the two perils of behavior-based personalization. In particular, I will relax the following assumptions of the main analysis: (1) a firm does not have better information about its own clientele than its rival, (2) there is
no switching cost, (3) firms can target a design to a specific segment, and (4) the first-period
designs are abandoned in period two.

5.1 When A Firm Knows More about Its Customers than Its Rival

In the main model, the amount of consumer preference information is fully captured by the
indifferent consumer $\hat{x}$. Therefore, a firm has no information advantage over its rival. Two
model features have contributed to this symmetric information outcome. First, a firm observes
no further consumer preference information besides purchase histories. Second, firms can infer
who the rival’s customers are by recognizing their own. I revisit these assumptions in order.

5.1.1 Perfect Discrimination within A Firm’s Clientele

Consumer database technologies may help a firm gather more information about its customers
besides purchase histories. Firms can collect customer demographic information through user
accounts, conduct post-purchase satisfaction surveys, or gauge customer preferences by analyzing
their product search behaviors (Hauser et al. 2009). To model this type of information
asymmetry between firms, I assume that each firm observes the exact locations of its old cus-
tomers. I continue to assume that personalization is free, which is plausible if investments in
flexible manufacturing are sunk costs. As a result, in the second period firms will implement
first-degree discrimination by offering each old customer a personalized design that perfectly
suits her taste (Thisse and Vives 1988), a strategy called “mass personalization” in practice.

In period two, a firm’s information advantage about its customer base translates into a
competitive edge. Take firm A’s clientele $[0, \hat{x})$ as an example. Although firm B can still serve
this segment with product $b_n$ (or even multiple products), with mass personalization firm A
can undercut firm B for every consumer $x$ in $[0, \hat{x})$ by offering her a price slightly lower than
$p_{b_n} + t(x - b_n)^2$. Therefore, firm B cannot make a positive profit from firm A’s clientele. For
simplicity I assume that firm B does not offer product $b_n$, as any infinitesimal manufacturing
cost will strictly prevent firm B from doing so. It follows that firm A will charge each of its
previous customers a price equal to her reservation price $v$; so will firm B for its own customers.
The second-period prices are higher than the static level and hence higher than those in the main model, which negates the first peril of behavior-based personalization.\textsuperscript{10}

Note that in period two each firm benefits from having a larger customer base, in contrast to the main model where both firms benefit from an asymmetric split of the market. Specifically, firms’ discounted total profits are

\[
\pi_A = p_a \hat{x} + \beta v \hat{x}, \quad \pi_B = p_b (1 - \hat{x}) + \beta v (1 - \hat{x})
\]

where the indifferent consumer \( \hat{x} \) is obtained by substituting \( \delta = 0 \) into Equation (5). This is because, knowing that they will receive zero surplus in period two regardless of their purchase histories, consumers in period one will maximize their current utility as if they were myopic. It follows that first-period competition is equivalent to a static location-price game with an additional profit margin of \( \beta v \). We know from Section A-2.1 of the Appendix that this extra margin does not affect firms’ period-one equilibrium product designs, which are the same as in the static case: \( a^* = -1/4 \), and \( b^* = 5/4 \). However, it intensifies the price war, with the first-period equilibrium prices lower than the static level of \( 3t/2 \):

\[
p_a^* = p_b^* = \frac{3}{2} t - \beta v.
\]

Moreover, first-period prices are lower than those in the main model, and the difference widens with firm patience.\textsuperscript{11} That is, firms’ ability to perfectly discriminate among their customers exacerbates the second peril of behavior-based personalization. The more forward-looking firms are, the deeper the price cut they take. First-period prices may even be negative if consumers’ intrinsic value \( v \) is sufficiently high. Practically, a firm may recruit customers by paying them cash incentives. An example is cell phone carriers who compete to sign up users by offering free phones and cash back bonuses.

\textsuperscript{10}Proof: A necessary condition for full market coverage is that \( v > 3t/2 \), the price of the static location-price game.

\textsuperscript{11}Proof: In the main model \( v \) must be no smaller than \( p_a^* + t(1/2 - a^*)^2 \) to ensure that the marginal consumer at 1/2 is willing to buy, where \( p_a^* \) is given by Equation (A-7) of the Appendix. It is then straightforward to show that \( p_a^* \) of the main model is greater than \( 3t/2 - \beta v \) and that the difference increases with \( \beta \).
Since firms in period one “compete away” any extra consumer surplus they can extract through perfect discrimination, they are \textit{ex ante} worse off compared with the static benchmark (unless $\beta = 0$ in which case firms are indifferent). In addition, it can be shown that firms’ discounted total profit of $3t/4$ is lower than that in the main model for a non-empty set of parameters. That is, firms can also be worse off than if they can only recognize consumers’ purchase histories. The following proposition summarizes the results. The proof holds by construction.

\textbf{Proposition 3} \textit{Firms’ ability to perfectly discriminate among their own customers negates the first peril of behavior-based personalization but exacerbates the second peril. Firms are \textit{ex ante} (weakly) worse off than in the static case, and can be worse off than if they only recognized consumers’ purchase histories.}

These results suggest that finer discrimination does not always help. Technologies which allow firms to collect individual customers’ taste information may end up eroding industry profits. The results are also useful in understanding whether firms, in a broader context, want to offer multiple products in the first period. If a firm privately observes its own customers’ choices, by offering multiple products in period one it can gain better knowledge about its customers than its rival. As the above analysis suggests, firms might then have greater incentive to compete for a larger market share, which may intensify first-period competition. Future research can formally model this tradeoff.

\textbf{5.1.2 New Generation of Consumers}

New generations of consumers may enter the market over time, especially in expanding product categories (Villas-Boas 1999, 2004). Firms may not be able to distinguish between new-generation consumers and the rival’s previous customers. Formally, I assume that in the second-period market a fraction $\gamma$ of consumers come from the new generation. That is, I normalize the mass of new-generation consumers as $\gamma/(1-\gamma)$, while the mass of old-generation consumers continues to be 1. I also assume that new-generation consumers are uniformly distributed along
the Hotelling interval. Firms offer personalized designs \(a_o\) and \(b_o\) to their old customers, and \(a_n\) and \(b_n\) to whoever did not buy their products in period one.

I begin by analyzing the second period. Let \(\hat{x}_n\) denote the location of the new-generation consumer who is indifferent between the two firms. Such an indifferent consumer exists, with \(\hat{x}_n\) solving \(p_{an} + t(x - a_n)^2 = p_{bn} + t(x - b_n)^2\). Let \(\hat{x}_A\) denote the consumer in firm \(A\)'s customer base who is indifferent between buying \(a_o\) and \(b_n\), and let \(\hat{x}_B\) denote the consumer in firm \(B\)'s clientele indifferent between \(a_n\) and \(b_o\). (If firm \(A\)'s old customers all prefer \(a_o\), let \(\hat{x}_A = \hat{x}\). Other boundary cases are similarly specified.) Firms’ second-period profits are

\[
\begin{align*}
\pi_{A2} &= p_{an} \hat{x}_n \gamma / (1 - \gamma) + p_{an} (\hat{x}_B - \hat{x}) + p_{ao} \hat{x}_A, \\
\pi_{B2} &= p_{bn} (1 - \hat{x}_n) \gamma / (1 - \gamma) + p_{bn} (\hat{x} - \hat{x}_A) + p_{bo} (1 - \hat{x}_B).
\end{align*}
\]  

(13)

Given these profit functions, I can derive the second-period equilibrium in the same way as in the main model. The results involve high-order polynomials, which I report graphically in Figure 3 to facilitate interpretation. For illustrative purpose, I set \(t = 1\) and \(\hat{x} = 1/2\).

**Figure 3:** Second-Period Equilibrium Differentiation and Prices with the Entry of New-Generation Consumers \((t = 1, \hat{x} = 1/2)\)

As a larger fraction of period-two consumers are from the new generation, the personalized designs for new customers \((a_n\) and \(b_n\)) become more differentiated. Intuitively, serving the new generation whose tastes are broadly distributed increases differentiation. Meanwhile, serving
the new generation diverts a firm’s focus from poaching the rival’s clientele, thus allowing the rival to offer a more tailored design to its old customers. When the fraction of the new generation $\gamma$ is sufficiently high, the products for firms’ old customers ($a_o$ and $b_o$) become less differentiated than those for the new customers. This result is the opposite of the main model, where old customers receive more “outlandish” designs in the second period. Finally, the prices of all personalized designs increase with $\gamma$; the new generation mitigates the competition between the products for new customers, which in turn softens the competition over each firm’s clientele. Therefore, the entry of a new generation mitigates the first peril of behavior-based personalization by raising second-period prices above the level in the main model.

I next ask how the new generation affects the first-period equilibrium. Figure 4 presents first-period product differentiation and prices where $t = 1$, $\beta = 0.5$, and $\delta = 0.5$. Other values of $\beta$ and $\delta$ suggest similar patterns. A larger fraction of new-generation consumers increases differentiation and reduces prices in the first period. These results can be interpreted as follows. As discussed above, when the fraction of the new generation increases, firms become less interested in poaching, and are consequently more likely to retain their old customers. This shifts forward-looking firms’ first-period imperative from suppressing preference information to growing the customer base. The implications are two-fold: aggressive designs are less likely to

![Figure 4](image-url)
be accommodated by the rival, leading to greater differentiation compared to the main model; however, firms compete more intensively in prices for market share. The latter effect is further aggravated by the fact that old-generation consumers’ price sensitivity increases with \( \gamma \). This is because as more new-generation consumers enter the market, old-generation consumers are more likely to get poaching offers that are independent of their first-period choices, and are thus more responsive to period-one prices compared to the main model. The net result is that first-period prices and profits are lower than that in the main model. I summarize these findings below. The proof holds by construction.

**Proposition 4**  
*The entry of a new generation of consumers mitigates the first peril of behavior-based personalization but exacerbates the second peril.*

This finding parallels the result of Section 5.1.1 where firms can perfectly discriminate within their clientele: when there is gain from having served a larger clientele, firms will compete more intensively in the first period for market share, which dissipates first-period profits.

### 5.2 Consumer Loyalty and Inertia

Firms may benefit from a large customer base not only by knowing their own customers better, but also through customer brand loyalty or inertia (see Chen 1997; Taylor 2003). In Section A-7 of the Appendix, I extend the main model by assuming that consumers incur a cost when switching to another firm in the second period. I find that a firm has a competitive advantage on its turf in period two: it is able to offer a design better tailored to its customers and charge a higher price than its rival. In doing so, firms also earn a higher second-period equilibrium profit than in the main model, thus mitigating the first peril of behavior-based personalization.

In period one, firms will try to expand their turf to increase their period-two profits. Indeed, it can be shown that the first-order effect of market share on a firm’s second-period profit is positive around \( \hat{x} = 1/2 \), and is increasing in the switching cost. This result is different from the main model, where the same first-order effect is zero such that firms are interested in asymmetric market shares rather than large market shares. Consistent with this intuition,
first-period equilibrium differentiation and prices both decline with switching cost, exacerbating the second peril. I summarize these results below (see the Appendix for proof).

**Proposition 5**  Consumer loyalty and inertia mitigates the first peril of behavior-based personalization but exacerbates the second peril.

These results are in line with common findings of the switching cost literature: firms can charge a higher price to consumers who are “locked in” due to the difficulty of switching; however, to compete for the surplus from locked in consumers firms escalate price wars early on (see Fudenberg and Villas-Boas 2007 for a review of this literature).

### 5.3 Consumer Self-Selection

The main model assumes that firms have perfect targeting abilities in the sense that they can limit a consumer’s access to a specific product. Firms can then implement third-degree discrimination in period two based on consumers’ past choices. This assumption is common in the literature, and is relevant to some industries. A familiar example is Amazon, whose personalized product recommendations can be thought of as a form of targeted offers—with search costs, a consumer may not access the products not recommended to her. Pazgal and Soberman (2008) note similar practices—Air Canada offers double frequent flyer miles exclusively to newly registered members, and Scandinavian Airlines provides automatic flight information to travelers who sign up for wireless access at specific airports. In general, targeted offers are more frequently seen in categories such as travel, telecommunications, credit cards, catalogue retail, and Internet retail.

In other industries, the norm might be to provide a full menu of products to all consumers and let them self-select. For the market setting I analyze, self-selection means consumers will have equal access to all four personalized designs in the second period. However, this is equivalent to the case of proliferation in which firms compete over the whole market without the aid of segmentation information. By Lemma 2, each firm will offer a standard design to all consumers to avoid intra-firm cannibalization. That is, consumer self-selection offsets the effect
of personalization by making competition more “global,” so that each firm earns the static profit of $3t/4$ in the second period. Since purchase histories no longer affect firm profits and consumer surplus in period two, first-period competition reduces to the static location-price game. I summarize the results in the following proposition. The proof holds by construction.

**Proposition 6** *Consumer self-selection obviates both perils of behavior-based personalization.*

*Firms solve a static location-price game in both periods.*

Some industries’ intrinsic characteristics endow firms with weak targeting power. Meanwhile, targeted offers may antagonize consumers and may spur arbitrage activities. Interestingly, Proposition 6 suggests that firms could be better off under these seemingly adverse market conditions. The general message is that finer discrimination does not always benefit firms, either in the form of first-degree discrimination (Section 5.1) or third-degree discrimination (i.e., targeted offerings). However, it should be noted that if the targeting ability can be endogenously acquired, then firms in equilibrium may do so and offer targeted personalized designs (Lemma 1). In this case, the prisoner’s dilemma uncovered in the main model will again emerge, and firms will again incur the perils of behavior-based personalization.

### 5.4 Classic Designs

In some industries it is infeasible to abandon old product designs for every consumption cycle. In particular, brand image concerns may require the provision of a timeless classic design to the existing clientele. Firms’ personalization problem in this situation becomes a product line extension problem. To capture this market feature, I modify the main model by assuming that in period two each firm’s old customers continue to receive the product they bought in period one, either $a$ or $b$. In addition, each firm offers a new design, $a_n$ and $b_n$, to poach its rival’s customers. However, since prices are often easier to change than product design, I allow the firms to charge their old customers a different price than what they paid in the first period. I denote the first-period prices of $a$ and $b$ as $p_{a1}$ and $p_{b1}$, and the second-period prices as $p_{a2}$ and $p_{b2}$. In each period firms set prices simultaneously after observing each other’s design choices.
In the second period, firms choose the new designs taking the classic designs as given, anticipating the price equilibrium that follows. Firms’ commitment to the classic designs thus allows them to be the incumbent over their customer base. Intuitively, this incumbent advantage is maximized if a firm is located at the center of its turf. This effect might lead firms to locate aggressively towards the market center in period one. However, there is a countervailing effect, that aggressive designs will trigger deep price cuts from the rival firm. To see this, suppose firm A unilaterally moves \( a \) slightly closer to the market center, which also increases \( \hat{x} \). Firm B will react by cutting prices to regain some market share even if it is myopic. If firm B is forward-looking, it will cut prices even more. This is because when the classic design \( a \) is closer to the market center, firm B will earn lower poaching profits in period two, and therefore will prefer to reduce \( \hat{x} \) and shrink the size of the unprofitable poaching market (mathematically, it can be verified that \( \partial^2 \pi_B / \partial a \partial \hat{x} < 0 \)). As a result, aggressive classic designs can ignite price wars worse than in the static case. Interestingly, this force is the opposite of the main model, where forward-looking firms accommodate aggressive designs to reduce the first peril.

Figure 5 presents equilibrium classic designs and their first-period prices as a function of firm patience \( \beta \). The differentiation between the classic designs is higher than the static level of \( 3/2 \) and increases with firm patience. This outcome suggests that the cost of aggressive designs outweighs the benefit as forward-looking firms resort to intensive price wars to protect their second-period profits. First-period prices are also above the static level of \( 3t/2 \) and increase with firm patience. The high prices are partly attributed to the high differentiation between the classic designs, and partly due to patient consumers’ lower price sensitivity as discussed in the main analysis. Overall, first-period profits are higher than in the main model and increase with firm patience. This effect negates the second peril of behavior-based personalization.

Last, Figure 6 presents the second-period equilibrium. The new designs are less differentiated than the classic designs, and the degree of differentiation between new designs decreases with firm patience. Naturally, as forward-looking firms position the classic designs far from the market, in period two they can poach their rival’s customers by offering a new design better tailored to their tastes. In doing so, firms charge higher prices for the new designs.
than for the classic designs, and earn the majority of their second-period profits from poaching new customers. Moreover, it can be verified that firms’ total second-period profits are higher than the value of $3t/16$ in the main model, thus mitigating the first peril of behavior-based personalization. I summarize these results below.

**Proposition 7**  Firms’ commitment to retaining a classic design for their old customers mitigates the first peril of behavior-based personalization and negates the second peril.
In summary, Section 5 explores whether firms can circumvent the perils of behavior-based personalization in more general settings. I find that attenuating one peril often exacerbates the other. In particular, although firms can improve their second-period profits by gaining better information about their customers or by exploiting consumer loyalty and inertia, to grow a larger clientele they compete more intensively in period one. Nevertheless, there do exist market conditions that help firms reduce the perils of behavior-based personalization, such as consumer self-selection and commitment to classic designs for previous customers.

Finally, since the symmetric equilibrium marks the lowest second-period profits, various forms of market asymmetry may mitigate the first peril of behavior-based personalization and thus affect the second peril. In particular, I investigate asymmetric patience between firms (for example, due to unequal access to the credit market) in Section A-8.1 of the Appendix. I find that asymmetric firm patience does attenuate the first peril, although the more patient firm fares worse in period one. I also consider heterogeneous consumer patience in Section A-8.2 of the Appendix. Firms are better off with a larger segment of forward-looking consumers because consumers’ effort to induce behavior-based personalization counters firms’ incentive to avoid it, thus weakening the second peril, a result consistent with Proposition 2.

6 Empirical Implications

In this section I summarize the findings of this paper in light of their empirical implications. One key prediction is that behavior-based personalization may hurt the profits of competing firms. This prediction contains two aspects. Cross-sectionally, firm profits can be lower in industries where behavior-based personalization prevails than in industries where it is infeasible, other things being equal. Longitudinally, firm profits are lower in mature markets than in new markets, with products becoming less differentiated and prices declining over time. This prediction challenges the common belief that personalization contributes to the bottom line by better meeting consumer needs (see Arora 2008 for a review). I argue that personalization does deliver greater value to consumers, but damages profits by intensifying firm competition.

The second result centers on how much firms know about their own customers. If firms can
obtain fine-grained preference information about individual customers, there could emerge mass personalized products at high prices during the mature stage of the market. However, firms compete intensively at the early stage, and might offer cash incentives to attract customers. That is, what firms know about their customers may imply different price trends in an industry. I predict prices to increase over time when firms are equipped to analyze their customers at an individual level, but decline when firms can only recognize consumers’ purchase histories.

Third, in stable industries with negligible entry of new-generation consumers, I expect that firms will offer their old customers more “extreme” product designs that symbolize the firm’s core image, and serve their new customers with more mainstream designs that cater to average tastes. However, I expect the opposite in expanding industries with a heavy influx of new-generation consumers. Moreover, the impact of the new generation on prices depends on the stage of the market—the analysis suggests that prices will decrease with the proportion of new-generation consumers in the early days, but increase with it during the mature stage.

A fourth prediction of the model is that there will be less personalization if consumers can self-select between personalized designs than if firms can target different consumers with different products. Moreover, firm profits are higher if consumers can self-select. This prediction is counterintuitive because it implies that, other things being equal, firms can be worse off with better targeting technologies such as direct mail, back-of-receipt offers, and personalized product recommendation systems.

Last, whether firms maintain a classic design implies different market dynamics. Early-stage product designs are expected to be more differentiated if firms will retain them for their old customers than if firms are free to subsequently replace them with new designs. People would normally imagine long-lived classic designs to be more nuanced and tailored to mainstream tastes, and seasonal designs to be more avant-garde. However, the analysis predicts that classic designs should be sufficiently differentiated to epitomize their distinct brand personalities, while new designs should be moderately positioned to target mainstream buyers.
7 Concluding Remarks

This paper identifies two perils of behavior-based personalization in competitive markets. First, by the same force that behavior-based price discrimination intensifies price wars, competitive use of consumer purchase histories in product design commoditizes the marketplace (the first peril of behavior-based personalization). With endogenous product design, there is yet another peril: had a market leader served all consumers on day one, purchase histories would have contained no information about consumer preferences, which could have eliminated the first peril. However, knowing that their rivals are willing to accommodate a market leader, firms are more likely to offer a mainstream design on day one, which damages differentiation (the second peril of behavior-based personalization). Based on this understanding, I explore how alternative market conditions affect both perils. I find that firms’ better knowledge about their own customers and switching costs mitigate the first peril but exacerbate the second. On the other hand, consumer self-selection and the need for classic designs help reduce both perils.

This paper suggests a perspective to understanding the era of product personalization. There are a number of paths to extend this research. It would be interesting to investigate consumer co-production, which will shed light on the optimal mix of firm-supplied personalization and consumer-initiated customization. It would also be interesting to study platforms’ incentives to provide purchase-based product recommendations, given that such recommendations may influence the competition between participating sellers. Future research can also analyze the effects of personalized advertising based on consumer purchase histories.
References


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Online Appendix of “The Perils of Behavior-Based Personalization”

A-1 Proof of Lemma 1

This paper analyzes the case in which firms cannot commit to ignoring consumer purchase histories in making pricing and product decisions.\textsuperscript{A-1}

**Personalized pricing:** Under uniform pricing, a firm charges the same price to all segments. Under personalized pricing, it may charge different prices to different segments. Hence, given product locations of both firms and given any pricing policy of the rival, profit maximization under uniform pricing is a constrained version of the optimization problem under personalized pricing. Therefore, the maximized profit with personalized pricing is always (weakly) higher.

**Personalized product designs:** Suppose \(N\) consumer segments are identified. Firm A’s second-period profit is additive separable in its profit from each segment, and is written as

\[
\pi_{A2} = \sum_{i=1}^{N} p_{a_i} D_{a_i}(a_i, b_i, p_{a_i}, p_{b_i})
\]

where \(D(\cdot)\) is the demand function. Since both firms will personalize their prices for each segment, the equilibrium prices for segment \(i\) only depends on the locations of the two products targeted at that segment: \(a_i\) and \(b_i\). Hence, A’s second-period profit can be written as

\[
\pi_{A2} = \sum_{i=1}^{N} p_{a_i}^*(a_i, b_i) D_{a_i}(a_i, b_i, p_{a_i}^*(a_i, b_i), p_{b_i}^*(a_i, b_i))
\]

Note that the functional forms of \(p_{a_i}^*(\cdot, \cdot)\) and \(p_{b_i}^*(\cdot, \cdot)\) are invariant to their argument values. In other words, firm A’s *objective function* is the same under standard and personalized product design. It follows that standard product design is a weakly dominated strategy as it corresponds to a constrained version of the same optimization problem. The same logic applies to firm B.

\textsuperscript{A-1}This setup is different from models that allow firms to commit to a pricing policy. For example, Guo and Zhang (2010) consider a two-stage game in which firms can commit to either uniform pricing or localized pricing in period one, and then set prices in period two.
A-2  Proof of Lemma 2 and Lemma 3

I first derive the equilibrium when each firm offers one product, and then show that each firm will effectively launch only one product even if offering multiple products is costless.

A-2.1 Equilibrium When Each Firm Offers One Product (Lemma 3)

Assuming full market coverage, consumer $x$ buys product $a$ if and only if $p_a + t(x - a)^2 < p_b + t(x - b)^2$. Therefore, the cutoff consumer satisfies $\hat{x} = (p_b - p_a)/2t(b - a) + (a + b)/2$. Given product locations $a$ and $b$, firm $A$ solves the problem of $\max_{p_a} \pi_A = p_a \cdot D_A$ where $D_A = (\hat{x} - Z)m/L$. In the price equilibrium $p_a^* = 2tL(b - a)D_A/m$. Similarly, $p_b^* = 2tL(b - a)D_B/m$, where $D_B = (Z + L - \hat{x})m/L$. As a result, in the price equilibrium $\hat{x}^* = (a + b + 4Z + 2L)/6$, $p_a^* = t(b - a)(a + b - 2Z + 2L)/3$, and $p_b^* = t(b - a)(-a - b + 2Z + 4L)/3$. The second-order condition in prices holds trivially.

Anticipating the sub-game price equilibrium, firm $A$ solves the problem of $\max_a \pi_A = 2tL(b - a)D_A^2/m$. It follows that $\partial \pi_A / \partial a = 2tL[(b - a)2D_A \partial D_A / \partial a - D_A^2]/m$. Rearranging terms, $\text{sign}(\partial \pi_A / \partial a) = \text{sign}(-3a + b + 2Z - 2L)$. Similarly, $\text{sign}(\partial \pi_B / \partial b) = \text{sign}(a - 3b + 2Z + 4L)$. If product locations can fall outside the Hotelling interval $[Z, Z + L]$, the equilibrium locations are solved from the first-order conditions:

$$a^* = Z - \frac{1}{4}L, \quad b^* = Z + \frac{5}{4}L \quad (A-1)$$

It follows that $D_A^* = D_B^* = m/2$, $p_a^* = p_b^* = 3tL^2/2$, and $\pi_A^* = \pi_B^* = 3tL^2m/4$. The same results are also independently derived by Schultz (2004).

A discussion of the above first-order approach is in order. Given the price equilibrium, $\pi_A$ is a cubic function with a negative sign attached to $a$. It might therefore appear that firm $A$ will choose $a = -\infty$ to maximize its profit. However, the equilibrium choice of $a$ is bounded below; given any $b$, if firm $A$ is located too far from the market center, it will not be able to achieve a positive demand at a positive price. In fact, it can be shown that for any $b$, there exists an upper bound and lower bound for $a$, such that firm $A$ cannot earn a positive profit for
any $a$ outside these bounds, and earns zero profits for both boundary values of $a$. Therefore, firm $A$’s equilibrium choice of $a$ must come from solutions to its first-order condition if such solutions exist and generate a positive profit for firm $A$. The same argument holds for firm $B$.

If product locations are constrained within the Hotelling interval $[Z, Z + L]$, the equilibrium involves corner solutions of the location game. Specifically, sign$(\partial\pi_A/\partial a|_{a=Z})=$sign$(b - Z - 2L) < 0$ because $b \leq Z + L$. Similarly, $\partial\pi_B/\partial b|_{b=Z+L} > 0$. Therefore, the equilibrium product locations are:

$$a^c = Z, \quad b^c = Z + L \quad (A-2)$$

where the superscript $c$ stands for “constrained.” It follows that $D_A^c = D_B^c = m/2$, $p_a^c = p_b^c = tL^2$, and $\pi_A^c = \pi_B^c = tL^2 m/2$.

A-2.2 In Equilibrium Each Firm Offers One Product (Lemma 2)

Firms first simultaneously decide on the number and location of products to launch, and then simultaneously set prices observing the locations of all products. A product is referred to as being effectively launched if it has positive demand in equilibrium. Suppose both firms can offer multiple adjacently located products.\(^{A-2}\) Let $a$ and $b$ be the locations of products closest to the rival firm’s array of products, and $p_a$ and $p_b$ be their prices. Let $\hat{x}$ denote the consumer indifferent between $a$ and $b$. To effectively launch an additional product in its turf, firm $A$ must be able to sell this product to some consumer at $x \in [Z, a)$. The price of this additional product hence cannot exceed $p_a$ plus the transportation costs from $x$ to $a$, otherwise consumer $x$ will purchase $a$ instead. Therefore, prices of firm $A$’s additional products in its turf are bounded above by the total cost curve of product $a$ (see Figure A-1). Correspondingly, firm $A$’s profits are bounded above by $\bar{\pi}_A = [p_a(\hat{x} - Z) + \int_Z^a t(a - x)^2 dx]m/L$. The same logic applies to firm $B$.

\(^{A-2}\)Technically, products from different firms may be interlaced. However, it can be shown that in equilibrium each firm positions its products adjacently. The intuition is as follows: When one firm positions its products adjacently, it can coordinate the prices to avoid intra-firm cannibalization. However, when products from different firms are interlaced, the negative price externality between adjacent products intensifies inter-firm price competition.
Figure A-1: Firm A’s Profit by Offering an Array of Products

Notes: By offering one product at $a$, firm A earns a profit equal to the shaded area. By offering additional products on its turf, firm A’s additional profit cannot exceed the checkered area.

Note that the sub-game price equilibrium is the same as in the one-product-per-firm case from Section A-2.1, where $p_a$, $p_b$, and $\hat{x}$ are solely determined by the locations of $a$ and $b$. That is, firm A sets price $p_a$ as if it confronts firm B with a single product $a$, and extracts additional consumer surplus from its turf by launching additional products. Product location $a$ determines the size of this extra surplus. The same result holds for firm B. Also note that, to effectively launch multiple products, firm A must ensure $a > Z$, and firm B must ensure $b < Z + L$.

Suppose both firms offer multiple products. Plugging in the equilibrium $p_a$ and $\hat{x}$ from the sub-game price competition, I obtain $\bar{\pi}_A = \frac{(b-a)(a+b-2Z+2L)^2}{6} + (a-Z)^3 tm/3L$. However, for any $Z < a \leq b < Z + L$, $\bar{\pi}_A$ is lower than $16tm(b-Z+L)^3/243L$, the profit firm A could have earned by offering one product $a = (b + 2Z - 2L)/3$ in best response to $b$ (see Section A-2.1). Intuitively, although firm A can extract extra consumer surplus by launching multiple products on its turf, doing so requires that firm A put product $a$ strictly inside the Hotelling interval. The loss of differentiation between $a$ and $b$ more than offsets the additional consumer surplus extracted. The same logic applies to firm B. We can similarly show that a situation in which only one firm offers multiple products does not arise in equilibrium.
A-3 Proof of the Existence of Indifferent Consumer $\hat{x}$

Consumer $x$ incurs a discounted total cost of $c_a(x) = t(x-a)^2 + p_a + \delta \min[t(x-a_o)^2 + p_{a_o}, t(x-b_n)^2 + p_{b_n}]$ when buying product $a$; and a discounted total cost of $c_b(x) = t(x-b)^2 + p_b + \delta \min[t(x-a_n)^2 + p_{a_n}, t(x-b_o)^2 + p_{b_o}]$ when buying product $b$. Let $d(x) = c_a(x) - c_b(x)$. Consumer $x$ will choose product $a$ iff $d(x) < 0$. Note that $d(x)$ is continuous in $x$. Let $I_1 = 1$ when $t(x-a_o)^2 + p_{a_o} < t(x-b_n)^2 + p_{b_n}$ and 0 otherwise. Also, let $I_2 = 1$ when $t(x-a_n)^2 + p_{a_n} < t(x-b_o)^2 + p_{b_o}$ and 0 otherwise. The derivative of $d(x)$, when it exists, is

$$d'(x) = 2t \{ (b-a) + \delta[I_1(x-a_o) + (1-I_1)(x-b_n)] - \delta[I_2(x-a_n) + (1-I_2)(x-b_o)] \}$$

Since $a_o \leq b_n$ and $a_n \leq b_o$, $d'(x) \geq 2t [(b-a) + \delta(x-b_n) - \delta(x-a_n)] = 2t [(b-a) - \delta(b_n-a_n)]$. It can be verified that in equilibrium $b - a > \delta(b_n-a_n)$. Hence, $d(x)$ strictly increases in $x$.

If $d(\hat{x}) = 0$ then $d(x) < 0$ for all $x < \hat{x}$, and $d(x) > 0$ for all $x > \hat{x}$. Therefore, there exists a cutoff consumer $\hat{x}$ such that consumers in $[0, \hat{x})$ all strictly prefer product $a$ while consumers in $(\hat{x}, 1]$ strictly prefer $b$.

A-4 Proof of Proposition 2

A-4.1 The Price Equilibrium

I will focus on the symmetric equilibrium in pure strategies. Therefore, I first derive the interior solutions of the price equilibrium by the first-order approach, and then check whether the second-order condition holds.

Solving the first-order conditions $\partial\pi_A/\partial p_a = 0$ and $\partial\pi_B/\partial p_b = 0$ simultaneously yields the inner solutions of equilibrium prices given $a$ and $b$:

$$p_a^*(a,b) = \frac{t}{3}(b-a)(2+a+b) + \frac{25}{16} \delta t + \frac{4\beta t(b-a)(1-a-b)}{16(b-a) - 24\beta + 25\delta}$$

$$p_b^*(a,b) = \frac{t}{3}(b-a)(4-a-b) + \frac{25}{16} \delta t - \frac{4\beta t(b-a)(1-a-b)}{16(b-a) - 24\beta + 25\delta}$$

(A-3)

Since $\partial^2\pi_A/\partial p_a^2 = \partial^2\pi_B/\partial p_b^2 = 16[18\beta - 25\delta - 16(b-a)]/t[16(b-a) + 25\delta]^2$, when $b - a >$
(18\beta - 25\delta)/16 the second-order conditions of the price equilibrium are satisfied. As we will see from Section A-4.2, these second-order conditions are indeed satisfied in the symmetric product equilibrium.

**A-4.2 The Product Equilibrium**

Substituting the equilibrium prices (Equation (A-3)) into \(\pi_A\) and \(\pi_B\), and solving \(\partial \pi_A(a, b)/\partial a = 0\) and \(\partial \pi_B(a, b)/\partial b = 0\) simultaneously, I obtain the symmetric product locations

\[
\begin{align*}
a^* &= \frac{1}{64} \left( 8 - 18\beta + 25\delta - \sqrt{324\beta^2 - 180\beta(8 + 5\delta) + (24 + 25\delta)^2} \right) \\
b^* &= \frac{1}{64} \left( 56 + 18\beta - 25\delta + \sqrt{324\beta^2 - 180\beta(8 + 5\delta) + (24 + 25\delta)^2} \right)
\end{align*}
\]

(A-4)

To prove that the above location choices indeed arise in equilibrium, it remains to check unilateral deviations to the boundaries. Given \(b^*\), if firm \(A\) deviates such that \(\hat{x}(a', b^*) = 0\), then \(\pi_A(a', b^*) = 3\beta t/4\), which is always lower than \(\pi_A(a^*, b^*)\). On the other hand, if \(A\) deviates such that \(\hat{x}(\tilde{a}, b^*) = 1\), then it has to relocate to \(\tilde{a} = (8 - \sqrt{16(b^* - 2)^2 + 72\beta - 75\delta})/4\). Correspondingly, \(\pi_A(\tilde{a}, b^*) \leq \pi_A(a^*, b^*)\) if and only if

\[
0 \leq \beta \leq \bar{\beta}(\delta) = \frac{1}{810} \left( 1789 + 1125\delta - 29\sqrt{2881 + 2250\delta} \right)
\]

(A-5)

It can be shown that when Condition (A-5) holds, \(a^*\) and \(b^*\) always have real roots. Note that \(b^* - a^* > (18\beta - 25\delta)/16\) under Condition (A-5), so that the second-order conditions of the price equilibrium are indeed satisfied. Figure A-2 presents Condition (A-5) graphically.

It follows from Equation (A-4) that firms split the market evenly in the first period:

\[
\hat{x}^* = \frac{1}{2}
\]

(A-6)

As a result, personalization arises in the second period.
Figure A-2: Parameter Conditions for a Symmetric First-Period Equilibrium to Exist

\[ \begin{align*}
0 & \leq \delta, \\
0 & \leq \beta,
\end{align*} \]

A-4.3 Comparative Statics

Let \( \Delta = b^* - a^* \) denote equilibrium first-period differentiation. It is straightforward to show that \( \partial \Delta / \partial \beta < 0 \), \( \partial \Delta / \partial \delta \geq 0 \) (with \( \partial \Delta / \partial \delta = 0 \) if and only if \( \beta = 0 \)). Finally, plugging Equation (A-4) back into Equation (A-3), I obtain the first-period equilibrium prices:

\[
p^*_a = p^*_b = \frac{t}{32} \left( 24 + 18\beta + 25\delta + \sqrt{324\beta^2 - 180\beta(8 + 5\delta) + (24 + 25\delta)^2} \right) \tag{A-7}
\]

It can be verified that \( \partial p^*_a / \partial \beta < 0 \), and \( \partial p^*_a / \partial \delta > 0 \).

A-5 Proof of Corollary 2

In the second period equilibrium, consumers in \([0, 1/4]\) buy product \( a^*_o = -1/8 \) at price \( p^*_a = 3t/8 \). Among them, the consumer at \( 1/4 \) incurs the highest total cost of \( p^*_a + t(1/4 - a^*_o)^2 = 33t/64 \). This total cost, however, is lower than both the equilibrium price of the static location-price game and the lowest value of the equilibrium first-period price. Therefore, in period two consumers over \([0, 1/4]\) are both better off than in the static case and better off than in the first period. Due to symmetry, the same conclusion holds for all consumers in \([0, 1]\).

In the second-period equilibrium, firms charge the same price of \( 3t/8 \) for all personalized
designs. This price is lower than both the equilibrium price of the static location-price game and the lowest value of the equilibrium first-period price. Therefore, in a symmetric equilibrium, firms are worse off in period two both than in the static case and than in the first period.

A-6 When Products Must Locate within the Hotelling Line

In this section I consider the possibility that firms must locate their products within the Hotelling line. Section A-2.1 shows that in a static location-price game over the interval \([Z, Z + L]\), the equilibrium locations are \(a^* = Z - L/4\) and \(b^* = Z + 5L/4\) if locations are unconstrained, and \(a^* = Z\) and \(b^* = Z + L\) if locations are constrained within the interval. From the first-order conditions we also know that \(a^* = Z\) and \(b^* = Z + 4L/3\) if locations are left-constrained (\(Z \leq a \leq b\)), and that \(a^* = Z - L/3\) and \(b^* = Z + L\) if locations are right-constrained (\(a \leq b \leq Z + L\)). Applying these results to the second period of the game, I derive the equilibrium locations as a function of the first-period indifferent consumer \(\hat{x}\):

\[
\begin{align*}
a_n^* &= 0, \\ a_o^* &= 0, \\ b_n^* &= 4\hat{x}/3, \\ b_o^* &= 1 \quad \text{if } \hat{x} \in [0, 1/4) \\
a_n^* &= 4\hat{x}/3 - 1/3, \\ a_o^* &= 0, \\ b_n^* &= 4\hat{x}/3, \\ b_o^* &= 1 \quad \text{if } \hat{x} \in [1/4, 3/4] \\
a_n^* &= 4\hat{x}/3 - 1/3, \\ a_o^* &= 0, \\ b_n^* &= 1, \\ b_o^* &= 1 \quad \text{if } \hat{x} \in (3/4, 1/4]
\end{align*}
\] (A-8)

The second-period equilibrium prices follow directly from the formulas in Section A-2.1. For example, when \(\hat{x} \in [1/4, 3/4]\), \(p_{a_n}^* = 32t(1 - \hat{x})^2/27\), \(p_{a_o}^* = 40t\hat{x}^2/27\), \(p_{b_n}^* = 32t\hat{x}^2/27\), and \(p_{b_o}^* = 40t(1 - \hat{x})^2/27\). Firms’ corresponding second-period profits are

\[
\begin{align*}
\pi_{A2}(\hat{x}) &= [25\hat{x}^3 + 16(1 - \hat{x})^3]8t/243 \\
\pi_{B2}(\hat{x}) &= [16\hat{x}^3 + 25(1 - \hat{x})^3]8t/243
\end{align*}
\] (A-9)

The profit functions reveal a similar mechanism as in the main model: to maximize their second-period profits, forward-looking firms want to suppress consumer preference information by having a market leader in period one, although each firm prefers to be the market leader itself.

Firms’ second-period profit functions for \(\hat{x} \in [0, 1/4)\) and \(\hat{x} \in (3/4, 1/4]\) can be similarly
specified. It is then straightforward to derive the first-period equilibrium. In summary, similar to the main model, there exists a symmetric equilibrium where

\[
a^* = \max \left(0, \frac{1}{8} - \frac{41}{162} \beta + \frac{35}{108} \delta - \frac{1}{648} \sqrt{26896 \beta^2 - 68880 \beta \delta - 132840 \beta^2 + 44100 \delta^2 + 102060 \delta + 59049} \right)
\]

\[
b^* = \min \left(1, \frac{7}{8} + \frac{41}{162} \beta - \frac{35}{108} \delta + \frac{1}{648} \sqrt{26896 \beta^2 - 68880 \beta \delta - 132840 \beta^2 + 44100 \delta^2 + 102060 \delta + 59049} \right)
\]

First-period product differentiation, as measured by \(b^* - a^*\), decreases with firm patience \(\beta\) and increases with consumer patience \(\delta\) when \(a^*\) and \(b^*\) are interior solutions.

In the above symmetric equilibrium, first-period prices are

\[
p_{a}^* = p_{b}^* = \left(\frac{3}{4} + \frac{23}{81} \beta + \frac{35}{54} \delta + \frac{1}{324} \sqrt{26896 \beta^2 - 4920 \beta (14 \delta + 27) + 9 (70 \delta + 81)^2} \right) t \quad \text{if } a^* > 0 \text{ and } b^* < 1
\]

\[
p_{a}^* = p_{b}^* = \left(1 - \frac{2}{5} \beta + \frac{35}{27} \delta \right) t \quad \text{if } a^* = 0 \text{ and } b^* = 1
\]

It can be verified that first-period equilibrium prices decrease with firm patience and increase with consumer patience, the same result as in the main model.

**A-7 Consumer Loyalty and Inertia**

Besides revealing consumer preferences, past purchases can also affect payoffs directly through brand loyalty or inertia. In this section I extend the main model by allowing consumers to incur a cost \(s \geq 0\) when switching to another firm in the second period.

I start with period two. Consider one of firm A’s previous customers \(x \in [0, \hat{x}]\). This consumer will prefer staying with firm A and buying product \(a_o\) to switching to firm B for \(b_n\) if and only if \(p_{a_o} + t(x-a_o)^2 \leq p_{b_n} + t(x-b_n)^2 + s\). It follows that the demand for product \(a_o\) is \((p_{b_n} - p_{a_o} + s)/2t(b_n-a_o) + (a_o + b_n)/2\), which increases with switching cost. I can similarly specify the demand for the other three personalized designs in period two. Following the solution strategy detailed in Section A-2.1 of the Appendix, period-two equilibrium locations
are derived as:\footnote{A-3}

\[
\begin{align*}
a^*_o &= -\frac{\hat{x}}{4} + \frac{s}{4t\hat{x}}, \quad b^*_n = \frac{5\hat{x}}{4} + \frac{s}{3t\hat{x}}, \quad a^*_n = \frac{5\hat{x} - 1}{4} - \frac{s}{3t(1 - \hat{x})}, \quad b^*_o = \frac{5 - \hat{x}}{4} - \frac{s}{3t(1 - \hat{x})}.
\end{align*}
\]

Note that with a positive switching cost, \(a^*_o\) is closer to the center of firm A’s customer base \(\hat{x}/2\) than \(b^*_n\). Similarly, \(b^*_o\) is closer to the center of firm B’s clientele \((1 + \hat{x})/2\) than \(a^*_n\). This location advantage increases with \(s\). Switching costs also give firms greater pricing power over their old customers. This can be seen from the second-period equilibrium prices:

\[
\begin{align*}
p^*_{a_o} &= \frac{3}{2}t\hat{x}^2 + \frac{2}{3}s, \quad p^*_{b_n} = \frac{3}{2}t\hat{x}^2 - \frac{2}{3}s, \quad p^*_{a_n} = \frac{3}{2}t(1 - \hat{x})^2 - \frac{2}{3}s, \quad p^*_{b_o} = \frac{3}{2}t(1 - \hat{x})^2 + \frac{2}{3}s.
\end{align*}
\]

Each firm is able to charge a premium price of \(2s/3\) to its old customers, but offers a “discount” of \(2s/3\) to attract its rival’s customers. In doing so, firms earn a higher second-period profit in equilibrium than in the main model: when \(\hat{x} = 1/2\), firms’ second-period profits are \(3t/16 + 16s^2/27t\). Therefore, switching costs mitigate the first peril of behavior-based personalization.

Since firms in period two enjoy a competitive advantages on their turf, they should try to increase their turf in period one. Indeed, it can be shown that the first-order effect of market shares on second-period profits is positive around \(\hat{x} = 1/2\) and is increasing in the switching cost: \(d\pi^*_A(\hat{x})/d\hat{x}|_{\hat{x}=1/2} = -d\pi^*_B(\hat{x})/d\hat{x}|_{\hat{x}=1/2} = 4s/3\). Therefore, switching costs are expected to intensify first-period competition. Indeed, first-period equilibrium differentiation and prices both decline with higher switching cost, as illustrated in Figure A-3.

\section{A-8 Heterogeneous Patience}

In this section, I extend the main model to explore the impact of heterogeneous patience on market dynamics. I first analyze asymmetric discount factors between firms, and then study how firms react if there is a segment of myopic consumers.
A-8.1 Asymmetric Patience between Firms

A multitude of factors affect firm patience and possibly lead to different firm discount factors. Without loss of generality, let firm $A$ be the relatively patient firm with discount factor $\beta$. Denote firm $B$’s discount factor as $d\beta$, where $d \in [0,1]$ measures the extent of symmetry in firm patience. The second-period equilibrium is derived in the same way as in the main model, with firm profits solely relying on the first-period indifferent consumer $\hat{x}$, which is determined by Equation (5). However, firms’ discounted total profits reflect their different discount factors:

$$\pi_A = p_a\hat{x} + \beta \pi_{A^2}(\hat{x}), \quad \pi_B = p_b(1 - \hat{x}) + d\beta \pi_{B^2}(\hat{x}) \quad (A-12)$$

Figure A-4 presents the first-period equilibrium locations, prices and profits as a function of the degree of symmetry in firm patience. Notably, the relatively patient firm $A$ is located further away from the market center, gains a smaller market share, charges a lower price, and earns a lower profit than firm $B$. The greater asymmetry in firm patience (smaller $d$), the larger firm $A$’s relative disadvantage. Counter-intuitively, firm $A$ is worse off in the first period by being more forward-looking.
Figure A-4: First-Period Equilibrium Product Locations, Prices, and Profits When Firms Are Asymmetric in Patience ($t = 1, \beta = 0.5, \delta = 0.5$)

This result mirrors the prediction of Proposition 2 that first-period differentiation and prices both decrease with firm patience. The same mechanism underlying Proposition 2 applies here as well: anticipating the detriment of personalization in period two, a forward-looking firm has the incentive to split the first-period market unevenly. The more forward-looking a firm is, the more likely it will respond with mild prices in period one when the rival firm offers an aggressive design close to the market center. This impatient rival firm will consequently adopt a more aggressive design, occupy a larger market share, and charge a higher price. Interestingly, both firms’ second-period profits improve as a result of asymmetric market shares. The following proposition states the results.

**Proposition A-1** Asymmetric patience between firms mitigates the first peril of behavior-based personalization. The more patient firm earns a smaller profit in period one.

**A-8.2 Segment of Myopic Consumers**

In this section I investigate heterogeneous patience in a different way by allowing a fraction $m$ of consumers to be myopic (with discount factor being zero) and a fraction $1 - m$ to be forward-looking with discount factor $\delta > 0$. I assume that myopic consumers are uniformly distributed in the market. As the main analysis reveals, consumers’ degree of patience affects
their price and product sensitivities in the first period. Therefore, when offered the same set of products in period one, the indifferent forward-looking consumer may not be at the same location as the indifferent myopic consumer. I denote these two indifferent consumers as $\hat{x}_f$ and $\hat{x}_m$ respectively.

In the second period, consumer patience no longer affects product choices. As a result, there is one single cutoff consumer from firm $A$’s clientele, $\hat{x}_A$, who is indifferent between $a_o$ and $b_n$. Suppose $\hat{x}_A \in [0, \min(\hat{x}_f, \hat{x}_m)]$. (Results where $\hat{x}_A \in (\min(\hat{x}_f, \hat{x}_m), \max(\hat{x}_f, \hat{x}_m)]$ can be similarly derived.) It follows that the demand for $a_o$ equals $\hat{x}_A$ and that the demand for $b_n$ equals $m\hat{x}_m + (1-m)\hat{x}_f - \hat{x}_A$. But once we rewrite $m\hat{x}_m + (1-m)\hat{x}_f$ as $\hat{x}$, the equilibrium choices of $a_o$ and $b_n$ are the same as in the main model. The same logic applies to the competition over firm $B$’s clientele. That is, in the second period firms compete as if on every point of the Hotelling line there is a representative consumer whose level of patience is a weighted-average of 0 and $\delta$. Firms’ second-period profits hence follow directly from Equation (2):

$$\pi^*_A(\hat{x}_f, \hat{x}_m) = \pi^*_B(\hat{x}_f, \hat{x}_m)$$

(A-13)

In the first period, the indifferent myopic consumer is simply $\hat{x}_m = (p_b - p_a) / 2t(b - a) + (a + b) / 2$. However, choices of forward-looking consumers depend on product offerings in the second period, which in turn depend on the fraction of myopic consumers as described above. Therefore, the cutoff value $\hat{x}$ is derived by solving the following conditions simultaneously. First, the forward-looking consumer $\hat{x}_f$ is indifferent between choosing $a$ and $b$: $p_a + t(\hat{x}_f - a)^2 + \delta \min[p_{a_o}^* + t(\hat{x}_f - a_o)^2, p_{b_o}^* + t(\hat{x}_f - b_o)^2] = p_b + t(\hat{x}_f - b)^2 + \delta \min[p_{a_n}^* + t(\hat{x}_f - a_n)^2, p_{b_n}^* + t(\hat{x}_f - b_n)^2]$. Second, period-two equilibrium designs and prices are functions of $\hat{x}$ as specified in Section 4.1.2. Third, the representative indifferent consumer reflects a weighted average: $\hat{x} = m\hat{x}_m + (1-m)\hat{x}_f$. The resulting representative indifferent consumer is derived as:

$$\hat{x} = \frac{a + b}{2} + \frac{(8 - \frac{2m\delta}{b-a})(p_b - p_a)}{25(29m)\delta t + 16t(b-a)} + \frac{25\delta(1-a-b)(1-m)}{(50-58m)\delta + 32(b-a)}$$

(A-14)

As long as $b - a > \delta/4$, which holds in equilibrium, the coefficient of $p_b - p_a$ in the second
Figure A-5: First-Period Equilibrium Differentiation and Prices with a Segment of Myopic Consumers \((t = 1, \beta = 0.25, \delta = 1)\)

Myopic consumers’ high price sensitivity should lead to lower prices in the first period. Figure A-5 confirms this intuition.\(^{A-4}\) Equilibrium first-period prices decrease with the fraction of myopic consumers. Additionally, as the fraction of myopic consumers rises, first-period equilibrium differentiation first decreases and then increases. This observation can be interpreted from two countervailing forces: on the one hand, myopic consumers’ higher product sensitivity induces firms to adopt more aggressive designs; on the other hand, there is a greater need to differentiate and soften the escalating price competition.

Following the symmetric first-period equilibrium, the indifferent consumers from the two segments coincide: \(\hat{x} = \hat{x}_m = \hat{x}_f = 1/2\). Firms offer the same products and earn the same profits in period two as in the main model. Therefore, both firms are \(\text{ex ante}\) worse off when there is a larger segment of myopic consumers, again consistent with Proposition 2. The following proposition summaries these results.

\(^{A-4}\)In the figure I set \(\beta = 0.25\) because a symmetric first-period equilibrium may not exist when both \(\beta\) and \(m\) take large values (Figure A-2 of the Appendix shows the parameter range for symmetric equilibria to exist).
Proposition A-2  A larger segment of myopic consumers does not affect the first peril of behavior-based personalization but exacerbates the second peril.

References
