Directional Limits on Persistent Gravitational Waves Using LIGO S5 Science Data

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Directional Limits on Persistent Gravitational Waves Using LIGO S5 Science Data

Introduction.—One of the most ambitious goals of gravitational-wave (GW) astronomy is to measure the stochastic gravitational-wave background (SGWB), which can arise through a variety of mechanisms including amplification of vacuum fluctuations following inflation [1], phase transitions in the early Universe [2], cosmic strings [3] and pre-big bang models [4,5]. Astrophysical sources of the SGWB include the superposition of unresolved pointlike sources as well as stochastic backgrounds. We perform two directional searches for persistent GWs using data from the LIGO S5 science run: one optimized for pointlike sources and one for arbitrary extended sources. Finding no evidence to support the detection of GWs, we present 90% confidence level (C.L.) upper-limit maps of GW strain power with typical values between \( \frac{2}{C_0^2} \times 10^{-20} \) and \( \frac{5}{C_0^2} \times 10^{-35} \) strain^2 Hz^\(-1\) sr^\(-1\) for pointlike and extended sources, respectively. The latter result is the first of its kind. We also set 90% C.L. limits on the narrow-band root-mean-square GW strain from interesting targets including Sco X-1, SN 1987A and the Galactic center as low as \( \frac{25}{C_0^2} \times 10^{-25} \) in the most sensitive frequency range near 160 Hz.

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sources such as core-collapse supernovae [6], neutron-star
instabilities [7], binary mergers [8] and persistent emission
from neutron stars [9].

We present the results of two analyses using data from the
LIGO S5 science run: a radiometer analysis optimized for
pointlike sources and a spherical-harmonic decomposi-
tion (SHD) analysis, which allows for arbitrary angular
distributions. This work presents the first measurement of
the GW sky in a framework consistent with an arbitrary
extended source.

**Detectors.** — We analyze data from LIGO’s 4 and 2 km
detectors (H1 and H2) in Hanford, WA, and the 4 km
detector (L1) in Livingston Parish, LA during the S5 science
run (Nov. 5, 2005—Sep. 30, 2007). During S5, both H1 and L1 reached a strain sensitivity of $3 \times 10^{-23}$
strain Hz$^{-1/2}$ in the most sensitive region between
100–200 Hz [10] and collected 331 days of coincident
H1L1 and H2L1 data. S5 saw milestones including limits
on GWs from the Crab pulsar surpassing those inferred
from the Crab’s spin down [11], as well as limits on the
isotropic SGWB surpassing indirect limits from big bang
nucleosynthesis and the cosmic microwave background
[12]. This work builds on [12,13].

**Methodology.** — Following [13,14] we present a fram-
ework for analyzing the angular distribution of GWs. We
assume that the GW signal is stationary and unpolarized,
but not necessarily isotropic. It follows that the GW energy
density $\Omega_{GW}(f)$, can be expressed in terms of the GW
power spectrum, $P(f, \Omega)$:

$$\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df} = \frac{2\pi^2}{3H_0^2} f^3 \int d\Omega P(f, \Omega). \quad (1)$$

Here $f$ is frequency, $\Omega$ is sky location, $\rho_c$ is the critical
density of the Universe and $H_0$ is Hubble’s constant. We
further assume that $P(f, \Omega)$ can be factored (in our analy-
sis band) into an angular power spectrum, $\mathcal{P}(\Omega)$, and a
spectral shape, $H(f) = (f/f_0)^\beta$, parameterized by the
spectral index $\beta$ and reference frequency $f_0$. We set $f_0 =
100$ Hz to be in the sensitive range of the LIGO
interferometers.

We measure $P(\Omega)$ for two power-law signal models. In
the cosmological model, $\beta = -3$ ($\Omega_{GW}(f) = \text{const}$),
which is predicted, e.g., for the amplification of vacuum
fluctuations following inflation [15]. In the astrophysical
model, $\beta = 0$ ($H(f) = \text{const}$), which emphasizes the
strain sensitivity of the LIGO detectors.

We estimate $P(\Omega)$ two ways. The radiometer algorithm
[13,16,17] assumes the signal is a point source character-
ized by a single direction $\hat{\Omega}_0$ and amplitude, $\eta(\hat{\Omega}_0)$:

$$P(\hat{\Omega}) = \eta(\hat{\Omega}_0) \delta^2(\hat{\Omega}, \hat{\Omega}_0). \quad (2)$$

It is applicable to a GW sky dominated by a limited number
of widely separated point sources. However, as the number of
point sources is increased, however, the beam pattern will cause
the signals to interfere and partly cancel. Thus, radiometer
maps do not apply to extended sources. Since pointlike
signals are expected to arise from astrophysical sources, we use $\beta = 0$ for the radiometer analysis.

The spherical-harmonic decomposition algorithm is
used for both $\beta = -3$ and $\beta = 0$. It allows for an extended
source with an arbitrary angular distribution, characterized
by spherical-harmonic coefficients $P_{lm}$ such that

$$P(\hat{\Omega}) = \sum_{lm} P_{lm} Y_{lm}(\hat{\Omega}). \quad (3)$$

The series is cut off at $l_{\text{max}}$, allowing for angular scale
$\sim 2\pi/l_{\text{max}}$. The flexibility of the spherical-harmonic algo-
rithm comes at the price of somewhat diminished sensitiv-
ity to point sources, and thus the algorithms are
complementary.

We choose $l_{\text{max}}$ to minimize $\sigma(\hat{\Omega}) \tilde{A}$ where $\sigma(\hat{\Omega})$ is the
uncertainty associated with $P(\hat{\Omega})$ and $\tilde{A}$ is the typical
angular area of a resolved patch of sky [18]. I.e., we max-
imize the sensitivity obtained by integrating over the typi-
cal search aperture. We thereby obtain $l_{\text{max}} = 7$ and 12 for
$\beta = -3$ and $\beta = 0$, respectively.

Both algorithms can be framed in terms of a “dirty map” $X_{\nu}$, which represents the signal convolved with the Fisher matrix, $\Gamma_{\mu\nu}$ [14]:

$$X_{\nu} = \sum_f \gamma_{\nu}(f, t) \left( \frac{\tilde{H}(f)}{P_1(f,t)P_2(f,t)} \right) C(f,t) \quad (4)$$

$$\Gamma_{\mu\nu} = \sum_f \gamma_{\mu}(f, t) \left( \frac{\tilde{H}^2(f)}{P_1(f,t)P_2(f,t)} \right) \gamma_{\nu}(f,t). \quad (5)$$

Here both Greek indices $\mu$ and $\nu$ take on values of $lm$ for
the SHD algorithm and $\hat{\Omega}$ for the radiometer algorithm.
$C(f, t)$ is the cross spectral density generated for each
interferometer pair. $P_1(f, t)$ and $P_2(f, t)$ are the individual
power spectral densities, and $\gamma_{\mu}(f, t)$ is the angular
decomposition of the overlap reduction function $\gamma(\hat{\Omega}, f, t)$, which characterizes the orientations and frequency re-
sponse of the detectors [14]:

$$\gamma_{\mu}(f, t) = \int_{S^2} d\Omega \gamma(\hat{\Omega}, f, t)e_{\mu}(\hat{\Omega}) \quad (6)$$

$$\gamma(\hat{\Omega}, f, t) = \frac{1}{2} F_1^A(\hat{\Omega}, t) F_2^A(\hat{\Omega}, t) e^{i2\pi f \tilde{\xi} \hat{\Omega} / c \cdot \Delta \tilde{\xi}/c}. \quad (7)$$

$F_1^A(\hat{\Omega}, t)$ characterizes the detector response of detector $I$
to a GW with polarization $A$, $e_{\mu}(\hat{\Omega})$ is a basis function, $c$
is the speed of light and $\Delta \tilde{\xi}_{12} \equiv \tilde{\xi}_1 - \tilde{\xi}_2$ is the difference
vector between the interferometer locations [14].

In [14] it was shown that the maximum-likelihood esti-
mators of GW power are given by $P = \Gamma^{-1} X$. The inver-
sion of $\Gamma$ is complicated by singular eigenvalues associated
with modes to which the Hanford-Livingston (HL) detector
network is insensitive. This singularity can be handled two ways. The radiometer algorithm assumes the signal is pointlike, implying that correlations between neighboring pixels can be ignored. Consequently, we can replace $\Gamma^{-1}$ with $(\hat{\Gamma}_\Omega^{-1})$ to estimate the point source amplitude $\eta(\Omega)$.

The SHD algorithm targets extended sources, so the full Fisher matrix must be taken into account. We regularize $\Gamma$ by removing a fraction $F$ of the modes associated with the smallest eigenvalues, to which the HL network is relatively insensitive. By removing some modes from the Fisher matrix, we obtain a regularized inverse Fisher matrix, $\Gamma^{-1}$, thereby introducing a bias discussed below.

We thereby obtain estimators

$$\hat{\eta}_\Omega = (\Gamma^{-1}_\Omega)^{-1}X_\Omega$$

$$\hat{P}_{lm} = \sum_{l'm'}(\Gamma^{-1}_R)_{l'm'l'm'}X_{l'm'}$$

with uncertainties

$$\sigma^{\text{rad}}_\Omega = (\Gamma^{-1}_\Omega)^{-1/2}$$

$$\sigma^{\text{sph}}_{lm} = |(\Gamma^{-1}_R)_{lm,lm}|^{1/2}.$$ (11)

We refer to $\hat{P}_{\Omega} = \sum_{lm} \hat{P}_{lm} Y_{lm}(\Omega)$ as the “clean map” and $\hat{\eta}_\Omega$ as the “radiometer map.” $\hat{P}_{\Omega}$ has units of strain$^2$ Hz$^{-1}$ sr$^{-1}$ whereas $\hat{\eta}_\Omega$ has units of strain$^2$ Hz$^{-1}$.

In choosing $F$, we balance the sensitivity to the kept modes with the bias associated with the removed modes. In practice, we do not know the bias associated with $F$, which depends on the unknown signal distribution $\mathcal{P}(\hat{\Omega})$. Therefore, we choose $F$ to produce reliably reconstructed maps with minimal bias for simulated signals. Following [14], we use $F = 1/3$, which was shown to be a robust choice for simulated signals including maps characterized by one or more point sources, dipoles, monopoles and an extended source clustered in the galactic plane.

The likelihood function for $\mathcal{P}(\hat{\Omega})$ at each point in the sky can be described as a normal distribution with mean $\hat{P}_{\Omega}$ and width $\sigma^{\text{sph}}_{lm}$. Regularization introduces a signal-dependent bias. Without knowing the true distribution of $\mathcal{P}(\hat{\Omega})$, it is impossible to know the bias exactly, but it is possible to set a conservative upper limit by assuming that on average the removed modes contain no more GW power than the kept modes. Thus, we calculate $\hat{P}_{lm}$ setting eigenvalues of removed modes to zero, whereas $\sigma^{\text{sph}}_{lm}$ is conservatively calculated setting eigenvalues of removed modes to the average eigenvalue of the kept modes. These upper limits are $\approx 25\%$ greater than they would be if we used the same regularization scheme for $\sigma^{\text{sph}}_{lm}$ and $\hat{P}_{lm}$.

In the case of the SHD algorithm, we also calculate [14],

$$\hat{C}_l = \frac{1}{2l+1} \sum_{lm} |\hat{P}_{lm}|^2 - (\Gamma^{-1}_R)_{lm,lm}.$$ (12)

which describe the angular scale of the clean map. The subtracted second term makes the estimator unbiased so that $\langle \hat{C}_l \rangle = 0$ when no signal is present. The noise distribution of $\hat{C}_l$ is highly non-Gaussian for small values of $l$, and so the upper limits presented below are calculated numerically. The $\hat{C}_l$ are analogous to similar quantities defined in the context of temperature fluctuations of the cosmic microwave background [19].

The analysis was performed blindly using the S5 stochastic analysis pipeline. This pipeline has been tested with hardware and software injections, and the successful recovery of isotropic hardware injections is documented in [12]. The recovery of anisotropic software injections is demonstrated in [14]. We parse time series into 60 s, Hann-windowed, 50%-overlapping segments, coarse-grained to achieve 0.25 Hz resolution. We apply a stationarity cut described in [13], which rejects $\approx 3\%$ of the segments. We also mask frequency bins associated with instrumental lines (e.g., harmonics of the 60 Hz power, calibration lines and suspension-wire resonances) as well as injected, simulated pulsar signals. For $\beta = -3, 0$ we include frequency bins up to 200, 500 Hz, so that $\sigma(\hat{\Omega})$ is within $\approx 2\%$ of the minimum possible value. Thirty-three frequency bins are masked, corresponding to 2% of the frequency bins between 40–500 Hz used in the broadband analyses.

In order to determine if there is a statistically significant GW signature, we consider the highest signal-to-noise ratio (SNR) frequency bin or sky-map pixel. We calculate the expected noise probability distribution of the maximum SNR given many independent trials (in a spectral band) and given many dependent trials (for a sky map).

For $N$ independent frequency bins, the probability density function, $\pi(\rho_{\text{max}})$, of maximum SNR, $\rho_{\text{max}}$, is

$$\pi(\rho_{\text{max}}) \propto \left[1 + \text{erf}(\rho_{\text{max}}/\sqrt{2})\right]^{N-1} e^{-\rho_{\text{max}}^2/2}.$$ (13)

The Gaussianity of $\hat{P}_{\Omega}$ and $\hat{\eta}_\Omega$, calculated by summing over many $O(500)$ independent segments, is expected to arise due to the central limit theorem [20]. Additionally, we find the Gaussian-noise hypothesis to be consistent with time-slide studies, wherein we perform the cross-correlation analysis with unphysical time shifts in order to wash out astrophysical signals and thereby obtain different realizations of detector noise.

The distribution of maximum SNR for a sky map is more subtle due to the nonzero covariances between estimators for different patches on the sky. For this case, we calculate $\pi(\rho_{\text{max}})$ numerically by simulating many realizations of dirty maps with covariances described by the Fisher matrix $\Gamma$.
FIG. 1 (color online). Top row: SNR maps for the three different analyses: SHD clean map $\beta = -3$ (left), SHD clean map $\beta = 0$ (center), and radiometer $\beta = 0$ (right). All three SNR maps are consistent with detector noise. The $p$ values associated with each map’s maximum SNR are (from left to right) $p = 25\%$, $p = 56\%$, $p = 53\%$. Bottom row: The corresponding 90% C.L. upper-limit maps on strain power in units of strain$^2$Hz$^{-1}$sr$^{-1}$ for the SHD algorithm, and units of strain$^2$Hz$^{-1}$ for the radiometer algorithm.

Following [12], we marginalize over the H1, H2, and L1 calibration uncertainties [12], which were measured to be 10%, 10%, and 13%, respectively [21]. Using a prior, taken to be flat above $P(\tilde{\Omega}) = 0$, we obtain Bayesian upper limits at 90% C.L. [22].

Results.—Figure 1 shows sky maps for SHD $\beta = -3$ (left), SHD $\beta = 0$ (center), and the radiometer $\beta = 0$ (right). The top row contains SNR maps. The maximum SNR values are 3.1 (with significance $p = 25\%$), 3.1 ($p = 56\%$), and 3.2 ($p = 53\%$), respectively. These $p$ values take into account the number of search directions and covariances between different sky patches. Observing no evidence of GWs, we set upper limits on GW power. The evidence of GWs, we set upper limits on GW power. The 90% confidence level (C.L.) upper-limit maps are given in the bottom row. For SHD $\beta = -3$, the limits are between $5 - 31 \times 10^{-49}$ strain$^2$Hz$^{-1}$sr$^{-1}$; for SHD $\beta = 0$, the limits are between $6 - 35 \times 10^{-49}$ strain$^2$Hz$^{-1}$sr$^{-1}$; and for the radiometer $\beta = 0$, the limits are between $2 - 20 \times 10^{-50}$ strain$^2$Hz$^{-1}$. Since the radiometer and SHD maps have different units—strain$^2$Hz$^{-1}$ and strain$^2$Hz$^{-1}$sr$^{-1}$ respectively—one must scale the SHD map by the typical diffraction limited resolution $\tilde{A} = 0.1$ sr to perform an approximate comparison.

The strain power limits can also be expressed in terms of energy flux per unit frequency [13]:

$$\hat{F}(f, \tilde{\Omega}) = \left(3.18 \times 10^{43} \frac{\text{erg}}{\text{cm}^2 \text{s}}\right) \frac{f}{(100 \text{ Hz})^{1/2}} \hat{P}(\tilde{\Omega}).$$

(Radiometer energy flux is obtained by replacing $\hat{P}(\tilde{\Omega})$ with $\hat{\tilde{\Omega}}$. The corresponding values are $2 - 10 \times 10^{-6} (f/100 \text{ Hz})^{-1} \text{erg cm}^2 \text{s}^{-1} \text{Hz}^{-1} \text{sr}^{-1}$ and $2 - 11 \times 10^{-6} (f/100 \text{ Hz})^2 \text{erg cm}^2 \text{s}^{-1} \text{Hz}^{-1} \text{sr}^{-1}$ for the SHD method, and $6 - 60 \times 10^{-8} (f/100 \text{ Hz})^3 \text{erg cm}^2 \text{s}^{-1} \text{Hz}^{-1} \text{sr}^{-1}$ for the radiometer. The radiometer limits constitute a factor of $\sim 30$ improvement over the previous best [13].)

Figure 2 shows 90% C.L. upper limits on the $C_i$. Since the $\hat{P}_{lm}$ have units of strain$^4$Hz$^{-2}$sr$^{-2}$, the $C_i$ have the somewhat unusual units of strain$^4$Hz$^{-2}$sr$^{-2}$.

Sco X-1 is a nearby (2.8 kpc) low-mass x-ray binary likely to include a neutron-star spun up through accretion. Its spin frequency is unknown. It has been suggested that the accretion torque is balanced by GW emission [23]. The Doppler line broadening due to the orbital motion is smaller than the chosen $\delta f = 0.25$ Hz bin width for frequencies below $\approx 930$ Hz [24]. At higher frequencies, the signal is certain to span two bins. We determine the maximum value of SNR in the direction of Sco X-1 to be 3.6 ($p = 73\%$ given $\Omega(7000)$ independent frequency bins) at $f = 1770.50$ Hz. Thus in Fig. 3 (first panel) we present limits on root-mean-square (RMS) strain, $h_{\text{RMS}}(f, \tilde{\Omega})$, as a function of frequency in the direction of Sco X-1 (RA, dec) = (16.3 hr, 15.6°). These limits improve on the previous best by a factor of $\sim 5$ [13]. RMS strain is related to narrow-band GW power via

$$h_{\text{RMS}}(f, \tilde{\Omega}) = [\eta(f, \tilde{\Omega}) \delta f]^{1/2},$$

FIG. 2 (color online). Upper limits on $C_i$ at 90% CL vs $l$ for the SHD analyses for $\beta = -3$ (left) and $\beta = 0$ (right). The $\hat{C}_i$ are consistent with detector noise.
and is better suited for comparison with searches for periodic GWs [25] (see also [26]). These limits apply to a circularly polarized signal from a pulsar whose spin axis is aligned with the line of sight. The limits constrain the RMS strain in each bin as opposed to the total RMS strain from Sco X-1, which might span two bins.

We also look for statistically significant signals associated with the Galactic Center (RA, dec) = (17.8 hr, −29°) and SN 1987A (RA, dec) = (5.6 hr, −69°). The maximum SNR values are 3.5 (p = 85%) at f = 203.25 Hz and 4.3 at (p = 7%) 1367.25 Hz, respectively. Limits on RMS strain are given in Fig. 3.

In summary, no evidence was found to support the detection of either extended or pointlike GW sources. However, the clean maps in Fig. 1 represent the first effort to look for anisotropic extended sources of GWs. With the ongoing construction of second-generation GW interferometers [27–30], we expect to achieve strain sensitivities that will test plausible astrophysical and cosmological models. The new framework presented here is expected to serve as the paradigm for future stochastic analyses.

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The data were first processed with $l_{\text{max}} = 20$; the present method of choosing $l_{\text{max}}$ was then adopted \textit{a posteriori} to more accurately model the network’s angular resolution.

Work is ongoing to take this effect into account.

A prior constructed from [13] would be nearly flat anyway since the strain sensitivity has improved 10-fold since S4.

A prior constructed from [13] would be nearly flat anyway since the strain sensitivity has improved 10-fold since S4.