Bernauer et al. Reply:

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Bernauer et al. Reply: The criticism of our results in the preceding Comment [1] is based on the “modern and complete calculations” of the Coulomb correction of Sick and Trautmann [2] going back to the work of Lewis published in 1956 [3]. This calculation is done in “second Born approximation,” i.e., two-photon exchange (TPE) without intermediate excited states. The integral describing the TPE has been evaluated numerically [4] and it is apparently this code on which Arrington’s Coulomb corrections are based.

In order to quantify the influence of TPE on our results we have chosen the modern analytical integration by Borisyuk and Kobushkin (Ref. [6] of the Comment) lending itself to an easy calculation. Figure 1 shows Fig. 1 of the Comment overlayed with these calculations for the same $Q^2$. It demonstrates the variance of TPE calculations also indicated by a remark in the caption of this figure in the Comment. All calculations go to the same curve in the limit $Q^2 \to 0$ given by Eq. (1) of the Comment.

The uncertainty is also demonstrated in Fig. 2 of Ref. [5] showing five theoretical calculations of TPE, which are mutually inconsistent, with all but one disagreeing with the null experimental TPE effect at $Q^2 = 2.5$ (GeV/c)$^2$.

Though that work concerns polarization variables and not a Rosenbluth separation, the diagrams of TPE are based on QED and have to be valid for both.

All this is not surprising since the unconstrained part of the TPE amplitude resulting from the off-shell internal structure of the nucleon does cause considerable variance at present among the different TPE calculations. Such calculations require knowledge beyond on-shell form factors, and imply as well dispersion effects resulting from the excitation spectrum of the nucleon.

Nevertheless, we have applied the calculation of Borisyuk and Kobushkin to our data and refitted. Figure 2 shows the ratio of the electric over the magnetic form factors $\mu_p G_E/G_M$ with the spline ansatz. We also determined the radii with all models for $G_E$ and $G_M$ as in our Letter. The averaged result changes $\langle r^2 \rangle^{1/2}$ without → with TPE correction:

\[
\langle r_E^2 \rangle^{1/2} = 0.879(8) \to 0.876(8) \text{ fm},
\]

\[
\langle r_M^2 \rangle^{1/2} = 0.777(17) \to 0.803(17) \text{ fm}.
\]

We wrote in our Letter: These radii have to be taken with the applied corrections in mind. While the effect of the Coulomb correction used is compatible with other studies (see references in our Letter) a more sophisticated theoretical calculation may affect the results slightly.

Finally, the statements about our systematic errors are wrong. The statistical contributions to the point-to-point systematic errors are shown to be Gaussian and are therefore taken together with the statistical counting error (innermost error band in our Letter). The systematic uncertainties due to the angular dependences are linearly added to this statistical error and shown by the second band. For the

outermost band the Coulomb correction has been varied by ±50%. For details see Ref. [8] of the Comment.

In summary, the criticism of the Comment neglects the uncertainty of the TPE corrections and exaggerates the quantitative effect at small $Q^2$. We hold that we are well advised to apply only the Coulomb correction of the unique limit at $Q^2 = 0$. In the detailed follow-up paper we intend to present the experimental effect of TPE in a way making a comparison to theoretical calculations possible without reanalysis of the data.

We are indebted to Marc Vanderhaeghen for advising us on TPE corrections.

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