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<td>As Published</td>
<td><a href="http://dx.doi.org/10.1061/(ASCE)0733-9429(2007)133:1(70)">http://dx.doi.org/10.1061/(ASCE)0733-9429(2007)133:1(70)</a></td>
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<tr>
<td>Publisher</td>
<td>American Society of Civil Engineers (ASCE)</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Wed Mar 16 02:02:08 EDT 2016</td>
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<tr>
<td>Citable Link</td>
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Integral Model of a Multiphase Plume in Quiescent Stratification

B. C. Crounse1; E. J. Wannamaker2; and E. E. Adams3

Abstract: The writers present a one-dimensional integral model to describe multiphase plumes discharged to quiescent stratified receiving waters. The model includes an empirical submodel for detrainment, and the capability to include dispersed phase dissolution. Model equations are formulated by conservation of mass, momentum, heat, dissolved species concentration, and salinity, and allow the tracking of dissolved material and changes in plume density due to solute density effects. The detrainment (or peeling) flux, \( E_p \), is assumed to be a function of the dispersed phase slip velocity, \( u_p \), the integrated plume buoyancy, \( B \), and the momentum of the entrained plume fluid, characterized by the fluid velocity, \( u_i \), given by the general relationship \( E_p = \varepsilon (u_p / u_i)^3 (B_i / u_i^2) \). The parameter \( \varepsilon \) is calibrated to laboratory experimental data. Because \( E_p \) is based on a force balance, this algorithm allows numerical models to reproduce a wide range of characteristic plume behavior. Such a predictive algorithm is important for applying models to field scale plumes, especially where chemical processes within the plume may alter plume buoyancy (and hence peeling behavior), as in the case of a CO2 droplet plume used for ocean sequestration of CO2.

DOI: 10.1061/(ASCE)0733-9429(2007)133:1(70)

CE Database subject headings: Bubbles; Buoyant jets; Plumes; Two phase flow; Stratification.

Introduction

Numerical modeling of multiphase plumes is motivated by the description of field scale events such as reservoir destratification (Schladow 1993; Lemckert and Imberger 1993), deep ocean CO2 injection (Adams et al. 1997; Alendal and Drange 2001), and oil well blowouts (Yapa and Zheng 1997; Johansen 2000). Entrainment of stratified water by these plumes causes negative buoyancy and, eventually, detrainment or “peeling” of the entrained water from the dispersed phase. In the case of CO2 plumes, the solute density effect caused by CO2 dissolution enhances this negative buoyancy effect. This paper presents a hybrid double plume model after the one introduced by Asaeda and Imberger (1993), with a new parameterization of peeling and equations based on conservation of stratifying agent rather than buoyancy, which makes it simpler to include the density effects of dissolving bubbles. The parameter, \( \varepsilon \), introduced as part of the peeling algorithm, is calibrated with experimental data. The detrainment algorithm is shown to match experimental data over a wide range of plume behavior.

Model Formulation

The spatial evolution of a two-phase plume in stratification is controlled by four primary processes: buoyant forces acting upon the bubbles and plume water, dissolution of the bubbles, turbulent entrainment of ambient water into the plume, and buoyant detrainment, called peeling. Although the dispersed phase can be gas bubbles, liquid droplets, or solid particles, we will frequently simply use the generic term bubbles.

Integral models reduce the three-dimensional plume flow to a one-dimensional problem by assuming a profile shape for each flux variable that is independent of height. Although the similarity assumption is not strictly valid for two-phase plumes in stratification, models based on similarity have been used successfully (Asaeda and Imberger 1993; Schladow 1992; Wüest et al. 1992; Turner 1986; McDougall 1978). Here, we use top-hat profiles (variables are assumed constant over the plume width) for both the inner, rising plume of water and bubbles, and for the outer, falling annular plume of water only. Asaeda and Imberger (1993) used a similar assumption for their analysis of a double plume. Fig. 1 shows a schematic of the double-plume model control volume and associated fluxes.

We formulate the model in terms of the governing flux variables. The mass flux of bubbles, \( W_b \), is given by their number flux, \( N_b \), their nominal diameter, \( d_b \), and their density, \( \rho_b \), yielding

\[
W_b(z) = \frac{1}{2} \pi d_b^2(z) N_b(z) \rho_b(z) = Q_b(z) \rho_b(z)
\]

(1)

The size and density of bubbles are tracked in a bubble submodel that accounts for dissolution and phase changes. Denoting \( X \) as the cross-sectional fraction of the inner plume occupied by bubbles, we define the volume flux, \( Q \), of plume water as

\[
Q(z) = \int_0^b (1 - X(z)) u(z) 2 \pi r dr = \pi b_i^2 u_i
\]

(2)

where \( u \) = average water velocity and \( b = \) plume width. The subscript \( i \) indicates an inner plume value. The momentum flux, \( M \),
includes the momentum of both the bubbles and the plume water
\[ M_i(z) = \gamma \int_0^{b_i} (1 - X(z)) u_i(z) \rho_i(z) z 2\pi dr + \gamma \int_0^{b_i} X(z) u_i(z) + u_i(z)^2 \rho_i 2\pi dr \]
where \( u_i = \) bubble slip velocity and \( \gamma = \) momentum amplification term, introduced by Milgram (1983) to account for the fact that the model formulation implicitly ignores turbulent momentum transport. Except possibly near the source, \( X \ll 1 \), and for bubble plumes \( \rho_b < \rho_i \); thus because \( u_i = O(u_i) \), the second term in Eq. (3) can generally be ignored giving
\[ M_i = \gamma \rho_i \pi b_i^2 u_i^2 = \gamma Q_o \mu . \]
The buoyant forces generating the plume result from changes in density. For this model, density is tracked through changes in salt flux, \( S \), heat flux, \( J \), and the dissolved species flux including ambient and bubble dissolution contributions, \( C \). The salt flux is defined from the local plume salinity, \( s \), such that
\[ S_i(z) = Q_i(z) \rho_s s(z) \] (4)
The heat flux of the plume is defined from the local water temperature, \( T \), yielding
\[ J_i(z) = Q_i(z) \rho_c c_p(z) T_i(z) \] (5)
where \( c_p = \) heat capacity of the fluid. Finally, the dissolved species flux is defined from the local dissolved species concentration, \( c \),
\[ C_i(z) = Q_i(z) c(z) \] (6)
Thus, Eqs. (1)–(6) define the model state variables for the inner plume.

The state variables for the outer plume are nearly identical. The primary difference is that, because the outer plume is assumed to be annular, the volume flux of the outer plume is defined as
\[ Q_o(z) = \pi (b_o^2 - b_i^2) u_o \] (7)
where the subscript \( o \) indicates an outer plume value. Defining \( z \) as the upward spatial coordinate and specifying that the outer plume flow downward, the velocity \( u_o \) is negative and \( u_i \) is positive. Using Eq. (7) and changing the subscripts in Eqs. (1)–(6) from \( i \) to \( o \) yield the flux equations for the outer plume.

The plume develops by exchanging fluid with the ambient and by exchanging fluid between the inner and outer plumes. The entrainment hypothesis, introduced by Morton et al. (1956), states that the entrainment flux across a turbulent shear boundary is proportional to a characteristic velocity in the turbulent layer. The entrainment in counterflowing streams is less well understood. Here we have followed the convention introduced by McDougall (1978) for his coflowing double plume model, and adopted by Asaeda and Imberger (1993) for their coflowing double plume model, and defined three entrainment fluxes (entrainment flow rate per unit length): \( E_i \) enters from the ambient or from the outer plume into the inner plume, \( E_o \) enters from the inner plume into the outer plume (the left facing arrow in Fig. 1), and \( E_u \) enters from the ambient into the outer plume. These fluxes can be related to plume velocities by
\[ E_i(z) = 2\pi b_o \alpha_o (|u_i - u_o|) \] (8)
\[ E_o(z) = 2\pi b_i \alpha_i |u_o| \] (9)
\[ E_u(z) = 2\pi b_i \alpha_i |u_o| \] (10)
where the absolute values on plume velocities are used to make all entrainment fluxes positive. The \( \alpha \)'s are entrainment coefficients, with values of \( \alpha_o = 0.055, \alpha_i = 0.11, \) and \( \alpha_o = 0.11 \) (Asaeda and Imberger 1993). Note that at any given time there is only entrainment into or out of the inner plume but not both; with the above coefficients \( (2\alpha_i = \alpha_o) \) the net entrainment into the inner plume is \( 2\pi \rho_b \alpha_o (|u_o| - |u_o|) \) which gives no exchange if the inner and outer plume velocities are equal in absolute magnitude.

A final exchange equation accounts for buoyant detrainment, which has been modeled in a variety of ways. Liro et al. (1992) assumed that a fixed fraction of plume fluid was ejected when the net buoyancy flux across the plume approached zero. McDougall (1978), Schladow (1992), and Asaeda and Imberger (1993) assumed that all of the plume fluid detrained when the net momentum approached zero. Experiments suggest that peeling is better predicted when the net momentum approaches zero, but that the process does not occur instantaneously (Socolofsky 2001; Socolofsky and Adams 2003). For this model, a self-regulating peeling criterion is introduced. We know that peeling occurs when the drag from the bubbles can no longer support the negative buoyancy of the fluid. A simple parameterization that produces behavior that is similar to experiments gives the peeling flux as
\[ E_p(z) = \varepsilon \left( \frac{u_o(z)}{u(z)} \right)^2 \left( \frac{B(z)}{u_o(z)} \right) \] (11)
where \( \varepsilon = \) nondimensional “peeling parameter” calibrated below, and \( B_i = \) buoyancy flux, defined as
\[ B_i(z) = g Q_i(z) (\rho_o(z) - \rho_i(z)) \] (12)
where \( \rho_i = \) ambient density. The relationship in Eq. (11) makes it easier for outer plumes to overlap and thus makes it possible to simulate the continuous peeling nature of Type 3 plumes, as described in Socolofsky (2001) and Socolofsky and Adams (2005). Fig. 2 shows these characteristic plume types, and indicates the
change in peeling behavior as the dimensionless bubble slip velocity \( U_N = \frac{u_t}{(BN)^{1/4}} \) increases.

With these definitions, the plume conservation equations can be readily defined. From mass conservation, we have

\[
\frac{dQ_i}{dz} = E_i - E_o - E_p \tag{13}
\]

\[
\frac{dQ_o}{dz} = -E_i + E_o + E_p + E_u \tag{14}
\]

Momentum conservation states that the momentum changes in response to the applied forces, which gives the following equations:

\[
\frac{dM_i}{dz} = g \left( \frac{Q_b}{\left(u_i + u_b\right)} (\rho_o - \rho_b) + \pi b_i^2 (\rho_o - \rho_i) \right) + E_i \rho_o u_i - E_o \rho_o u_i - E_p \rho_p u_i \tag{15}
\]

\[
\frac{dM_o}{dz} = -g \pi (b_o^2 - b_i^2) (\rho_o - \rho_p) + E_o \rho_o u_o - E_i \rho_i u_o - E_p \rho_p u_i \tag{16}
\]

The conservation of salt, heat, and dissolved tracer flux follow from the mass conservation equation, yielding for the inner plume

\[
\frac{dS_i}{dz} = E_s s_o - E_s s_i - E_p s_i \tag{17}
\]

\[
\frac{dJ_i}{dz} = c_i \rho_i (E_i T_o - E_o T_i - E_p T_i) + \frac{dW_b}{dz} \Delta H_{\text{diss}} \tag{18}
\]

\[
\frac{dC_i}{dz} = E_c c_o - E_c c_i - E_p c_i - \frac{dW_b}{dz} \tag{19}
\]

and for the outer plume

\[
\frac{dS_o}{dz} = -E_s s_o + E_s s_i + E_p s_i + E_p s_o \tag{20}
\]

\[
\frac{dJ_o}{dz} = c_o \rho_o (E_i T_o + E_o T_i + E_p T_i + E_p T_o) \tag{21}
\]

\[
\frac{dC_o}{dz} = -E_c c_o + E_c c_i + E_p c_i + E_p c_o \tag{22}
\]

The last term in Eq. (18) accounts for the energy released by dissolving bubbles. The densities \( \rho_o \) and \( \rho_i \) are determined by an equation of state which is a function of \( s, T, \) and \( c. \) \( dW_b/dz \) is calculated by the bubble submodel.

The model begins with integration of the inner plume from the point of release to the point where the bubbles dissolve or the water surface is reached. Once the inner plume integration is complete, the outer plume segments are integrated. The integration of each outer plume section continues until the momentum flux approaches zero. Then, the next outer plume section is initialized and integrated. This cycle repeats until the solution converges to a steady result (typically ten iterations).

We note that the model assumes quiescent ambient conditions. In practice there will always be some current, but if the magnitude is small, it will have little influence on plume dilution or height of rise. Socolofsky and Adams (2002) describe a series of experiments to investigate the impact of ambient currents on bubble plumes. In the absence of ambient stratification, they found that the entrained plume water separates from the dispersed phase above a certain separation height that depends on the ambient current speed, as well as plume buoyancy and bubble slip velocity. For bubble plumes in an ambient that is both stratified and flowing, they argue that the plume is “crossflow dominated” if this separation height is less than the initial plume trap height, and “stratification dominated” if the separation height is greater than the trap height. The present theory would apply only in the latter case.

### Processes and Properties

Submodels describe physical and chemical properties of the plume phases, behavior of bubbles, droplets, and particles, and buoyant forces.

### Seawater

The model depends on knowledge of several properties of ambient water at any given depth. Our focus has been on seawater, so these include the ambient salinity, temperature, and pressure, as well as properties that are functions of these three quantities. The model can use density and temperature profiles provided by the user, or it can be used to calculate properties for seawater given temperature, depth, and salinity. The most important independent property is in situ density, which is calculated using the United Nations Educational, Scientific, and Cultural Organization (UNESCO) equation of state for seawater (Gill 1982).

### Dispersed Phase

There are several physical and chemical properties of the bubble material which are relevant to the current work. These include density, solubility in seawater, molar volume in seawater, surface tension, and the rate of diffusion in seawater. Although \( \text{CO}_2 \) has been the only chemically active dispersed phase implemented to date, with knowledge of these properties any dispersed phase can be modeled. Air is the dispersed phase used for the calibration studies outlined below. Because air-bubble plumes are generally deployed in shallow environments, the density of air is modeled as an ideal gas

\[
\rho_{\text{air}} = \frac{M_{\text{air}} P}{RT} \tag{23}\]

where \( M_{\text{air}} = 29.0 \text{ g/mol} \) = effective molecular weight of air and \( R = \text{ideal gas constant}. \)

### Bubble Dynamics

Plume behavior is primarily controlled by the amount of buoyant force acting on the bubbles. As the bubbles will rise at an approximately constant velocity relative to the plume water, the buoyant force is essentially balanced by drag force on the bubbles. As the bubbles move, the drag forces do work on the ambient liquid, so the end result is that the buoyant force of the bubbles is transferred to the water in the plume.

Two important factors determine the fate of material in the dispersed phase: Slip velocity and dissolution rate. These factors are determined primarily by the size and shape of the bubbles, as well as the material properties of the bubbles and receiving water.
Slip Velocity
There are many correlations available for predicting a bubble’s slip velocity (Clift et al. 1978; Zheng and Yapa 2000). The challenge is to predict the bubble drag coefficient, \(C_d\), defined by the equation

\[ F = C_d \rho_b u_b^2 \]  

where \(F\) = mean-drag force acting on the bubble. The present work includes correlations given in Clift et al. (1978) for individual fluid bubbles (or droplets) or solid particles translating in a still medium, although it is reasonable to expect that the presence of neighboring bubbles and a turbulent flow field could alter slip velocity (Esmaeili and Tryggvason 1999).

Mass Transfer
The rate of mass transfer, or dissolution, of dispersed phase from a bubble to dissolved form can be described by the empirical Ranz–Marshall equation

\[ \frac{dm_b}{dt} = -\pi d_b^2 K(c_i - c) \]  

where \(K\) = mass transfer coefficient [L/T]; \(c_i\) = surface concentration or solubility of dispersed phase [M/L]; and \(c\) = concentration of dissolved dispersed phase in the vicinity of the bubble [M/L].

Substituting \(m_b = \pi d_b^2 \rho_b \) into Eq. (25) and assuming \(\rho_b\) to be constant yields

\[ \frac{dd_b}{dt} = -2K\frac{(c_i - c)}{\rho_b} \]  

Thus, the rate of bubble shrinkage (the most direct experimental observation) is affected by two components: the mass transfer coefficient and the dispersed phase solubility. Multiplying Eq. (25) by the bubble number flux describes the rate of change of bubble mass flux. Also, in order to be consistent with the governing equations of the plume model, this differential equation should describe a change over distance rather than time. This is achieved by dividing Eq. (25) by the nominal bubble velocity, \((u_i + u_b)\):

\[ \frac{dW_b}{dz} = -N_b \pi d_b^2 K \frac{(c_i - c)}{u_i + u_b} \]  

What remains is the determination of \(K\), which can be correlated with bubble properties. The present model uses correlations for rigid particles, given by Clift et al. (1978) as a function of particle Reynolds and Schmidt numbers, though other correlations could be used. Mass transfer coefficients for rigid particles are lower than those for pure bubbles or droplets which is appropriate for a deep ocean CO2 release in which mass transfer from a liquid droplet would likely be inhibited by hydrate formation (Hirai et al. 1997) and surfactant effects.

Bubble Spread
Previous investigators observed that the width of the bubble core, the region of flow actually containing bubbles, is sometimes smaller than the width of the upward flowing continuous phase. This has been modeled by defining a spreading ratio, \(\lambda\). For a bubble plume to be self-similar, \(\lambda\) must be constant. However, it is not clear that it is constant for real plumes. Further, \(\lambda\) seems to vary significantly across experiments, ranging from 0.3 (Ditmars and Cederwall 1974) to 1.0 (Chesters et al. 1980), probably due to differences in bubble slip velocity (Socolofsky 2001) and experi-

Material Amplification
Introduced in Eq. (3), \(\gamma\) accounts for the fact that the plume flow is turbulent. The instantaneous vertical plume velocity may be decomposed into mean and turbulent quantities: \(u = \bar{u} + u'\). Values of \(\gamma > 1\) account for the momentum flux associated with \(\bar{u}u'\). Although Milgram (1983) has presented a correlation for \(\gamma\) based on a phase distribution number, it was found that the current model results were insensitive to changes in \(\gamma\), and hence it has been held constant at 1.1.

Initial Conditions
The model requires initial values for its state variables. The initial bubble mass flow rate, and thus the droplet buoyancy flux, are known, and the initial droplet diameter must be input. However, because the flow near the release is not well understood, quantities such as the initial plume volume and momentum fluxes cannot be calculated with great accuracy. Fortunately, plume behavior is quite insensitive to the initial volume and momentum fluxes, provided that they are small (Liro et al. 1992; Milgram 1983). Following Liro et al. (1992), the current model is initialized by assuming that the initial flow is approximately that of a pure plume (driven purely by buoyancy), so that the initial volume and momentum fluxes (in the inner plume) are

\[ Q_i^{\text{init}} = \frac{6}{5} \left[ \frac{9 \pi \alpha_1 B z_0^5}{10} \right]^{1/3} \]  

\[ M_i^{\text{init}} = \left[ \frac{81 \pi \alpha_1 B^2 z_0^4}{100} \right]^{1/3} \rho_i \]  

where \(z_0\) = small vertical distance, taken to be 10 times the diameter of the release orifice, which is a design variable; and \(B\) = buoyancy of the bubbles.

Comparison with Data
The model has been compared against measurements in plumes of varying complexity. In general, optimal agreement requires slight adjustment of the model entrainment coefficients and the peeling parameter, suggesting some uncertainty in model formulation, but Crouse (2000) and Wannamaker (2002) found that the set of entrainment coefficients reported in Asaeda and Imberger (1993), \(\alpha_i = 0.055\), \(\alpha_s = 0.11\), and \(\alpha_c = 0.11\), gave acceptable agreement with available data. These coefficients are used herein. Calibration of the peeling parameter \(\varepsilon\) is discussed below. Unless stated otherwise, the calibrated value of \(\varepsilon = 0.007\) has been used herein.

Laboratory and field data for the trap height of bubble and sediment plumes in stratification were available from Asaeda and Imberger (1993), Lemkert and Imberger (1993), Reingold (1994), and Socolofsky (2001). The measured intrusion height of the first peeling event, \(h_p\), decreases with the dimensionless slip velocity, \(U_p = u_i/(BN)^{1/4}\) as indicated in Fig. 3. The model correctly captures this trend, but the considerable scatter in the data makes it impossible to claim model validation.

Socolofsky (2001) reported volume fluxes for his laboratory multiphase plumes. Here we compare the model output with the
first intrusion flux, $Q_i$. Fig. 4 shows that the model captures the decline of $Q_i$ with $U_N$, but again there is considerable scatter in the data.

To simulate rising CO$_2$ droplets in a generally downward stream of CO$_2$-enriched seawater, Harrison (2001) introduced air at the bottom of a water tank and observed the upward rise of bubbles against a falling stream of dense brine released near the tank surface. The goals were to test the postulated model structure in which both positively buoyant bubbles and negatively buoyant plume water rise upward in the inner core and to compare modeled and observed plume velocities. Injected dye clearly showed the entrained water rising within the central core, and Fig. 5 indicates reasonable agreement between measured and modeled plume velocities in both inner and outer plumes. Conditions depicted in Fig. 5 correspond to an upward flow of air of 100 mL/min and a downward flow of brine (specific gravity = 1.14) of 715 mL/min yielding buoyancies for air and brine of approximately equal absolute value.

Calibration of the peeling parameter, $\epsilon$, of particular interest to this study, is based on experimental data from Socolofsky (2001) and Socolofsky and Adams (2003). Using dye injected near the bubble source, they determined the fraction of plume water, $f$, that detrained in the first peeling event as a function of $U_N$ (refer to Fig. 8).

To provide a comparison with Socolofsky’s data, the integral model was used to predict plume behavior for a given $U_N$ and $\epsilon$. From the model output, the fraction of plume water that detrained in the first peeling event was calculated as

$$f = \frac{Q_1 - Q_2}{Q_1}$$

(30)

where $Q_1$ = inner plume volume flux just before the onset of the first peel and $Q_2$ = inner plume volume flux after completion of the first peeling event. Fig. 6 depicts these fluxes using representative model output for three values of $\epsilon$, with $Q_1=Q_p+Q_2$.
The peeling fraction, $f$, was then calculated from Eq. (30) and plotted versus $\varepsilon$ and $U_N$ (Fig. 7). Superimposed on the model results are the values of $f$ measured by Socolofsky, with corresponding error bars. Considering the experimental error, a value for $\varepsilon$ of 0.005–0.01 seems acceptable. Hence, when not stated otherwise, a value of 0.007 has been used herein.

Using values of $\varepsilon=0.001$, 0.005, and 0.01, the peel fraction was calculated for $U_N$ ranging from 0.5 to 3.5 (see Fig. 8). The experimental results generally lie between $\varepsilon=0.005$ and 0.01, again suggesting that a value of $\varepsilon=0.007$ is reasonable. Note that the values of $U_N$ span the range of deep-water plume types, which include Type 2 plumes with distinct peels, Type 3 plumes with overlapping, random peels from Asaeda and Imberger (1993), as well as Type $1^*$ plumes with peeling dispersed phases from Socolofsky (2001) and Socolofsky and Adams (2003). Hence, we conclude that this modeling algorithm reasonably predicts the peeling behavior of multiphase plumes in deep water.

**Conclusions**

An integral double plume model has been developed to describe the behavior of bubble plumes discharged into quiescent, stratified, ambient waters. The model differs from earlier models, primarily in its inclusion of a submodel to describe the continuous peeling or detrainment of rising plume fluid as it encounters increasing adverse density gradients. Model entrainment coefficients presented by Asaeda and Imberger (1993), plus a calibrated detrainment parameter $\varepsilon$, allow the model to simulate the range of observed plume types as well as reproduce quantitative trends in measured plume parameters.

**Acknowledgments**

This study was supported by the National Energy Technology Laboratory, U.S. Department of Energy (Grant No. DE-FG26-98FT40334) and the Ocean Carbon Sequestration Program, Biological and Environmental Research (BER), U.S. Department of Energy (Grant No. DE-FG02-01ER63078). MIT graduate student Tim Harrison helped build and conduct some of the experiments.

**Notation**

The following symbols are used in this paper:

- $B$ = initial bubble buoyancy flux ($m^3/s$);
- $B_i$ = inner plume buoyancy flux ($m^3/s$);
- $b_i$ = inner plume width (m);
- $b_o$ = outer plume width (m);
- $C_D$ = bubble drag coefficient (—);
- $C_i$ = inner plume dissolved species flux (kg/s);
- $C_o$ = outer plume dissolved species flux (kg/s);
- $c_a$ = ambient dissolved species concentration (kg/m$^3$);
- $c_i$ = inner plume dissolved species concentration (kg/m$^3$);
- $c_o$ = outer plume dissolved species concentration (kg/m$^3$);
- $c_p$ = heat capacity of fluid (J/kg/$^\circ$C);
- $c_s$ = bubble surface concentration (kg/m$^3$);
- $d_b$ = bubble diameter (m);
- $E_a$ = entrainment flux from ambient to outer plume ($m^3/s$);
- $E_i$ = entrainment flux from ambient or outer plume to inner plume ($m^3/s$);
- $E_o$ = entrainment flux from inner plume to outer plume ($m^3/s$);
- $E_p$ = detrainment or peeling flux ($m^3/s$);
- $F$ = mean-drag force acting on a particle (kg m/s$^2$);
- $f$ = fraction of plume water detrained in a peeling event (—);
- $g$ = gravitational constant (m/s$^2$);
- $h_T$ = plume trap height (m);
- $J_i$ = inner plume heat flux (J/s);
- $J_o$ = outer plume heat flux (J/s);
- $K$ = bubble mass transfer coefficient (m/s);
- $M_{air}$ = molecular weight of air (g/mol);
\[ M_i = \text{momentum flux of inner plume (kg m}^2\text{s}^{-1}) \];
\[ M_o = \text{momentum flux of outer plume (kg m}^2\text{s}^{-1}) \];
\[ m_b = \text{bubble mass (kg)} \];
\[ N = \text{ambient stratification frequency (1/s)} \];
\[ N_b = \text{number flux of bubbles (1/s)} \];
\[ P = \text{pressure of atm (atm)} \];
\[ Q_b = \text{dispersed phase volume flux (m}^3\text{s}^{-1}) \];
\[ Q_i = \text{inner plume volume flux (m}^3\text{s}^{-1}) \];
\[ Q_o = \text{outer plume volume flux (m}^3\text{s}^{-1}) \];
\[ Q_p = \text{peeling flux (m}^3\text{s}^{-1}) \];
\[ Q_1 = \text{plume flux before a peeling event (m}^3\text{s}^{-1}) \];
\[ Q_2 = \text{plume flux after a peeling event (m}^3\text{s}^{-1}) \];
\[ R = \text{ideal gas constant [L atm/(K mol)]} \];
\[ r = \text{plume radius (m)} \];
\[ S_i = \text{inner plume salinity flux (kg/s)} \];
\[ S_o = \text{outer plume salinity flux (kg/s)} \];
\[ s_i = \text{ambient salinity (kg/m}^3\text{)} \];
\[ s_o = \text{inner plume salinity (kg/m}^3\text{)} \];
\[ s_p = \text{outer plume salinity (kg/m}^3\text{)} \];
\[ T = \text{fluid temperature (K)} \];
\[ T_i = \text{temperature of inner plume (°C)} \];
\[ T_o = \text{temperature of outer plume (°C)} \];
\[ U_s = \text{dimensionless slip velocity (—)} \];
\[ u = \text{turbulent velocity (m/s)} \];
\[ u_m = \text{turbulent mean velocity (m/s)} \];
\[ u_r = \text{turbulent velocity fluctuation (m/s)} \];
\[ u_b = \text{bubble slip velocity (m/s)} \];
\[ u_i = \text{inner plume velocity (m/s)} \];
\[ u_o = \text{outer plume velocity (m/s)} \];
\[ v_b = \text{bubble volume (m}^3\text{)} \];
\[ W_b = \text{mass flux of bubbles (kg/s)} \];
\[ X = \text{cross-sectional fraction of inner plume occupied by bubbles (—)} \];
\[ z = \text{vertical coordinate, positive upward (m)} \];
\[ z_r = \text{release distance (m)} \];
\[ \alpha = \text{entrainment coefficient corresponding to } E_o (—) \];
\[ \alpha_i = \text{entrainment coefficient corresponding to } E_i (—) \];
\[ \alpha_o = \text{entrainment coefficient corresponding to } E_o (—) \];
\[ \gamma = \text{momentum amplification term (—)} \];
\[ \Delta H_\text{dis} = \text{heat of dissolution of dispersed phase (J/kg)} \];
\[ \epsilon = \text{detainment parameter (—)} \];
\[ \lambda = \text{bubble core spreading ratio (—)} \];
\[ \mu = \text{fluid viscosity (kg/ms)} \];
\[ \rho_a = \text{ambient density (kg/m}^3\text{)} \];
\[ \rho_b = \text{density of air (kg/m}^3\text{)} \];
\[ \rho_p = \text{dispersed phase density (kg/m}^3\text{)} \];
\[ \rho_i = \text{inner plume density (kg/m}^3\text{)} \];
\[ \rho_o = \text{outer plume density (kg/m}^3\text{)} \];
and
\[ \rho_r = \text{reference density (kg/m}^3\text{)} \].

References


