Measurement of the top-quark mass in the lepton+jets channel using a matrix element technique with the CDF II detector

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MEASUREMENT OF THE TOP-QUARK MASS

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32 University of Michigan, Ann Arbor, Michigan 48109, USA
33 Michigan State University, East Lansing, Michigan 48824, USA
34 Institution for Theoretical and Experimental Physics, ITEP, Moscow 117259, Russia
35 University of New Mexico, Albuquerque, New Mexico 87131, USA
36 Northwestern University, Evanston, Illinois 60208, USA
37 The Ohio State University, Columbus, Ohio 43210, USA
38 Okayama University, Okayama 700-8530, Japan
39 Osaka City University, Osaka 588, Japan
40 University of Oxford, Oxford OX1 3RH, United Kingdom
41a Istituto Nazionale di Fisica Nucleare, Sezione di Padova-Trento, I-35131 Padova, Italy
41b University of Padova, I-35131 Padova, Italy
42 LPNHE, Université Pierre et Marie Curie/IN2P3-CNRS, UMR7585, Paris, F-75252 France
43 University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
44a Istituto Nazionale di Fisica Nucleare Pisa, I-56127 Pisa, Italy
44b University of Pisa, I-56127 Pisa, Italy
44c University of Siena, I-56127 Pisa, Italy
44d Scuola Normale Superiore, I-56127 Pisa, Italy
45 University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA
46 Purdue University, West Lafayette, Indiana 47907, USA
47 University of Rochester, Rochester, New York 14627, USA
48 The Rockefeller University, New York, New York 10065, USA
49a Istituto Nazionale di Fisica Nucleare, Sezione di Roma 1, I-00185 Roma, Italy
49b Sapienza Università di Roma, I-00185 Roma, Italy
50 Rutgers University, Piscataway, New Jersey 08855, USA
51 Texas A&M University, College Station, Texas 77843, USA
52a Istituto Nazionale di Fisica Nucleare Trieste/Udine, I-34100 Trieste, Italy
52b University of Udine, I-33100 Udine, Italy
53 University of Tsukuba, Tsukuba, Ibaraki 305, Japan
54 Tufts University, Medford, Massachusetts 02155, USA
55 University of Virginia, Charlottesville, Virginia 22906, USA
56 Waseda University, Tokyo 169, Japan

a Deceased.
b Visitor from Istituto Nazionale di Fisica Nucleare, Sezione di Cagliari, 09042 Monserrato (Cagliari), Italy.
c Visitor from University of California Irvine, Irvine, CA 92697, USA.
d Visitor from University of California Santa Barbara, Santa Barbara, CA 93106, USA.
e Visitor from University of California Santa Cruz, Santa Cruz, CA 95064, USA.
f Visitor from CERN, CH-1211 Geneva, Switzerland.
g Visitor from Cornell University, Ithaca, NY 14853, USA.
h Visitor from University of Cyprus, Nicosia CY-1678, Cyprus.
i Visitor from Office of Science, U.S. Department of Energy, Washington, DC 20585, USA.
j Visitor from University College Dublin, Dublin 4, Ireland.
k Visitor from University of Fukui, Fukui City, Fukui Prefecture, Japan 910-0017.
l Visitor from Universidad Iberoamericana, Mexico D.F., Mexico.
m Visitor from Iowa State University, Ames, IA 50011, USA.
n Visitor from University of Iowa, Iowa City, IA 52242, USA.
o Visitor from Kinki University, Higashi-Osaka City, Japan 577-8502.
p Visitor from Kansas State University, Manhattan, KS 66506, USA.
q Visitor from University of Manchester, Manchester M13 9PL, United Kingdom.
r Visitor from Queen Mary, University of London, London, E1 4NS, United Kingdom.
s Visitor from University of Melbourne, Victoria 3010, Australia.
t Visitor from Muons, Inc., Batavia, IL 60510, USA.
u Visitor from Nagasaki Institute of Applied Science, Nagasaki, Japan.
v Visitor from National Research Nuclear University, Moscow, Russia.
w Visitor from University of Notre Dame, Notre Dame, IN 46556, USA.
x Visitor from Universidade de Oviedo, E-33007 Oviedo, Spain.
y Visitor from Texas Tech University, Lubbock, TX 79069, USA.
z Visitor from Universidad Técnica Federico Santa María, 110v Valparaíso, Chile.
aa Visitor from Yarmouk University, Irbid 211-63, Jordan.
bb On leave from J. Stefan Institute, Ljubljana, Slovenia.
A measurement of the top-quark mass is presented using Tevatron data from proton-antiproton collisions at center-of-mass energy $\sqrt{s} = 1.96$ TeV collected with the CDF II detector. Events are selected from a sample of candidates for production of $t\bar{t}$ pairs that decay into the lepton + jets channel. The top-quark mass is measured with an unbinned maximum likelihood method where the event probability density functions are calculated using signal and background matrix elements, as well as a set of parametrized jet-to-parton transfer functions. The likelihood function is maximized with respect to the top-quark mass, the signal fraction in the sample, and a correction to the jet energy scale (JES) calibration of the calorimeter jets. The simultaneous measurement of the JES correction ($\Delta_{\text{JES}}$) amounts to an additional in situ jet energy calibration based on the known mass of the hadronically decaying $W$ boson. Using the data sample of 578 lepton + jets candidate events, corresponding to 3.2 fb$^{-1}$ of integrated luminosity, the top-quark mass is measured to be $m_t = 172.4 \pm 1.4(\text{stat} + \Delta_{\text{JES}}) \pm 1.3(\text{syst})$ GeV$/c^2$.

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The top-quark mass, $m_t$, is an intrinsic parameter of the standard model (SM) of particle physics and is of particular importance due to its strikingly large value. As a result, the top quark has a large effect on radiative corrections to electroweak processes and has a Yukawa coupling to the Higgs boson mass, which may provide insight into the standard model (SM) of particle physics and is of particular importance.

The Higgs boson mass, $m_H$, is not predicted by the SM, but constraints on its value can be derived from the calculation of radiative corrections to the $W$ boson mass, $m_W$, and from the values of other precision electroweak variables [2]. These corrections depend primarily on $\ln m_H$ and $m_t^2$, and thus precision measurements of $m_W$ and $m_t$ provide important constraints on $m_H$.

The dominant top-quark production process is pair production via the strong interaction. At Fermilab’s Tevatron, this process is initiated by $p\bar{p}$ collisions at center-of-mass energy $\sqrt{s} = 1.96$ TeV. Because of its large mass, the top quark decays rapidly with lifetime $\tau_t \sim 10^{-25}$ s [3]—fast enough that it has essentially no time to interact and may be considered as a free quark. This allows a direct measurement of its mass from the daughter particles from its decay, and as a result $m_t$ has the lowest relative uncertainty of all of the quark masses [4].

In the SM top quark decays via the weak interaction, predominantly to $W$ bosons and $b$ quarks as $t \bar{t} \rightarrow W^+ b W^- \bar{b}$. W bosons decay into lower-mass fermion-antifermion pairs: a charged lepton and a neutrino ($W^+ \rightarrow \ell^+ \bar{\nu}_\ell$ or $W^- \rightarrow \ell^- \nu_\ell$), “leptonic decay”; or an up-type quark and a down-type quark ($W^+ \rightarrow q\bar{q}'$ or $W^- \rightarrow \bar{q}q'$), “hadronic decay.” The result presented here uses the lepton + jets decay channel (with $q\bar{q}' b\ell \bar{\nu}_\ell b$ or $\ell \nu_\ell b q\bar{q}' b$ in the final state), where one of the two $W$ bosons decays leptonically into an electron or a muon, and the other decays hadronically. All the quarks in the final state evolve into jets of hadrons. Events with tau leptons are not selected directly, but may contribute a few percent of the total sample via leptonic cascade decays or fake jets. The most recent $m_t$ measurements obtained at the Tevatron using the lepton + jets topology are reported in Ref. [5], while the results of an earlier version of the present analysis using 955 pb$^{-1}$ of integrated luminosity are reported in Ref. [6]. The distinctive feature of this analysis is the use of matrix element calculations to describe the dominant background contribution. The result presented here uses a more than 3 times larger data sample than the earlier version, and employs a more detailed likelihood function.

The leptons and jets resulting from the top-antitop quark pair $(t\bar{t})$ decay are detected in the CDF II general-purpose particle detector that is described in detail elsewhere [7]. Azimuthally and forward-backward symmetric about the beam line, the detector contains a high precision particle tracking system immersed in a 1.4 T magnetic field and surrounded by calorimetry, with muon detectors on the outside. A right-handed spherical coordinate system is employed, with the polar angle $\theta$ measured from the proton beam direction, the azimuthal angle $\phi$ in the plane perpendicular to the beam line, and the distance $r$ from the center of the detector. Transverse energy and momentum are defined as $E_T = E \sin \theta$ and $p_T = p \sin \theta$, where $E$ and $p$ denote energy and momentum. Pseudorapidity is defined as $\eta = -\ln \tan(\theta/2)$.

This measurement makes use of CDF II data collected between February 2002 and August 2008, representing approximately 3.2 fb$^{-1}$ of integrated luminosity. The event selection criteria (Table I) are tuned to select the lepton + jets final-state particles, requiring that each event must have exactly one high-$E_T$ electron or high-$p_T$ muon, exactly four high-$E_T$ jets, and a significant amount of missing $E_T$, $\not{E}_T$ [8], characteristic of the undetected neutrino. Jets are reconstructed using a cone algorithm [9], with the cone radius $\Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = 0.4$. At least one of the four jets must be identified as originating from a $b$ quark via the SECVTX algorithm [10], which detects displaced
MEASUREMENT OF THE TOP-QUARK MASS

**TABLE I.** Event selection criteria.

<table>
<thead>
<tr>
<th>Event Type</th>
<th>Selection Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>$E_T &gt; 20$ GeV, $</td>
</tr>
<tr>
<td>or Muon</td>
<td>$p_T &gt; 20$ GeV/c, $</td>
</tr>
<tr>
<td>Jets</td>
<td>$E_T &gt; 20$ GeV, $</td>
</tr>
</tbody>
</table>

Four jets: at least one from a $b$ quark event

Background events are assumed to be described by a matrix element for $W +$ jets production, which is calculated using a sum of 1286 $W + 4$-partons amplitudes for 592 subprocesses encoded in the VECBOS MC event generator [19]. This approximation does mean that there are some events that, in principle, are not described by either $P_s$ or $P_b$, including non-$W$, single top, diboson, $Z +$ jets, and $W + b\bar{b}$ + 2-partons events, as well as $W +$ jets events from $W + 0$, 1-, 2-, and 3-partons processes. However, studies with MC simulated events show that the ratio $P_b/P_s$ calculated for all of these event types is similar to that for $W + 4$-partons events, and that, in practice, such events mostly contribute to the likelihood function via the $P_t$ term and do not add any more bias than the $W + 4$-partons events or than the poorly reconstructed $t\bar{t}$ events themselves [20]. Any residual bias in the measured top-quark mass is removed at the end, as described later in the paper.

The signal and background p.d.f.s, $P_s$ and $P_b$, are constructed in analogous fashions, starting with the appropriately normalized parton-level differential cross section [4], $d\sigma_s$ or $d\sigma_b$, which is then convolved with parton distribution functions (PDFs) and a jet-to-parton transfer function $W(k, x)$. $P_s$ is thus given by

$$P_s(k; m_t, \Delta_{\text{JES}}) = \frac{1}{n_{jp}} \sum_{\text{jet perm}} \frac{1}{A_s(m_t, \Delta_{\text{JES}})} \times \int d\sigma_s(x; m_t, x_{Bj}) dx_{Bj}^1 dx_{Bj}^2 W(k, x; \Delta_{\text{JES}}) f(x_{Bj}^1)(x_{Bj}^2).$$

where $x = (e_i, \hat{p}_i)$ represents the actual event parton-level kinematic quantities corresponding to the measured quantities $k$, and parameter $\Delta_{\text{JES}}$ is defined in a later paragraph. The PDFs $f(x_{Bj})$ define the probability density for a colliding parton to carry a longitudinal momentum fraction $x_{Bj}$ and are given by CTEQ5L [21]. $A_s$ is the mean acceptance function for signal events, a normalization term that is the consequence of the constriction of the phase-space of the integral by the event selection cuts and by the detector acceptance. The average over the jet permutations, $n_{jp}$, is

**TABLE II.** Number of expected signal and background events, corresponding to the total integrated luminosity of 3.2 fb$^{-1}$. The percentages are used when generating Monte Carlo simulated experiments.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Number of events</th>
<th>Percentage of total</th>
<th>Percentage of background</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t\bar{t}$ signal</td>
<td>425.0 ± 58.9</td>
<td>76.0%</td>
<td>⋮</td>
</tr>
<tr>
<td>$W +$ jets</td>
<td>92.6 ± 15.9</td>
<td>16.6%</td>
<td>69.0%</td>
</tr>
<tr>
<td>Non-$W$</td>
<td>25.0 ± 12.5</td>
<td>4.5%</td>
<td>18.7%</td>
</tr>
<tr>
<td>Single-top quark</td>
<td>6.6 ± 0.4</td>
<td>1.2%</td>
<td>4.9%</td>
</tr>
<tr>
<td>Diboson</td>
<td>6.0 ± 0.6</td>
<td>1.1%</td>
<td>4.5%</td>
</tr>
<tr>
<td>$Z +$ jets</td>
<td>3.9 ± 0.5</td>
<td>0.7%</td>
<td>2.9%</td>
</tr>
<tr>
<td>Total</td>
<td>559.2 ± 67.0</td>
<td>100%</td>
<td>⋮</td>
</tr>
<tr>
<td>Observed</td>
<td>578</td>
<td>⋮</td>
<td>⋮</td>
</tr>
</tbody>
</table>
due to ambiguity in assigning final-state jets to partons. The fact that the two light quarks in the final state are indistinguishable allows the reduction from the original 24 permutations to 12 in the expression for $P_{r}$, and the $b$-tagging information allows a further reduction to 6 assignments for events with one identified $b$ jet and 2 for events with both $b$ jets identified. In the similar expression for $P_{b}$, all 24 permutations are averaged.

The jet-to-parton transfer function $W(k, x)$ is a p.d.f. describing the probability density for an event with outgoing partons and charged lepton with $x$ to be measured as reconstructed $k$. The charged lepton is assumed to be well measured, allowing the use of a Dirac $\delta$ function to represent the mapping between its parton-level momentum, $\vec{p}_{t}$, and its reconstructed momentum, $\vec{p}_{t}$. For the four jets, the function is obtained by parametrizing the jet-to-parton mapping observed in fully simulated PYTHIA $t\bar{t}$ events. These events contain all of the information about the original partons as well as the measured jets. The simulation includes physical effects, such as radiation and hadronization, as well as the effects of measurement resolution and of the jet reconstruction algorithm. The parametrization is made in two parts that are assumed to be independent: the energy transfer function $W_{E}$, describing the jet energies $E$, and the angular transfer function $W_{A}$, describing the mapping for the jet angles. The jet-to-parton transfer function is thus given by

$$W(k, x; \Delta_{\text{JES}}) = \delta^{4}(\vec{p}_{t} - \vec{p}_{t})W_{A}\prod_{i=1}^{4}\left(\frac{1}{E_{i}p_{i}}W_{E}(E_{i}, \varepsilon_{i}; \Delta_{\text{JES}})\right). \quad (3)$$

The reconstructed jet energies, $E_{i}$, used in the function $W_{E}$ are not just the raw calorimeter energy deposits, but are first calibrated so that they represent the combined energies released in the calorimeter by the many particles constituting each jet. This is achieved using the CDF jet energy scale (JES) calibration [22], which is subject to a significant systematic uncertainty. The uncertainties of individual jet energy measurements, $\sigma(E_{i})$, are therefore correlated, and their fractional JES uncertainty, $\sigma(E_{i})/E_{i}$, is typically $\sim 3\%$. If this were included as a systematic uncertainty on the measured $m_{W}$, it would reduce the measurement precision drastically; in fact, each 1% of fractional JES uncertainty would add about 1 GeV/$c^{2}$ uncertainty to the measured $m_{W}$ [23]. However, such a treatment overestimates the uncertainty because the energies of the two daughter jets of the hadronically decaying $W$ boson can be constrained based on the known $W$ boson mass. Applying this constraint to all events in the data sample while allowing the jet energies to be shifted results in the in situ measurement of the JES correction, $\Delta_{\text{JES}}$, defined as the number of $\sigma(E_{i})$ values by which the energy of each jet is shifted in the likelihood fit. This effectively recalibrates the measured jet energies based on the known $W$ boson mass and replaces a large component of the JES systematic uncertainty with a much smaller statistical uncertainty on the $\Delta_{\text{JES}}$. The $\Delta_{\text{JES}}$ dependence of the jet energies is included in the parametrization of the function $W_{E}$. This parametrization is made in eight bins in pseudorapidity [$\eta$], separately for light and $b$ jets, using a sum of two Gaussians as a function of the difference between the parton energies and the corrected jet energies as measured in a sample of PYTHIA $t\bar{t}$ events that pass the same selection criteria as the data.

In an earlier version of this analysis [6], the jet-to-parton transfer functions for all jet angles were approximated by Dirac $\delta$ functions. The introduction of the function $W_{A}$ was motivated by a discrepancy noticed in simulated $t\bar{t}$ events in the 2-jet effective invariant mass of the hadronically decaying $W$ boson, $m_{W}$. Even when the true simulated parton-level jet energies are used, instead of the corresponding reconstructed detector-level values, the use of the measured jet angles rather than their parton-level values causes a significant shift of the reconstructed $m_{W}$ from its nominal value, as illustrated in Fig. 1.

There is also a negative skewness in the distribution for measured angles, and since parton-level jet energies are used, the observed effects are due to the differences between the measured angles and the parton-level angles alone. The peak of the $m_{W}$ distribution, when fit by a Breit-Wigner distribution, corresponds to a $W$ boson pole mass of 79.5 GeV/$c^{2}$, a $-0.9$ GeV/$c^{2}$ shift from its parton-level value of 80.4 GeV/$c^{2}$. This is found to be a result of a

![FIG. 1. The reconstructed 2-jet invariant mass of the hadronically decaying $W$ boson, $m_{W}$, for measured jet angles (solid line) and for parton-level angles (dotted line), obtained after assuming the primary parton energy as jet energy. For ease of comparison, the parton-level distribution is normalized so that the maxima of the two distributions are the same.](071105-6)
correlation between the measured jet directions: the measured angle, $\alpha_{12}$, between the two jets is, on average, reduced so that the two jets appear closer together than their parent partons, which can be seen in Fig. 2. Since the apparent $W$ boson mass is utilized to measure $\Delta_{\text{JES}}$ and thus calibrate the measured jet energies, a jet-to-parton transfer function describing the change in the angle $\alpha_{12}$ is important in making an accurate measurement of $\Delta_{\text{JES}}$ and thus the top-quark mass. The function $W_A$ also describes a much smaller correlation effect seen in the angle $\alpha_{WB}$ between the hadronic-side $b$ jet and the hadronically decaying $W$ boson. The function $W_A$ is thus parametrized using two different functions, $W_{12}^A$ and $W_{WB}^A$, describing the mappings for the angles $\alpha_{12}$ and $\alpha_{WB}$. The remaining angles describe resolution effects rather than the correlations and, due to computational constraints, are assumed to be well measured with their contributions to $W_A$ approximated by Dirac delta functions.

The functions $W_{12}^A$ and $W_{WB}^A$ are both fit using a sum of a skew-Cauchy distribution and two Gaussians, describing the change in the cosine of the relevant angle, $\Delta \cos(\alpha_{12})$ and $\Delta \cos(\alpha_{WB})$, from partons to measured jets. Since the correlation effects are stronger in jets that are closer together, the functions are parametrized in bins of $\cos(\alpha_{12})$ and $\cos(\alpha_{WB})$, respectively; one example for each function is shown in Fig. 2.

The $m_W$ distribution after convolution with the function $W_A$ is shown in Fig. 3. The skewness is removed and the mean value agrees well with the parton-level distribution.

The 20 integration variables (3 for each final-state particle and the $x_{Bj}$ for each initial state parton, assuming zero transverse momentum for the $t\bar{t}$ pair) in the expression for the signal and background p.d.f.s [Eq. (2)] are reduced to 16 by integrating over the 4-momentum conservation Dirac $\delta$ function inherent in the expression for $d\hat{\sigma}$. The charged lepton 3-momentum integration and all but two of the jet angular integrations are made trivial by the Dirac $\delta$ functions in the function $W(k, \chi)$, leaving 7 integration variables. In $P_x$, this is further reduced to 5 variables via a change of variables to the squared masses of the top quarks and by using the narrow-width approximation for the Breit-Wigner distributions of both top-quark decays in the $t\bar{t}$ matrix element. The integral is then evaluated using the VEGAS [24] adaptive Monte Carlo integration algorithm.

![Figure 2](image1.png)

**FIG. 2.** Examples of parametrization of the functions $W_{12}^A$ and $W_{WB}^A$ in the bins where $0.2 < \cos(\alpha_{12}) < 0.4$ and $0.2 < \cos(\alpha_{WB}) < 0.4$. The histograms show MC simulation events and the curves represent the parametrization.

![Figure 3](image2.png)

**FIG. 3.** The $m_W$ distribution for measured angles from Fig. 1 is plotted (solid line) after convolution with the function $W_A$. For ease of comparison, the parton-level distribution (dotted line) is normalized so that the maxima of the two distributions are the same.
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[6], which uses importance sampling, which means that the sample points are concentrated in the regions that make the largest contribution to the integral.

The treatment of $P_b$ is unchanged since the previous version of this analysis [6], except for the updated energy transfer function $W_E$. The integrand in the expression for $P_b$ is much more computationally intensive than for $P_s$ and a simplified Monte Carlo method of integration is employed, giving reasonable convergence with an execution time comparable to that of $P_s$. The simplifications used in this computation of $P_b$ include setting the function $W_A$ to a Dirac $\delta$ function for all angles, using a narrow-width approximation for the $W$ boson decay, and neglecting the $\Delta_{\text{JES}}$ dependence of the function $W_E$. Therefore, the value of $P_b$ for each event does not depend on the likelihood parameters $m_t$ and $\Delta_{\text{JES}}$, while $P_s$ is a two-dimensional function of those parameters [6]. In this approximation, the product of the background p.d.f. normalization terms [corresponding to the variables $\sigma_b \cdot A_b$ in Eq. (2)] is set to a constant, whose value is chosen to optimize the statistical sensitivity of the method, effectively providing an appropriate relative normalization with respect to $P_s$.

The log-likelihood function is given as a sum over the 578 events in the sample:

$$\ln L(k; m_t, \Delta_{\text{JES}}, \nu_{\text{sig}}) = \sum_{i=1}^{578} \ln[n_{\text{sig}}P_s(k_i; m_t, \Delta_{\text{JES}}) + (1 - \nu_{\text{sig}})P_b(k_i)].$$  \hspace{1cm} (4)

It is calculated on a two-dimensional $31 \times 17$ grid in $m_t$ and $\Delta_{\text{JES}}$, spanning $145 \leq m_t \leq 205 \text{ GeV}/c^2$ and $-4.8 \leq \Delta_{\text{JES}} \leq 4.8$, with a spacing between grid points of 2 GeV/$c^2$ in $m_t$ and 0.6 in $\Delta_{\text{JES}}$. To optimize computational time, the bin size is chosen to be as large as possible without appreciably affecting the fit result. The third likelihood parameter, the signal fraction parameter $\nu_{\text{sig}}$, is allowed to vary continuously (within the constraint $0 \leq \nu_{\text{sig}} \leq 1$), and the likelihood function is maximized with respect to $\nu_{\text{sig}}$ at each point on the grid using the MINUIT program [25]. The resulting surface described on the grid is the profile log-likelihood, maximized for $\nu_{\text{sig}}$. The top-quark mass, $m_t$, and the jet energy scale correction, $\Delta_{\text{JES}}$, are measured by making a two-dimensional parabolic fit to the surface, consistent with the expectation for the likelihood function to be Gaussian near its maximum. The maximum of the parabola gives the measured $m_t$ and $\Delta_{\text{JES}}$, while the measured $\nu_{\text{sig}}$ is taken from its value at the grid point of maximum likelihood. The estimated one-$\sigma$ statistical uncertainty of the measurement is represented by the ellipse corresponding to a change in log-likelihood $\Delta \ln L = 0.5$ from the maximum of the fitted parabola. The values of $m_t$ and $\Delta_{\text{JES}}$ are anticorrelated (Fig. 4). No correlation is observed between $\nu_{\text{sig}}$ and $m_t$ or $\Delta_{\text{JES}}$.

The accuracy of the measured $m_t$ and $\Delta_{\text{JES}}$, and their uncertainties, are checked using ensembles of MC simulated experiments, using the MC samples previously mentioned with the addition of 22 $t\bar{t}$ samples generated with values of $m_t$ between 161 and 185 GeV/$c^2$. The numbers of $t\bar{t}$ events and those of the various backgrounds are Poisson fluctuated around the values shown in Table II. Studies of the relationships between the known input simulation parameters and their corresponding measurements show no evidence of bias when a clean sample of MC simulated $t\bar{t}$ events is used, containing only lepton + jets events with correct jet-parton matching. However, the presence of signal events with jets which are poorly or incorrectly matched to partons and events which do not match the decay hypothesis biases the likelihood fit result and increases the pull width. The presence of background events also biases the fit, due to the backgrounds that are not well described by $P_b$ and the approximations in $P_b$. The bias is removed using a set of functions obtained from a fit to the MC simulation and parametrized in terms of the measured $\Delta_{\text{JES}}$ and $\nu_{\text{sig}}$ [20]. This amounts to adding 1.1 GeV/$c^2$ to the $m_t$ value produced by the likelihood fit and multiplying the uncertainty by 1.26 so that the pull width is consistent with unity. The systematic uncertainty due to this measurement calibration is small, as shown in Table III.

Despite the reduction from the in situ $\Delta_{\text{JES}}$ calibration, the remaining uncertainty from JES obtained by varying the parameters in JES [22] is among the largest systematic uncertainties of the measurement (Table III). Other significant systematic uncertainties are mainly a result of assumptions made in the simulation of the events that are used in the tuning and calibration of the measurement method. In most cases, they are evaluated by varying different aspects of the MC simulation, such as signal MC generator (PYTHIA versus HERWIG [26]), color

reconnection model tune (Apro versus ACRpro [27–29]), and parameters of initial and final-state radiation (ISR and FSR). A detailed description of the systematic effects has been published elsewhere [30]. The systematic uncertainties for each effect are added in quadrature, resulting in a total estimated systematic uncertainty of 1.3 GeV/c^2 (Table III).

The measurement is made using the data sample of 578 events, yielding

\[ m_t = 172.4 \pm 1.4 \text{(stat)} \pm 1.3 \text{(sys)} \text{ GeV/c}^2, \]

\[ m_t = 172.4 \pm 1.9 \text{(total)} \text{ GeV/c}^2, \]

with \( \Delta_{\text{JES}} = 0.3 \pm 0.3 \text{(stat)} \). The central value and the contour ellipses corresponding to the one-, two-, and three-\( \sigma \) statistical confidence intervals of the measurement are illustrated in Fig. 4. The overall statistical uncertainty on the measured top-quark mass is labeled “stat + \( \Delta_{\text{JES}} \)” because it includes the uncertainty on \( m_t \) due to the statistical uncertainty on the measured \( \Delta_{\text{JES}} \); i.e., the uncertainty is given by half of the full width of the one-\( \sigma \) contour of Fig. 4.

In conclusion, a precise measurement of the top-quark mass has been presented using CDF lepton + jets candidate events corresponding to an integrated luminosity of 3.2 fb\(^{-1}\). Using an improved matrix element method with an in situ jet energy calibration, the top-quark mass is measured to be \( m_t = 172.4 \pm 1.9 \text{ GeV/c}^2 \).

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Table III. Contributions to the total expected systematic uncertainty.

<table>
<thead>
<tr>
<th>Systematic</th>
<th>(GeV/c^2)</th>
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<td>Color reconnection</td>
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<td>b-jet energy</td>
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<td>Background</td>
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<td>ISR and FSR</td>
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<tr>
<td>Multiple hadron interactions</td>
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<td>PDFs</td>
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<td>Lepton energy</td>
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<td>Measurement calibration</td>
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<tr>
<td>Total</td>
<td>1.31</td>
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</tbody>
</table>

[8] A. Abulencia et al. (CDF Collaboration), Phys. Rev. D 74, 072006 (2006), we define \( \vec{E}_i = | \sum E_j \hat{n}_j | \), where \( i \) = calorimeter tower number with \( |\eta| < 3.6 \) and \( \hat{n}_i \) is a unit vector perpendicular to the beam axis and pointing at the \( i \)th calorimeter tower.