Timeout Order Abstraction for Time-Parametric Verification of Loosely Synchronized Real-Time Distributed Systems

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Timeout Order Abstraction for Time-Parametric Verification of Loosely Synchronized Real-Time Distributed Systems

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Abstract—We present timeout order abstraction (TO-abstraction), a technique to systematically abstract a given loosely synchronized real-time distributed system (LSRTDS) into an untimed model. We define the subclass of LSRTDS’s that we can apply TO-abstraction using a syntax template that represents a restriction to Tempo, the primary modeling language of TIOA [7]. The untimed model obtained from the abstraction is a classical finite state machine, and thus one can automatically verify temporal properties of the model using a conventional model-checker. We prove the soundness of the abstraction using simulation relation. From this result, we guarantee that any untimed safety property of the untimed model also holds for the original TIOA model.

We have applied TO-abstraction to a resource-sharing protocol and the DHCP Failover protocol. We verified untimed abstractions of them by bounded model-checking up to depth 20. We have also experimented with effectiveness of bug-finding using our technique by mutating particular parts of the original code. From this experiment, we found a complex bad execution that would have been very difficult to find by human or simulations.\(^1\)

1. INTRODUCTION

Analyzing correctness of real-time distributed systems is a challenging problem due to the combination of nondeterminism from process interleaving and timing constraints in the system executions. The class of Loosely-synchronized real-time distributed systems (LSRTDS’s) is an interesting subclass of real-time distributed systems. In this subclass, the processes or modules in the system are assumed to have loose synchronization, that is, there is an a priori known upper bound \(\varepsilon\) on the skew between local clocks in processes. Processes communicate time data (time-related information such as time stamps) with each other, and set their timeouts using time data. These timeouts are used to constrain processes’ behavior in such a way that the processes execute a certain designated action before or after other processes execute another designated action. For example, the DHCP Failover protocol [5], used for one of the two case studies in this paper, falls into this subclass. Due to a general assumption about evolution of local clock values and timeout setting using time data communication between processes, automatic exhaustive exploration (model-checking) techniques for LSRTDS’s has not been studied thus far in the community, as far as we know. In particular, existing timed and hybrid model-checkers cannot directly treat the subclass of LSRTDS’s that we treat in this paper. We tackle in this paper the verification problem for LSRTDS’s by providing a way of machine-assisted automatic analysis for a particular subclass of LSRTDS’s.

We present timeout order abstraction (TO-abstraction), a technique to systematically abstract an LSRTDS into an untimed model. We define the subclass of LSRTDS’s that we can apply TO-abstraction using a syntax template that represents a restriction to Tempo, the primary modeling language of TIOA [7]. The TIOA framework has been used to model and verify (with hand proofs) several real-time distributed systems and algorithms (for example, [8], [9], [3], [4], [5], [14]). TO-abstraction enables the user to conduct time-parametric verification of a given TIOA model described by the template in the sense that the local clock skew bound \(\varepsilon\) and the special timing-related constant that we explain later are treated as parameters of the system, and therefore, are not instantiated into concrete values. TO-abstraction performs a code-to-code conversion by which a model described using the template is converted into ordinary (untimed) I/O automaton code. The untimed model obtained by TO-abstraction is a finite-state machine, and thus one can automatically verify (untimed) temporal properties of the model using a conventional model-checker. We prove a simulation relation from the original TIOA model to the abstraction using the loose synchronization assumption described earlier. From this result, we guarantee that any untimed safety property of the untimed model also holds for the original TIOA model. This soundness theorem works as a meta theorem for the template: the theorem applies to every system described by the template.

The template can express interesting building blocks that processes in the system can use to communicate time data and set timeouts. For instance, a process can pick a time nonce, an arbitrary value that represents some future time on its local clock. The process sends the time nonce to other processes. By intelligently setting timeouts using the time nonce, one group of processes can time out after another group of processes have done so. A process may update its timeout time or variables using a “max” operation for pieces of time data. This max operation is typically used to conservatively update the estimate of other processes’ timeouts times. A process may check whether or not time data stored in their state variables or time data received from other processes has already “expired” (its local clock value already exceeds the value of the time data).

Contributions: There are three main contributions in the presented work.

First, we provide an abstraction technique, timeout order abstraction (TO-abstraction), for formal verification of a specific subclass of LSRTDS’s, which have not been studied in the context of automated formal verification. As far as we know, TO-abstraction presented in this paper is the first technique that enables the user to reduce a verification problem of a LSRTDS to a finite-state model checking problem. By conducting case studies, we have found that TO-abstraction is useful not just for verification (developing guarantees), but also for bug-finding. We have experimented with “mutating” parts of the code for a case study of the technique, and found, by model-checking the abstracted model, a counterexample that is complex and thus is arguably difficult to find by human or simulations.

\(^1\)A supplemental information about this paper (such as more detailed explanation of parts of the paper or proofs), and the code used for the case study can be obtained from the URL cited as [11].
Second, the template presented in the paper provides protocol designers with interesting building blocks for real-time protocols. In particular, the time-stamp-estimation trick that we explain in Section 2 gives the designers an interesting way of adding fault-tolerance to their protocols. This special trick is used in [5] without particularly mentioning its usefulness, intuition, or subtlety. We make this special usage explicit by having it as one of the building blocks that can be expressed by the template.

Third, we provide a case study on automatic verification of the DHCP Failover (DHCP-F) protocol. The protocol has been studied in [5] in the context of formal verification using manual (handwritten) proofs, but no study on automatic analysis of the protocol has been reported thus far. Our case study provides exhaustive exploration of scenarios of DHCP-F up to the execution length of 20 (20 discrete transitions, including sending and receiving messages, of the system) for the configuration of two clients and two servers.

Related work: There are several techniques that have been developed thus far for a reduction from real-time system verification to finite-state machine verification. The most famous technique is arguably the triangular-automaton-construction technique for timed automata, developed by Alur and Dill [1]. We have studied an abstraction technique, event order abstraction, for parametric timed verification by focusing on key event orders [10]. The class of LSRTDS’s that we treat by TO-abstraction cannot be expressed by such frameworks as Alur-Dill Timed Automata [1], Linear Hybrid Automata [6], or Time-Interval Automata that we use in [10]. This is mainly due to the fact that the loosely synchronized assumption allows very general evolution of local clock values: as long as the local clock value evolves increasingly and the loosely synchronized assumption (the skew bound among the local clocks) are satisfied, the evolution can be arbitrary (and thus can be non-linear). Therefore, we cannot directly benefit from the existing verification techniques developed for these frameworks. We consider this fact as one of the main reasons that LSRTDS’s have not been studied intensively in the automatic verification community.

The rest of this paper is organized as follows. Section 2 explains how we have found the template for TO-abstraction, and why the building blocks for time data communication expressed using the template are interesting. We discuss the usage of building blocks using small toy examples. In Section 3, we describe the settings of a distributed real-time system that we assume and the restricted syntax template for the TIOA programming language with which the modules in the system must be described in order to use TO-abstraction. Section 4 is devoted to presenting how we can abstract a TIOA model described using the template defined in Section 3 into a finite-state untimed model. In Section 5, we briefly explain how we can prove the soundness of TO-abstraction. In Section 6, we report applications of TO-abstraction to a resource-sharing protocol and the DHCP Failover protocol. We conclude in Section 7.

2 The Internet-draft version of DHCP-F does not consider this type of subtle arguments about loosely synchronized clocks, and thus how long a leading server has to wait to conservatively estimate time stamps other servers have picked is not clearly stated. We contacted the first author of [5], and were informed that this subtle special usage, which we will call the time-stamp-estimation trick, was proposed by him, discussed by the authors of [5], and then adopted in the model of DHCP-F used in [5].

2 A time nonce in an actual low-level implementation may be computed using, for example, a complex optimization and/or adaptive algorithm or a randomized algorithm. We just assume the most general (least restrictive) assumption: as long as its value is larger than the current clock of the process that picks the time nonce, it is a valid time nonce for our setting.
“done” message. If the message delay is relatively long compared to the resource usage time interval, inefficiency becomes a real
problem. Processes can use time nonces to loosely synchronize their behaviors, as Protocol 1 shows. Protocol 1 makes more
efficient use of the resource if the clock skew bound $\varepsilon$ is smaller than the message delay.

Protocol 1: $P_1$ picks the time until which it will use the resource, as a time nonce $TN_1$, and sends it to $P_2$. $P_1$ sets a
timeout at $TN_1$ and starts using the resource. When $P_2$ receives $TN_1$, it sets a timeout at $TN_1 + \varepsilon$. The skew bound $\varepsilon$ is used to
most conservatively estimate when $P_1$ times out, on $P_2$’s local
clock. When $P_2$ times out, it is guaranteed that $P_1$ has already
timed out. Therefore, $P_2$ can immediately use the resource. $P_2$
picks another time nonce $TN_2$, sets a timeout at $TN_2$, and sends $TN_2$ to $P_1$. Upon receipt of $TN_2$, $P_1$ sets its timeout to $TN_2 + \varepsilon$.
$P_1$ and $P_2$ repeat the same routine forever. □

Now we consider a different assumption for channels. Now the
channels are not stable, and contents of messages sent between
processes may become broken. When message contents become
broken inside the channel, processes can recognize that the contents
are broken (for example, using a check-sum). Under the above
assumption, we cannot use Protocol 1 – if information of $TN_1$ sent
from $P_1$ to $P_2$ becomes broken, there is no way $P_2$ can estimate
when $P_1$ finishes its job. One (not so smart) option is going back
to “done” message communications.

Using a time stamp with a constant waiting time instead of an
arbitrarily time nonce resolves this situation. Suppose processes a
priori share the value of the constant waiting time $u$, and the value
of $u$ is fixed.

Protocol 2: $P_1$ picks a time stamp $TS_1$, instead of an arbitrary
time nonce, and sets its timeout to $TS_1 + u$. Therefore, $P_1$ uses the resource for $u$ time units (measured on its local clock). $P_1$ sends
$TS_1 + u$ to $P_2$. If the above message sent to $P_2$ is not broken, $P_2$
sets its timeout to $TS_1 + u + \varepsilon$ (the received time data plus $\varepsilon$) as
in Protocol 1. If the message is broken, $P_2$ sets its timeout to the
special value $clock_2 + u + 2\varepsilon$, where $clock_2$ is the current value
of $P_2$’s local clock. When $P_2$ times out, it immediately starts using
the resource. It also picks a time stamp $TS_2$, sets its timeout to
$TS_2 + u$, and sends $TS_2 + u$ to $P_1$. $P_1$ and $P_2$ repeat the same
routine forever. □

Time-Stamp-Estimation Trick: The time setting using
$clock_2 + u + 2\varepsilon$ in Protocol 2 is the special timeout setting, the
time-stamp-estimation trick that we have mentioned in the introduction.
We explain in the following why this value can be used to estimates
conservatively when $P_1$ times out. We can interpret the value
$clock_2 + u + 2\varepsilon$ as $(clock_2 + \varepsilon) + u + \varepsilon$. If $clock_2 + \varepsilon$ is equal
to or greater than $TS_1$, then $P_2$ indeed succeeds in conservatively
estimating $P_1$’s timeout (if the message were not broken, $P_2$
would have set its timeout to $TS_1 + u + \varepsilon$). We consider in the following
the moment that $P_2$’s timeout is set to $clock_2 + u + 2\varepsilon$. From
the loose synchronization assumption, the value of $clock_2 + \varepsilon$ is
at least as large as the value of $clock_1$, the local clock of $P_1$.
Because $P_1$’s clock value is monotonically increasing, the current
value of $clock_1$ is greater than the value of $TS_1$, which is copied from
the past value of $clock_1$. Therefore, $clock_2 + \varepsilon \geq TS_1$, as
needed. The key to the above argument was that because of the
fact that $P_2$ received a (broken-content) message from $P_1$, $P_2$ was
sure that $P_2$ had already picked time nonce $TS_1$. The value of
$clock_2 + \varepsilon$ overestimates any time stamp that has been picked thus
far in physical time, and a process can perform this estimation by
looking at just its local clock.

We can extend time-sharing problem to two groups of processes,
as we describe in the following Protocol 3.

Protocol 3: Two groups of processes $\Pi_1$ and $\Pi_2$ share a re-
source, and any two processes in the different groups must not use
the resource at the same time. Processes in $\Pi_1$ first broadcast their
$TS + u$ values to $\Pi_2$, and processes in $\Pi_2$ wait until the
latest time among received $TS + u$ values. Upon receipt of
every $TS + u$ value, processes in $\Pi_2$ update an estimate of the
latest time using a ‘max’ operation: $estimation :=
\max(received_{TS,u}, estimation)$, and updates the timeout time
to $estimation + \varepsilon$. When a process receives a broken-content
message, it uses the time-stamp-estimation trick, by setting a
timeout at $\max(clock_1 + u + 2\varepsilon, old_timeout_time)$. When a process in $\Pi_2$ has received messages from all processes in $\Pi_1$ and has timed
out, it can use the resource and broadcasts a $TS + u$ value to $\Pi_1$.
$\Pi_1$ and $\Pi_2$ repeat the same routine forever. We can in addition give
processes the choice of choosing their own “customized” resource-
sharing time, instead of the constant time $u$: When broadcasting a
resource-sharing time to another group, a process may choose to
propose an arbitrary time using a time nonce, instead of $TS + u$.
In this case, that process starts using the resource only after it
gets acknowledgments from all processes in the different group
(because the time-stamp-estimation trick cannot be used in this
case).

Protocol 3 above may exhibit subtle corner-case scenarios that
are difficult to verify by human or simulations. For example, a
process $P_1$ waiting for acknowledgments for its “customized”
request may receive another “customized” request from a process
in the different group, instead of an acknowledgment to its own re-
quest. This scenario can happen when $P_1$’s sent message becomes
content-broken, and thus $P_1$ cannot receive acknowledgements
from all processes in the different group. Because of subtle
scenarios in Protocol 3 such as the one described above, the
implementer of the protocol will gain the benefit of automatic
analysis of the implemented protocol using TO-abstraction. We
use one possible implementation of the protocol as our first case
study in the paper.

3. SETTINGS AND TEMPLATE

In this section, we explain the settings of a distributed real-time
system that we assume and the syntax template that describes a
restricted subclass of the ordinary TIOA guarded-command-style
language. The basic idea of the template is to restrict use of time
data and timeouts to a special form for which TO-abstraction can
be applied. There is no restriction imposed for “untimed” part of
the language.

3.1. Processes and Modules

A system consists of a set of processes $\{P_i\}$ and channels
between processes $\{C_{ij}\}$.4 Processes in the system can be
heterogeneous – a process can execute a different program from that
of another. For example, processes may be split into two groups,
Severs and Clients: Processes in Servers run the same program,
and processes in Clients run a program that is different from the
server, but is the same for all Clients. Processes communicate with
each other via channels. A channel has a first-in-first-out (FIFO)
buffer in it to store process messages. The buffer size is bounded
(in order to conduct model-checking after the abstraction), and
a message sent to the channel when its buffer is full is simply
discarded. We assume that processes are loosely synchronized: for
any pair $(P_i, P_j)$ of processes, the deviation between the values of

4In the full version of this paper [13], We actually give the user an option to use
additional modules, environment and helper modules, to describe a more general
setting. Due to space limitation, we only talk about processes and channels in this paper.
their local clocks, $\text{clock}_i$ and $\text{clock}_j$, respectively, are bounded by an a priori known amount $\varepsilon$, as shown in the following inequality: $|\text{clock}_i - \text{clock}_j| < \varepsilon$. We assume that the $\varepsilon$ bound is known to every process as its parameter. We also assume that a constant positive real value $u$ is known to every process as its parameter. A TIOA that models a process in the system must have the following automation declaration: “Automaton $P_1(\varepsilon, u; \text{NonNegReal}, \nu; \text{OtherDiscreteParameters})”. In the above declaration, $\nu$ is a parameter that contains non-timing-related information, such as value initializations for “untimed variables”. We discuss untimed and timed variables in Section 3.3.

The entire system is represented by composing processes and channels in the sense of TIOA (an automata composition by synchronizing output and input actions with the same name. See [7] for more details). COMPOSITION below represents the composition.

$$S(\varepsilon, u, \{init_i\}) = (\Pi_{i \in \text{ProcessIDs}} P_i(\varepsilon, u, init_i)) \times (\Pi_{i,j \in \text{ProcessIDs}} C_{i,j})$$

The set $\{init_i\}$ in $S(\varepsilon, u, \{init_i\})$ represents a list of initialization parameters for each process $P_i$. Note that every process shares the same $\varepsilon$ and $u$ after the composition ($\varepsilon$ and $u$ are parameters of the entire system $S$).

### 3.2. Action Signatures of Process $P_i$

In this subsection, we present the form for the signatures (interfaces) of actions for process $P_i$. The system has output and input actions for message communication between processes, such as ‘send’ and ‘receive’. A ‘send’ action has the following signature: $\text{send}(j; \text{ProcessID}, \text{M:UntimedMessage}, \chi; \text{NonNegReal}, \kappa; \text{InteractionInst})$. This action represents that $P_i$ sends real-valued “time data” $\chi$ and “untimed data” $\text{M}$ to process $P_j$. We will explain in Section 3.5 the form of time data allowed in the template. Untimed data is an arbitrary value in a bounded domain. An interaction instance $\kappa$ is a special identifier of process interactions to distinguish different set of interactions. A ‘receive’ action has the same type of interface as a ‘send’, in order to match communications between processes. A ‘broadcast’ action, $\text{bcast}_i$, has a similar interface as $\text{send}_i$, but instead of a process ID $j$, it has a set of process IDs $J$ that represents a subset of processes to which the broadcast is performed. A process also has internal actions which are performed without communicating outside of the world.

A timeout action of $P_i$ is modeled as an output action, $\text{timeout}_i^k$, and has a very specific form for its precondition (the transition guard) so that the timeout happens at the time the local clock of $P_i$ hits a specified timeout time. The actual form of the precondition is described in Section 3.5.

To give the reader information about how templates are actually used in a real example, we present the Tempo code for our implementation of 3 that we use for one of the case studies. Due to space limitation, we present in Figure 1 only parts of the code. We explain some use of time data and timeout in this code. $\text{bcast}$, of ‘CONST’ in Figure 1 uses $\text{clock}_i + u$ for its broadcasting time data. It’s timeout time is set to the same value. Whereas, $\text{bcast}$, of ‘CUSTOM’ picks a new time nonce, and broadcasts it. It’s timed variable $\text{proposing_time_nonce}$ is set to this time nonce, but the timeout is not set yet (since it has not obtained acknowledgments). The timeout time is updated using the time-stamp-estimation trick and a ‘max’ operation when a process receives a ‘BROKEN’ message.

![Figure 1](tempo_code.png)

### 3.3. State Variables of Process $P_i$

State variables in a TIOA that represents process $P_i$ are split into the six groups shown in Table I. This explicit split of variables is the core of why we can apply TO-abstraction to the template.

<table>
<thead>
<tr>
<th>Group Name</th>
<th>Variables</th>
<th>Types of Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timed variables</td>
<td>${x^k}_{k=1}^n$</td>
<td>NonNegReal</td>
</tr>
<tr>
<td>Timeout variables</td>
<td>${t^k}_{k=1}^n$</td>
<td>NonNegReal</td>
</tr>
<tr>
<td>Timeout setting Booleans</td>
<td>${\text{timeout_is_set}^k}_{k=1}^n$</td>
<td>Boolean</td>
</tr>
<tr>
<td>Interaction Instance variables</td>
<td>${w^k}_{k=1}^n$</td>
<td>Integer</td>
</tr>
<tr>
<td>Untimed variables</td>
<td>${v^k}_{k=1}^n$</td>
<td>BoundedDomain$^k$</td>
</tr>
<tr>
<td>Local clock variable</td>
<td>$\text{clock}_i$</td>
<td>NonNegReal</td>
</tr>
</tbody>
</table>

**TABLE I**

State variables of process $P_i$

The first group consists of timed variables. These variables can store real-valued time data such as time nonces and time stamps (plus $u$), or the result of a ‘max’ operation of multiple pieces of time data. We will see the form of value assignments that can be used for timed variable in Section 3.5. The second group consists of timeout variables. These variables are special variables to set a time when a timeout action must be performed. They can store time data, possibly plus the slack $\varepsilon$, and the result of the time-stamp-estimation trick. The third group consists of timeout setting Booleans. These are Boolean variables that are used to indicate whether or not the timeout for $\text{timeout}_i^k$ is set. The fourth group consists of untimed variables. These variables can store any bounded-size information other than time data. An untimed variable can be, for example, a Boolean, a (bounded-size) counter, or a finite set of process IDs to aggregate the information about which processes have responded to $P_i$’s message. The last group consists of just one variable, $\text{clock}_i$, which represents the local clock of process $P_i$. The value of this variable evolves over real time. The actual evolution is defined by the trajectory definition discussed in Section 3.4.
3.4. Trajectory of Process $P_i$

We insist that a TIOA model for process $P_i$ have the following trajectory definition for evolution of the value of its local clock. The definition describes that 1. A local clock is monotonically increasing, and 2. When a local clock hits one of $t_i^k$ (a timeout time) and the timeout for $t_i^k$ is set, the timeout action must be performed before time elapses.

\[
\text{evolve: } d(\text{clock}_i) > 0 \quad \text{stop when: } \exists k : \text{clock}_i \geq t_i^k \land \text{timeout}_i\text{is_set}^k
\]

Note that the loose-synchronization assumption described earlier in this section cannot be described by the trajectory definition of one process, but is used in the soundness theorem as an assumption.

3.5. Transitions of Process $P_i$

In this subsection, we describe the template for transitions of process $P_i$. A transition is defined using its precondition (the guard) and effects. We have templates for both preconditions and effects. The template is defined formally using a BNF-like form. Due to space limitation, we cannot list the entire template in the paper. Instead, we explain important constructs in the template in this subsection. (See [13] for the entire template.)

**Timed Expression Template**: The templates for $\text{TimeDataExpression}$, $\text{TimeoutExpression}$, and $\text{TimeoutUpdateExpression}$ shown in TimedExpression are the most important templates in the entire template set. These includes a time nonce picking (new_time_nonce()), which computes a value greater than clocki, a future time, computation of a time stamp plus the constant $u$ (clocki + $u$), a ‘max’ operation, timeout setting using $\epsilon$, and the time-stamp-estimation trick (clocki + $u + 2\epsilon$). These expressions are used in templates for process transitions explained later in this section. See Figure 1 and its explanation again to observe that timed expressions in the code can be expressed using these templates.

\[
\begin{align*}
\text{TimeDataExpression} & := \text{new\_time\_nonce()} \mid \text{clock}_i + u \mid \chi \mid \text{max}(\text{TimeDataExpression}, \text{TimeDataExpression}) \\
\text{TimeoutExpression} & := \text{TimeDataExpression} \mid \text{TimeDataExpression} + \epsilon \mid \text{clock}_i + u + 2\epsilon \mid \text{max}(\text{clock}_i + u + 2\epsilon, \text{TimeDataExpression} + \epsilon) \\
\text{TimeoutUpdateExpression} & := \text{TimeoutExpression} \mid \text{max}(t_i^k, \text{TimeoutExpression}) \quad \text{(TimedExpression)}
\end{align*}
\]

**Precondition of Timeout Actions**: The precondition of timeout action $t_i^k$ must always have the following form: $\text{clock}_i = t_i^k \land \text{timeout}_i\text{is_set}^k$. The above condition states that a timeout action $t_i^k$ is enabled when the local clock of $P_i$ hits the specified timeout time $t_i^k$, and also the value of $\text{timeout}_i\text{is_set}^k$ is true.

**Precondition of ‘send,’ and ‘bcast,’ Actions**: Preconditions for $\text{send}_i(j, M, \chi, \kappa)$ and $\text{bcast}_i(j, M, \chi, \kappa)$ must include the binding of the sending time data $\chi$ in the following form: $\chi = \text{TimeDataExpression}$. Basically, a process can include any time data (which can be expressed by the template) in its sending messages.

**Effects of Actions**: The effects of an action are constructed by simple assignments to variables and if statements that include assignments or nested if statements.

The user can use expiration checks in the branch condition of if statements. These checks are in the form of $X \sim \text{clock}_i$ or $X + \epsilon \sim \text{clock}_i$, where $\sim$ is either $>$ or $\leq$, and $X$ is either $x_i^k$ or $\chi$ ($\chi$ is only for send, broadcast, and receive actions). Using an expiration check, a process can, for example, check whether received time data represents a future time, and thus can be validly set the time data as a timeout time (receive, of ‘CUSTOM’ in Figure 1 uses this check).

The values of untimed variables are assigned by arbitrary expressions that only use untimed variables. The values of timed variables are assigned by any form of $\text{TimeDataExpression}$, defined in TimedExpression.

A timeout is set and reset by $\text{TimeoutAssignment}$ defined in $\text{TimeoutASSIGN}$. $\text{TimeoutUpdateExpression}$ is defined in TimedExpression. We can see in $\text{TimeoutASSIGN}$ that an assignment to a timeout variable $t_i^k$ is always performed with a setting to $\text{timeout}_i\text{is_set}^k$.

\[
\text{TimeoutAssignment} :::= t_i^k := \text{TimeoutUpdateExpression}; \text{timeout}_i\text{is_set}^k := \text{true}; \mid t_i^k := 0; \text{timeout}_i\text{is_set}^k := \text{false}; \quad (\text{TIMEOUTASSIGN})
\]

4. Timeout Order Abstraction

Timeout order abstraction (TO-abstraction) enables the user to abstract a TIOA model described using the template explained in Section 3 into an untimed finite-state model. This abstraction is performed as a code-to-code conversion from a model described by the template into ordinary (untimed) I/O automata code. We first explain the overview of this technique.

4.1. Intuition behind Timeout Order Abstraction

Our goal for TO-abstraction is to obtain an untimed model that conservatively abstracts the original one. In order to obtain an untimed model, we remove the local clocks of processes, and maintain just the right amount of information that can retrieve correct timeout orders in the original timed model. The key idea is to not focus on real values of time nonces and time stamps, but instead use identifiers (IDs) of them (labels in other words). Identifiers are integers that distinguish different time nonces and stamps. We explain in more detail how IDs can be used by using toy examples described in Section 2.

In Protocol 1, time nonces are used to synchronize processes behavior. Processes control the ordering of timeouts using the relative difference of their timeout times measured on their local clocks: one process $P_i$ sets a timeout to a time nonce $TN_k$, and the other $P_j$ sets it to $TN_k + \epsilon$. Information that is sufficient to retrieve the fact that $P_j$ times out after $P_i$ has done so in any possible scenario, is that both processes use the same time nonce $TN_k$, and $P_i$’s timeout is set by adding the “slack” $\epsilon$ to the time nonce, but $P_j$’s timeout is set solely by the time nonce. To keep track of the above information, we can use the following abstraction: Picks of real-valued time nonces are replaced by picks of symbolic IDs of them. The $k$-th picked time nonce $TN_k$ in an execution of the original system is represented by ID (integer) $k$.

The timeout for $P_i$ set to $TN_k$ is symbolically represented.
by a pair \((k, \text{false})\), and the timeout time for \(P_2\) set to \(TN_k + \varepsilon\) is symbolically represented by a pair \((k, \text{true})\) – the Boolean values represent whether the slack \(\varepsilon\) is added or not. Then, we remove the local clocks of both processes. After this abstraction, even though we cannot access to the information of local clock values (since they are removed), we can carefully choose which timeouts must not occur at the current state of the untimed model, by looking at the symbolic representations: If \(P_i\)'s timeout is set to \((k, \text{false})\) and \(P_j\)'s timeout is set to \((k, \text{true})\), \(P_j\)'s timeout must not be performed, considering timeout orders that can possibly appear in the original timed model. The above discussion is the basic idea of time data abstraction and timeout order constraining, two of the three techniques that we use in combination for TO-abstraction.

We can apply the above idea of maintaining information that is sufficient to retrieve correct timeout orders to Protocol 2 as well. In this case, we use IDs of time stamps. Since processes always add \(u\) to time stamps, we can just keep track of the ID of a time stamp when the value computed from a time stamp plus \(u\) (in the form of \(TS_k + u\)) is communicated. We explain how we treat the time-stamp-estimation trick in the abstraction after we explain the abstraction for the max operation. In Protocol 3, processes use ‘max’ operations to conservatively update the timeout estimates. To represent a ‘max’, we use a set of IDs. For example, \(\max\{tn_1, tn_3, tn_4\}\) is represented by \(\{1, 3, 7\}\). Now \(tn_3\) is considered as \(\max\{tn_3\}\), and thus is represented by \(\{3\}\). Timeout setting \(\max\{tn_1, tn_3\} + \varepsilon\) is represented by \(\{(1, 3), \text{true}\}\). We can constrain timeout orders as follows: If for two sets of time nonce IDs \(IDS_1\) and \(IDS_2\), \(IDS_1 \supseteq IDS_2\) and two timeouts are set at \((IDS_1, \text{true})\) and \((IDS_2, \text{false})\), respectively, then the timeout at \((IDS_1, \text{true})\) is disabled. This is because \(IDS_1\) represents a larger value than \(IDS_2\). When time nonces and time stamps are used in the same protocol, we use two sets of integers (IDs) to represent one time data: \((\{1, 2\}, \{4\})\) represents \(\max(TN_1, TN_2, TS_4 + u)\), and \((\{3\}, \{2\}, \text{true})\) represents a timeout set at \(\max(TN_3, TS_2 + u) + \varepsilon\). We do not allow a timeout update using a slack-added timeout and non-slack timeout, since the updated timeout cannot be represented symbolically.\(^5\) We add a run-time check of consistency of timeout update in the abstraction, so that the abstracted model conservatively approximates the original.

The time-stamp-estimation trick using \(\text{clock}_i + u + 2\varepsilon\) is abstracted as \(\{(\{\}, \text{picked}_{\text{ts}, \text{id}, \text{set}, \text{true}}), \text{true}\}\), where \(\text{picked}_{\text{ts}, \text{id}, \text{set}, \text{true}}\) is the set of time stamp IDs that have been picked thus far in the current execution. \(\{(\{\}, \text{picked}_{\text{ts}, \text{id}, \text{set}, \text{true}}), \text{true}\}\) represents \(\max(\text{picked}_{\text{ts}, \text{set}}) + u + \varepsilon\) in the original model, where \(\text{picked}_{\text{ts}, \text{set}}\) is the set of time stamps picked thus far. This reflects the discussion for the time-stamp-estimation trick in Section 2 that \(\text{clock}_i + u + 2\varepsilon\) can be interpreted as \((\text{clock}_i + \varepsilon) + u + \varepsilon\), and \(\text{clock}_i + \varepsilon\) over-approximates every time stamp that has been picked in physical time.

4.2. Overview of Timeout Order Abstraction

We use three sub-techniques in combination for timeout order abstraction of a TIOA model described by the template explained in Section 3. The three techniques are: 1. Time data abstraction, 2. Timeout order constraining, and 3. Time data reuse and compression. The first two techniques are used to abstract the underlying real-time system into an untimed, but infinite-state model. The third technique is used to represent the infinite state space of the untimed abstraction using a finite one.

We first explain each technique briefly here, and will go into more details in subsections of this section.

[Time Data Abstraction]: First, time data abstraction abstracts away local clocks from a given TIOA model, and abstracts real-valued time data in the system using symbolic representations, as discussed in Section 4.1. In addition, all boolean conditions in the original model that require a local clock value to determine their truth-values are conservatively approximated.

[Timeout Order Constraining]: Using information from the symbolically represented timeout time, we constrain the order of timeouts, as we discussed in Section 4.1.

[Time Data Reuse and Compression]: An execution of the untimed model may require infinite number of new time nonce and/or time stamp IDs because the length of a model execution is typically infinite. Our basic strategy is to reuse a time stamp or a time nonce that is once taken by some process but is no longer used anywhere in the model.

4.3. Time Data Abstraction

By time data abstraction that we describe in this section, the user can abstract a TIOA model described using the template discussed in Section 3 into an untimed infinite-state model. We will consider how we can make this untimed model a finite-state model in Section 4.5.

The basic idea of time data abstraction is to represent time stamps and time nonces using positive integers that represent the “identifiers” of time stamps and time nonces, as discussed in Section 4.1. The type of timed variables (which store time data) after abstraction becomes: SymbolicTimeData = (set of Natural) × (set of Natural), where the first set represents time stamps and the second represents time nonces. And the type of timeout variables becomes: SymbolicTimeout = (set of Natural) × (set of Natural) × Boolean, where the last Boolean entry represents whether or not the “slack” value \(\varepsilon\) is added to the timeout value. For the above two types SymbolicTimeData and SymbolicTimeout, we define two functions \(\text{ts}_{\text{set}}\) and \(\text{tn}_{\text{set}}\) that get the first integer set and the second integer set, respectively, from a given symbolic representation of time data or a timeout value. We also use a predicate (a Boolean function) \(\text{slack}_{\text{added}}\) for SymbolicTimeout to access to the third entry (a Boolean value) of SymbolicTimeout.

To use symbolic representation of time data discussed above, time data abstraction conducts the following five processes.

1) Over-approximating the preconditions of timeout actions,
2) Symbolically representing of time stamps and time nonces using IDs,
3) Abstracting expiration checks, and
4) Abstracting the time-stamp-estimation trick, \(\text{clock}_i + u + 2\varepsilon\), and
5) Removing the local clock and the trajectory definition.

The transformation is formally defined as a function (a code-to-code conversion) from the TIOA syntax template defined in Section 3 to the ordinary I/O automata syntax. Due to space limitation, we cannot present the definition of this function in this paper, but the definition is described in [13]. In the following, we explain important parts of the abstraction.

**Over-approximating the Preconditions of Timeout Actions:**
The precondition of every timeout action, which has the form \(\text{clock}_i = t_i^k \land \text{timeout}_{\text{ts}}_{\text{set}, t_i^k}\), is over-approximated by replacing it with \(\text{timeout}_{\text{ts}}_{\text{set}, t_i^k}\).
Symbolically Representing Time Stamps and Time Nonces:

We treat time nonce IDs as a global variable `picked_tn_id_set` of positive integer type. `picked_tn_id_set` represents the set of time nonce IDs picked thus far. Now new `time_nonce()` function gives the smallest ID not in `picked_tn_id_set`. We perform bookkeeping for time stamp IDs similarly using `picked ts_id_set`. Every appearance of `clock_i + u` is replaced by new `time_stamp(ID())`, which gives the smallest ID not in `picked ts_id_set`.

**Abstraction of Execution Checks:**

Expiration checks of the form `clock_i > x` is abstracted using the same logic as we use to determine the order of timeouts, explained in Section 4.4.

Informally, if there is a timeout set at `y` and the value of `y` is at most `x`, then `clock_i > x` will not hold, considering the loose-synchronization assumption. If we cannot infer the truth value of the expiration check, then the abstraction uses a non-deterministic choice for the result of the expiration check.

Expiration checks of the form `clock_i > x` is approximated using a special list, called the expired time-data list, which stores time nonce and time stamp IDs picked thus far that are “strongly expired” – expired on all local clocks. Whether a time nonce/stamp ID `ID_i` is strongly expired is deduced from either of the following two facts: 1. A timeout is performed at `TD_i + ε`, and symbolic time data `TD_i` contains `ID_i`; 2. A non-deterministic choice for `clock_i > x + ε` results in evaluating the inequality to be true, and `x` includes `ID_i`. Using the expired list, we can infer the truth value of `clock_i > x` in the following case: if all IDs in `x` are strongly expired, then `clock_i > x` must be true. If we cannot infer the truth value of the expiration check, then the abstraction uses a non-deterministic choice.

**Rule for the timestamp-estimation trick:**

As we discussed in Section 4.1, the timestamp-estimation trick using `clock_i + u + 2ε` is abstractly represented by `(({}), picked ts_id_set, true).

Removing the local clock and the trajectory definition:

After the untiming abstraction conducted above, the definition of the transitions of the untimed system does not use local clocks `clock_i` at all. Thus, we can safely remove local clock variables and the trajectory definition.

4.4. Timeout Order Constraining

The untimed model after time data abstraction loosens the control over timeout orders in the original timed model because the abstraction removes local clocks of processes. Therefore, we need to put the correct timeout orders back into the untimed model by constraining timeout orders using the information from symbolically represented timeout time. To constrain timeout order, we allow the abstract scheduler that resolves non-determinism in the untimed model to disable specific timeouts. The scheduler determines the next action to be performed from the set of all enabled actions in the current state. Whether a specific action is enabled or not is determined solely by its precondition.

In addition to each precondition of actions, for timeout actions, the scheduler looks at symbolically represented timeout time to decide whether a particular timeout action is enabled or not. Considering the loose synchronization assumption, if a timeout action `timeout_i` is set to a time larger than the time for another timeout `timeout_i` by more than `ε`, then `timeout_i` occurs after `timeout_i` has done so. By rephrasing the above statement in symbolic representations, we obtain the following. Timeout action `timeout_i` of a process `i` is disabled (even when `timeout_is_set` is true) when the following condition holds: `∃ t_i ∈ Timeout_variables : (j ≠ i) ∧ (ts(t_i) ⊆ ts(t_k)) ∧ (ts(t_i) ⊆ ts(t_k)) ∧ ~slack_added(t_i) ∧ slack_added(t_k).

4.5. Time Data Reuse and Compression

The length of an execution of an untimed model obtained from time data abstraction can be infinite. This implies that an execution may require infinite number of new time nonces or time stamps.

The simplest strategy that we can follow to reduce the necessary number of time nonces and time stamps is to reuse a time stamp or a time nonce once taken by some process but no longer used in any time data in the model. This reuse would not change the behavior of the untimed model (in terms of its traces) since a time nonce or a time stamp is symbolically represented by its IDs, and for the untimed model, these IDs have no more information than just distinguishing an ID from another. In addition, we sometimes need a more elaborate technique that we call time data compression in order to effectively express multiple time nonces (or stamps) by one symbolic time nonce (or stamp).

5. Soundness of Timeout Order Abstraction

In this section, we briefly describe how we can prove the soundness of TO-abstraction that we have discussed in Section 4. (A more detailed proof appears in [13].) The soundness claim we guarantee is stated as Theorem 1.

**Theorem 1:** For any execution `α` of the original timed model with any instantiation of `ε` and `u`, there exists a corresponding execution `β` of the untimed model such that values of the untimed variables in the state after the `k`th (discrete) transition appearing in `α` are the same as those in the state after the `k`th transition appearing in `β`.

The key technique to use is a simulation relation from the original model to the untimed abstraction [7]. The basic idea of the simulation relation technique is to find, for each possible transition and trajectory of one automaton, a corresponding transition that have the “same effects”. For example, the maximum operation of the timed original model has the “same effect” of the symbolic maximum operation in untimed model. We split this simulation relation proof into two parts: 1. Simulation relation from the original model (we call the model `Orig`) to the infinite-state untimed model obtained after time data abstraction and timeout order constraint (we call this model `Inf`) and 2. Simulation relation from `Inf` to the finite-state untimed model obtained after time data reuse and compression (we call this model `Fin`).

**Proof Idea:** [Simulation relation from `Orig` to `Inf`]: Essentially, the two models executes the same execution, one using numerical time data, and the other using the symbolic representation of it.

The simulation relation is defined so that a state of `Orig` and a state of `Inf` is related when the values of all untimed variables in `Orig` are exactly equal to those in `Inf`, and also symbolic representations of time data in `Inf` “correctly” represent time data.

For example, we need this technique for the DHCP-F protocol. However, we do not need it for resource-sharing example described in Section 2.
in Orig. The notable difference between Orig and Inf raises from two points. The first point is the difference of when timeouts can be executed in Orig and Inf. In Inf, the local clocks are abstracted away, and instead, the scheduler conservatively disables timeouts that would violate the timeout order inferred by symbolic timeout times. We need to prove (and have actually proved in [13]) that when a timeout action is enabled in Orig, the same timeout action is also enabled in Inf. This fact comes from the observation that when the scheduler disables a timeout in Inf, the corresponding timeout in Orig has not yet reached. The second point is from the over-approximation of \( \text{clocks} + u + 2e \) using \( \text{max}(\text{picked}_{t,s,2d}, s) + u + e \) in Inf. We take into account this point in the simulation relation by relating a state \( s \) of Orig and a state \( r \) of Inf when the symbolic value of a timeout variable in \( r \) (when converted into the numeric value using the information in \( s \)) is not exactly equal, but is less than or equal to the value of the corresponding timeout variable in \( s \). We have succeeded to prove a simulation relation with this condition (which is weaker than exact matching of symbolic and numerical values). 

\[ \text{Simulation relation from Inf to Fin:} \text{ A proof for a simulation relation from Inf to Fin is relatively easy compared to that from Orig to Inf. The key idea is to add a table of corresponding time stamps (and time nonces) and reused/compressed version of them in Fin, so that Fin now emulates Inf in parallel to its usual execution. This table is just for proving correctness, and does not change the behavior of Fin. After this emulation is defined, we can easily relate Inf and Fin using the state values of Inf enumerated in Fin.} \]

6. CASE STUDIES

In this section, we briefly present two case studies of TO-abstraction. The first one is an implementation of the resource-sharing protocol described as Protocol 3. The second one is the DHCP Failover protocol. We used the SAL model-checker [2] developed by SRI for both case studied. For the presented case studies, we manually abstracted the timed model into the untimed model described in the SAL language. We tried to define every data structure in SAL as general as we can, so that the structure can be reused for other case studies. The actual code can be found in [12]. We are planning to develop an automatic conversion tool for TO-abstraction. The full model-checking using BDD was examined the effectiveness of bug-finding using TO-abstraction, we experimented with verification of DHCP-F with a slight change: We removed one condition for checking a sequence number to ensure that the received message is for the current round. After this mutation, we found a counterexample at depth 17 (the run time was 52851.28 seconds \( \sim 14.7 \) hours). This counterexample has a complex scenario, and we believe that it is very hard to find by human analysis. More details of this counterexample appear in [13].

7. CONCLUSION

We presented \textit{timeout order abstraction} (TO-abstraction), a technique to abstract a particular subset of loosely synchronized real-time distributed systems (LSRTDs’s) into a finite-state untimed model. By using this technique, the user can model-check the untimed abstraction to verify the original system, or find bugs in it. TO-abstraction is (as far as we know) the first automatic analysis tool for time-parametric verification of LSRTDs’s. The user can use only maximum operation for manipulating time data in the current version of TO-abstraction. Extension of allowable operations is future study.

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