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Transverse momentum dependent quark densities from Lattice QCD

B. U. Musch*, Ph. Hägler†, J. W. Negele** and A. Schäfer‡

*Theory Center, Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA
†Theoretische Physik T39, TU München, James-Franck-Straße 1, 85747 Garching, Germany
**Massachusetts Institute of Technology, 77 Massachusetts Avenue, Bldg. 6-315, Cambridge, MA 02139, USA
‡Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany

Abstract. We study transverse momentum dependent parton distribution functions (TMDs) with non-local operators in lattice QCD, using MILC/LHPC lattices. We discuss the basic concepts of the method, including renormalization of the gauge link. Results obtained with a simplified operator geometry show visible dipole deformations of spin-dependent quark momentum densities.

Keywords: transverse momentum; parton distribution functions; lattice; QCD

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INTRODUCTION

Generalized parton distribution functions (GPDs) and transverse momentum dependent parton distribution functions (TMDs) provide us with a picture of the internal quark distributions in a nucleon at the instant of an interaction, see illustration Fig. 1 a). GPDs and TMDs have their natural interpretation at large nucleon momentum (TMDs) provide us with a picture of the internal quark distributions in a nucleon at the instant of an interaction, see [4] for a review. A remaining theoretical problem concerns the precise form of the correlator defining TMDs in the continuum, see [5, 6] and references therein. In its basic form, it is given by [7]

$$\Phi^\Gamma_0(x,k_\perp;P,S;\epsilon) \equiv \int dk^- \int \frac{d^4l}{(2\pi)^4} e^{-ik^-l} \frac{1}{2} \left( P,S \right) \bar{q}(l) \Gamma \gamma_i \langle 0 \left| \Phi^\Gamma_q \right| P,S \rangle |_{k^+=P^+}$$

$$= \frac{1}{P^+} \int \frac{d(l-P)}{2\pi} e^{-i(l-P)x} \int \frac{d^4l}{(2\pi)^4} e^{ik_\perp \cdot l} \Phi^\Gamma_q(l,P,S;\epsilon) |_{l^+=0}$$

(1)

where \( \Gamma \) is a Dirac matrix. The Wilson line \( \gamma_i \) running along a continuous path \( \gamma_i \) from \( l \) to \( 0 \) ensures gauge invariance of the expression. For the SIDIS and Drell–Yan scattering process, the Wilson line extends to infinity along a direction \( \nu \) that needs to be chosen (almost) lightlike, such that the cross section factorizes into hard, perturbative parts and soft contributions, see, e.g., Ref. [8]. Based on its symmetry transformation properties, the above correlator can be parametrized in terms of TMDs [9, 10, 11], for example

$$2P^\Gamma_{TL} = \Phi^\Gamma_{q} \gamma^+ \gamma_i \gamma_i |_{l^+=0} = f_{1,q} + \lambda \frac{k_\perp \cdot S}{m_N} g_{1T,q} + \left[ S_i \frac{E_{q} k_i}{m_N} f_{1T,q} \right]_{\text{odd}},$$

(2)
FIGURE 1. a) Illustration of quark degrees of freedom in the nucleon at large momentum. b) Dipole-deformed $x$-integrated densities obtained with straight gauge links at a pion mass $m_\pi \approx 500\text{MeV}$. The insets display the spin polarization of the quarks (red arrow) and of the nucleon (blue arrow).

FIGURE 2. a) Representation of a straight Wilson line (dashed line) as a step-like product of link variables. b) Amplitude $\tilde{A}_2(l^2, 0)$ for up quarks at a pion mass $m_\pi \approx 500\text{MeV}$, using straight gauge links.

Here $\lambda$ is the longitudinal quark polarization, and $\Lambda$ and $S_z$ are longitudinal and transverse nucleon polarization, respectively. The leading-twist TMDs $f_{1T,q}$, $g_{1T,q}$, $f_{1T,q}$ are real-valued functions of $x$ and $k^2$. The “naively time-reversal odd” function $f_{1T,q}$ switches sign when comparing the SIDIS- with the Drell-Yan process, because the direction $v$ of the Wilson line changes from future- to past-pointing [12].

STRAIGHT LINK TMDS FROM THE LATTICE

In light of the uncertainties about the precise form of the continuum correlator, and to develop our methods, our first lattice studies employ a simple operator geometry that does not relate to a specific scattering process: We connect the quark fields with a direct, straight Wilson line. For the resulting “process-independent” TMDs, the T-odd functions such as the Sivers function $f_{1T,q}$ vanish exactly.

In our approach, we calculate matrix elements $\langle P, S | O | P, S \rangle$ from ratios of three- and two-point functions using the same techniques as GPD calculations by the LHPC collaboration in Ref. [13]. We also use the same sequential propagators and quark propagators, calculated by LHPC with domain-wall valence fermions on top of asqtad-improved staggered MILC gauge configurations [14, 15, 16] with $2+1$ quark flavors at a lattice spacing $a \approx 0.12\text{fm}$. The difference with respect to GPD calculations is that we directly insert the non-local operator $O \equiv \bar{q}(l) \Gamma \gamma^5 [G] \gamma(0)$ in our three-point function. The Wilson line $\gamma^5 [G]$ is approximated as a step-like product of HYP-smeared link-variables as illustrated in Fig. 2 a). See also Ref. [2, 3].

The connection between the matrix elements $\Phi^{\Gamma}$ and TMDs is established through a parametrization in terms of Lorentz-invariant amplitudes $\tilde{A}_i(l^2, i.P)$. For straight Wilson lines, we obtain in analogy to the parametrization in terms
of amplitudes $A_{i}(k^{2}, k · P)$ in Ref. [9] (here our sign conventions follow Ref. [11] with the substitution rule $k \rightarrow im_{N}^{2}l$):

$$\phi^{(2)} = 2 P^{\mu} \tilde{A}_{2} + 2i m_{N}^{2} \mu \tilde{A}_{5}, \quad \phi^{(1)} = -2 m_{N} S^{\mu} \tilde{A}_{6} - 2 i m_{N} P^{\mu} (1 · S) \tilde{A}_{7} + 2 m_{N}^{3} \mu (1 · S) \tilde{A}_{8}.$$ 

The TMDs are then obtained by

$$f_{1}(x, k_{1}^{2}) = 2 \int \frac{d \mu}{2\pi} \phi^{(2)}(l^{2}, l · P), \quad g_{1T}(x, k_{1}^{2}) = 4 m_{N}^{2} \partial_{k_{1}^{2}} \int \frac{d \mu}{2\pi} \phi^{(1)}(l^{2}, l · P).$$

In the equations above, $\phi$ only acts on $l · P$, while $\mathbb{M}$ only acts on $l^{2}$. Thus $x \leftrightarrow l · P$ and $k_{1}^{2} \leftrightarrow l^{2}$ are pairs of conjugate variables. Our Euclidean lattice approach is restricted to the determination of amplitudes $A_{i}$ for $l^{0} = -i t_{0} = 0$, i.e., to the region $l^{2} < 0, |l · P| < \sqrt{-l^{2}}|P|$, where $P$ is the selected three-momentum of the nucleon on the lattice. The limited range in $|l · P|$ prohibits us from a direct evaluation of $\phi$. However, first studies of $x$- and $k_{1}$-correlations are possible [17, 3]. Moreover, $x$-integrated TMDs and densities are directly accessible: Integrating Eq. (1) with respect to $x$ removes $\mathcal{Y}$ and sets $l · P$ to zero. Correspondingly, the $x$-integral of, e.g., $f_{1}$ becomes $\int_{-1}^{1} dx f_{1}(x, k_{1}^{2}) \equiv f_{1}^{(1)}(k_{1}^{2}) = 2 \mathbb{M} \tilde{A}_{2}(l^{2}, 0)$.

In Fig. 2 b), open symbols correspond to unrenormalized lattice data for $\tilde{A}_{2}(l^{2}, 0)$. To obtain results independent of our lattice spacing $a$ and our lattice action, we must renormalize our data. The Wilson line $\mathcal{W}[\mathcal{G}]$ introduces a length dependent renormalization factor $\exp(-\delta m \sqrt{-l^{2}})$ [18, 19, 20]. To fix $\delta m$, we follow the strategy of Refs. [21, 22], and match the renormalized static quark potential $V_{\text{ren}}(r) = V_{\text{str}}(r) + 2 \delta m$ to the string potential $V_{\text{str}}(r) = \sigma r - \pi/(12r)$ [23] at a matching point $r = 1.5 t_{0} \approx 0.7 \text{fm}$. In Fig. 3 a), we test the method for several lattice spacings $a$ on four MILC lattices with similar pion masses $m_{\pi} \approx 500 \text{MeV}$. The renormalized lattice data agree very well with each other and are approximated well by the string potential (red dashed curve) near the matching point, indicated by a vertical dashed line. The procedure implements a gauge-invariant renormalization condition that we can formulate as the demand that the static quark potential asymptotically approaches a straight line $\sigma r$ through the origin (shown as a red dashed line). In connection with TMDs, we lack at present an interpretation of this renormalization condition as a physical renormalization or factorization scale. In Figure 3 b), we check the applicability of the approach to Wilson lines by plotting $V_{\text{ren}}(l) = \ln(U_{l+a/2} U_{l+a/2}^{-1}/a + \delta m)$, where $U_{l}$ is the expectation value of the color trace of a straight Wilson line of length $l$ evaluated on a Landau gauge fixed ensemble, and where the length dependent renormalization has been carried out with the values $\delta m$ obtained from the static quark potential. Only at short lengths, $l \lesssim 0.25 \text{fm}$, we find significant differences between lattice data from different lattice spacings, a sign of lattice cutoff effects. For our TMD calculations discussed below we exclude data obtained in this region from our fits. For $l \gtrsim 0.25 \text{fm}$, we assume that renormalization of the lattice operator can be carried out as in the continuum, $A_{\gamma} = Z_{q, \gamma}^{-1} \exp(-\delta m \sqrt{-l^{2}}) O$, where the renormalization constants $Z_{q, \gamma}^{-1}$ and $\delta m$ are independent of the Dirac structure $\Gamma$ [19].

Figure 2 b) shows the renormalized lattice data for $\tilde{A}_{2}(l^{2}, 0)$ as solid data points. The curve and statistical error band correspond to a Gaussian fit to this data in the range $\sqrt{-l^{2}} \lesssim 0.25 \text{fm}$. Note that the renormalization constant $Z_{q, \gamma}^{-1}$ has been fixed (in the isovector, $u-d$-channel) such that the $x$-$k_{1}$-integrated Gaussian density of unpolarized quarks yields the correct total number of valence quarks, $\int d^{2} k_{1} f_{1}^{[1]}(u-d) = 1$. Similar fits for $\tilde{A}_{7}$ enable us to calculate the “worm-gear” function $g_{1T}^{[1]}$, and correspondingly, the dipole deformed $x$-integrated density $\rho_{\gamma}^{[1]}$ defined in Eq. (2) and shown in Fig. 1 b). While the widths of our distributions depend strongly on our renormalization condition for $\delta m$, the average
transverse quark momentum shift can be expressed in terms of ratios of the Gaussian amplitudes at $l^2=0$:
\[
(k_x)_{TL} \equiv \frac{\int d^2k_\perp k_x \rho_{TL}^{[1]}(k_\perp)}{\int d^2k_\perp \rho_{TL}^{[1]}(k_\perp)} \left|_{l_1=1, l_2=0} \right. = m_N \frac{\int d^2k_\perp k_x^2 / (2m_N^2) \rho_{TL}^{[1]}(k_\perp)}{\int d^2k_\perp \rho_{TL}^{[1]}(k_\perp)} = -m_N \frac{\tilde{A}_T(0,0)}{A_T(0,0)} = \left\{ \begin{array}{ll} 67(5) \text{ MeV} & \text{(up)} \\ -30(5) \text{ MeV} & \text{(down)} \end{array} \right.
\]
(errors statistical only). In these ratios, renormalization factors largely cancel. Reference [24] reveals a remarkable similarity of our results with a light-cone constituent quark model [25], despite the unphysically large quark masses employed in our lattice calculation: They find $(k_x)_{TL} = 55.8 \text{ MeV}$ for up-, and $(k_x)_{TL} = -27.9 \text{ MeV}$ for down-quarks.

**CONCLUSIONS AND OUTLOOK**

We have performed first lattice studies of TMDs using non-local operators with a simplified, straight gauge link. Resulting average momentum shifts $(k_x)_{TL}$ corroborate model results. An ongoing project with staple-shaped gauge links can potentially address TMDs specific to SIDIS or the Drell-Yan process, including T-odd functions responsible for single-spin asymmetries.

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