Driven Dynamics and Rotary Echo of a Qubit Tunably Coupled to a Harmonic Oscillator


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We have investigated the driven dynamics of a superconducting flux qubit that is tunably coupled to a microwave resonator. We find that the qubit experiences an oscillating field mediated by off-resonant driving of the resonator, leading to strong modifications of the qubit Rabi frequency. This opens an additional noise channel, and we find that low-frequency noise in the coupling parameter causes a reduction of the coherence time during driven evolution. The noise can be mitigated with the rotary-echo pulse sequence, which, for driven systems, is analogous to the Hahn-echo sequence.

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Circuit quantum electrodynamics implemented with superconducting artificial atoms and microwave resonators has emerged as a framework for studying on-chip light-matter interactions [1–3]. It has enabled a range of experiments including lasing [4], the creation [5–7] and detection [8] of arbitrary Fock states, and microwave photon-correlation measurements [9,10]. Microwave resonators also provide a means to couple distant qubits [11,12] and, in this role, have been used to implement quantum algorithms in superconducting circuits [13] and to develop quantum computer architectures [14]. However, the coupling of a qubit to a resonator also influences the qubit coherence, for example, by modifying its relaxation rate through the Purcell effect [15]. Such adverse effects can be avoided by making the qubit-resonator coupling tunable [16–19], which also provides advantages when coupling multiple qubits through a single resonator, since it allows the effective qubit-qubit interactions to be turned on and off.

In this work, we study the driven dynamics and the dephasing of a flux qubit [20] that is tunably coupled to a lumped-element harmonic oscillator [21]. We find that the resonator mediates an indirect driving field that interferes with the direct drive set by the qubit-antenna coupling, thereby modifying both the amplitude and the phase of the net driving-field. The tunable coupling allows the indirect driving to be switched off, but it also opens an additional channel for noise to couple into the system. Fluctuations in the coupling parameter translate into effective driving-field amplitude noise, which reduces the qubit coherence during driven evolution. We show that the qubit dephasing due to amplitude noise (whether due to tunable coupling or otherwise) can be mitigated by a rotary echo [22], a pulse sequence originally developed for nuclear magnetic resonance.

The device, shown in Fig. 1(a), consists of a flux qubit and a SQUID embedded in a two-mode LC resonant circuit. The diabatic states of the qubit correspond to clockwise or counterclockwise persistent currents in the qubit loop [dashed arrow in Fig. 1(a)], with energies controlled by the flux in the loop. The resonator mode of interest is the SQUID plasma mode, depicted by the two solid arrows in Fig. 1(a). The SQUID serves dual purposes: it acts as a tunable coupler between the qubit and the resonator, and it is also used as a sensitive magnetometer for qubit readout [23].

We have investigated two devices with similar layouts but slightly different parameters, both made of aluminum. Device A was designed and fabricated at MIT Lincoln Laboratory and device B was designed and fabricated at NEC. Figure 1(b) shows a spectroscopy measurement of device A versus applied flux, with the qubit flux detuning \( \Phi_{\text{qb}} \) defined as \( \Phi_{\text{qb}} = \Phi + \Phi_0/2 \) and \( \Phi_0 = h/2e \). The qubit frequency follows \( f_{\text{qb}} = \sqrt{\Delta^2 + \epsilon^2} \), where the tunnel coupling \( \Delta = 2.6 \text{ GHz} \) is fixed by fabrication and the energy detuning \( \epsilon = 2f_p\Phi_{\text{qb}}/h \) is controlled by the applied flux \( \Phi \) (\( f_p \) is the persistent current in the qubit loop). The resonator frequency is \( f_r \) around 2.3 GHz and depends only weakly on \( \Phi_{\text{qb}} \) and \( I_b \). In addition, there are features visible at frequencies corresponding to the sum and difference of the qubit and resonator frequencies, illustrating the coherent coupling between the two systems [3,24].

The system is described by the Jaynes-Cummings Hamiltonian [1,21,25]
Here, the drive amplitude $\varepsilon_{\text{mw}}$ experienced by the qubit becomes a combination of the drive $\varepsilon_{\text{direct}}$ due to direct coupling between antenna and qubit, and the drive $(\Delta \varepsilon / \partial I_b) I_{\text{mw}}$ mediated by the resonator. We get

$$\varepsilon_{\text{mw}} = \sqrt{\left[ \varepsilon_{\text{direct}}^{\text{mw}} + \cos \theta \left( \frac{\partial C_{\text{mw}}}{\partial I_b} I_{\text{mw}} \right) \right]^2 + \left[ \sin \theta \left( \frac{\partial \varepsilon_{\text{mw}}}{\partial I_b} I_{\text{mw}} \right) \right]^2},$$

(3)
where $\theta = \theta_d - \theta_r$ is the phase difference between the direct drive and the drive mediated by the resonator. The Rabi frequency due to the drive $e_{\text{mw}}$ depends on the qubit’s quantization axis, which changes with the static energy detuning $\varepsilon_{\text{dc}}$:

$$f_{\text{Rabi}} = \frac{\Delta}{2 \sqrt{\varepsilon_{\text{dc}}^2 + \Delta^2}}. \tag{4}$$

Fitting the data in Fig. 1(a) to Eqs. (3) and (4) allows us to extract the parameters $e_{\text{mw}}^\text{direct}$, $e_{\text{mw}}^\text{fr}$, and $\theta$. The different drive contributions are plotted together with the data in Fig. 2(a). The direct drive is independent of $I_b$, while the drive $I_{\text{mw}}^\text{fr}(\delta e/eI_b)$ mediated by the resonator increases linearly with $|I_b|$, which originates from the linear dependence of $g_1$ shown in Fig. 1(d). The minimum in Rabi frequency occurs at a value of $I_b$ slightly shifted from the point $I_b^\ast$ where $g_1 = 0$. This offset appears because of the phase difference $\theta$ between the two drive components. The fit gives $\theta = -75^\circ$, which is consistent with a resonator driven above its resonance frequency.

Figures 2(c) and 2(d) show how the two drive components depend on microwave frequency, measured by changing the static flux detuning $\Phi_{\text{qubit}}$ to increase the qubit frequency [see Fig. 1(b)]. The direct drive only depends weakly on frequency (due to cable losses), whereas the drive mediated by the resonator drops sharply as the qubit-resonator detuning increases. The black curve in Fig. 2(d) is the frequency response of a harmonic oscillator with $f_r = 2.3$ GHz and $Q = 100$, with amplitude normalized to match the data.

To further investigate how the presence of the resonator affects the qubit dynamics at large detunings, we performed measurements on device $B$. Figure 3(a) shows a spectrum of that device, where the qubit and the resonator mode ($f_r = 2$ GHz) are clearly visible. This device has a larger tunnel coupling ($\Delta = 5.4$ GHz), which allows us to operate the qubit at large frequency detuning from the resonator while still staying at $\varepsilon_{\text{dc}} = 0$, where the qubit, to first order, is insensitive to flux noise [28,29]. The qubit-resonator detuning corresponds to several hundred linewidths of the resonator, which is the regime of most interest for quantum information processing [12].

Figure 3(b) shows the Rabi frequency vs bias current $I_b$ of device $B$, measured at $f_{\text{qubit}} = \Delta = 5.4$ GHz and for two different values of the microwave drive current $I_{\text{mw}}^\text{antenna}$. Similarly to Fig. 2(a), the Rabi frequency clearly changes with $I_b$, but the dependence is weaker than in Fig. 2(a) because of the larger frequency detuning. Note that $f_{\text{Rabi}}$ scales linearly with $I_{\text{mw}}^\text{antenna}$ for all values of $I_b$. By fitting the data to Eqs. (3) and (4), we find $e_{\text{mw}}^\text{direct}/I_{\text{mw}}^\text{antenna} = 6.4$ MHz/µA, $I_{\text{fr}}/I_{\text{mw}}^\text{antenna} = 2.4$ nA/µA, and $\theta = -155^\circ$. The large phase difference $\theta$ for device $B$ causes the minimum in $f_{\text{Rabi}}$ to shift away from the point at $I_b^\ast = 5$ nA where the coupling $g_1 = 0$ [see Fig. 1(d)]. We attribute the large phase shift to influences from a second resonant mode, which is formed by the two $L$ and the two $C$ in the outer loop of Fig. 1(a) [17,30]. For sample $B$, this mode resonates around 5 GHz.

The results of Figs. 2 and 3 show that the microwave signal mediated by the resonator plays a significant role when driving the qubit, appearing already at moderate qubit-resonator coupling $g_1$ and persisting even when the two systems are far detuned. The design investigated here
allows the coupling to be turned off \( g_1 = 0 \) in Fig. 1(d)], but it comes with a drawback: the parameter used to control the coupling \( I_b \) in our setup also provides a way for low-frequency noise to enter the system. Consider the relation between the \( f_{\text{Rabi}} \) and \( I_b \) in Fig. 3(b): fluctuations \( \delta I_b \) near \( I_b = 0 \) will cause fluctuations in the amplitude of the drive field seen by the qubit, which will lead to dephasing during driven evolution.

To quantify the dephasing, we linearize the relation between the Rabi frequency and \( I_b \) close to \( I_b = 0 \) as \( f = f_0 [1 + r \delta I_b] \), where \( f_0 = f_{\text{Rabi}}(I_b = 0) \) and \( r = (\partial f_{\text{Rabi}}/\partial I_b) \). For \( f_0 = -1.28 \) (\( \mu \text{A} \)) \(^{-1} \) is given by Eqs. (3) and (4), or from Fig. 3(b). We model the fluctuations \( \delta I_b \) as normally distributed, with standard deviation \( \sigma_r \). Assuming the noise to be quasistatic, where the value of \( \delta I_b \) is constant during a single trial but differs from run to run \([31,32]\), we find that the Rabi oscillations decay as

\[
\int e^{-\delta I_b^2/(2\sigma_r^2)} \cos(2\pi f_0 [1 + r \delta I_b] t) d\delta I_b = e^{-2(\pi f_0 \sigma_r)^2} \cos(2\pi f_0 t).
\] (5)

In addition, the qubit energy relaxation time \( T_1 \) gives an exponential contribution to the Rabi decay, with time constant \( 4T_1/3 \) given by the Bloch equations. The Rabi decay also depends on the flux noise at the Rabi frequency, but this contribution can be disregarded when operating the qubit at \( \phi_{\text{dc}} = 0 \), where the qubit is insensitive to first-order flux noise \([33]\). The total decay envelope \( f(t) \) of the Rabi oscillations becomes

\[
f(t) = e^{-3(3T_1/4T_e)^2} e^{-t/T_e}, \quad \text{with} \quad T_e = 1/(\sqrt{2\pi f_0 \sigma_r}).
\] (6)

Note that the Gaussian decay constant \( T_e \) due to the effective amplitude fluctuations is inversely proportional to \( f_0 \), the average Rabi frequency. This is a consequence of having noise in the coupling between the qubit and the antenna; the effective amplitude fluctuations seen by the qubit will scale with the drive amplitude.

The red circles in Fig. 4(a) show the envelope of Rabi oscillations measured for \( f_{\text{Rabi}} = 65 \) MHz, together with a fit to Eq. (6). The qubit energy relaxation \( T_1 = 11.7 \) \( \mu \text{s} \) is known from separate experiments \([33]\), leaving \( T_e = 4.3 \) \( \mu \text{s} \) as a fitting parameter. In Fig. 4(b), we plot the Rabi decay time versus \( f_{\text{Rabi}} \), extracted from envelopes similar to Fig. 4(a). To capture both the exponential and the Gaussian decay, we plot the time \( T_e \) for the envelope to decrease by a factor \( 1/e \). For the lowest Rabi frequency, the decay time is within 25\% of the upper limit set by qubit relaxation, but it decreases with \( f_{\text{Rabi}} \), as expected from Eq. (6). The black solid line shows a fit to Eq. (6), giving a value of \( \sigma_r = 0.8 \) nA for the noise in \( I_b \). The effective fluctuations in the drive amplitude are \( r \sigma_r = 0.06 \% \). We cannot rule out that part of that noise may be caused by instrument imperfections.

Dephasing due to low-frequency fluctuations of the qubit frequency is routinely reduced by performing a Hahn-echo experiment \([34]\). Similarly, the fluctuations in drive field that cause dephase of the Rabi oscillations in Fig. 4(b) can be mitigated with the rotary-echo pulse sequence \([22]\), depicted in Fig. 4(c), which for driven systems is analogous to the Hahn-echo sequence. By shifting the phase of the drive by \( 180^\circ \) after a time \( t_p/2 \), any additional rotations, acquired due to slow fluctuations in the drive amplitude during the first half of the sequence, will cancel out during the reversed rotations in the second half of the sequence.

The blue squares in Fig. 4(a) show the decay of the rotary-echo sequence, measured for \( f_{\text{Rabi}} = 65 \) MHz. The rotary-echo data show a clear improvement compared to the Rabi decay for the same parameters [red circles in Fig. 4(a)]. We fit the rotary-echo data to Eq. (6) and plot the extracted decay times together with the results from Rabi measurements in Fig. 4(b). The rotary-echo signal outperforms the Rabi decay over the full range of Rabi frequencies, and reaches the upper limit set by qubit relaxation \((4T_1/3)\) at low frequencies. For intermediate frequencies, the rotary-echo decay times are slightly shorter than \( 4T_1/3 \); we attribute the reduced coherence times to fluctuations in \( I_b \) that occur on time scales comparable to the length of the pulse sequence. Noise at frequencies around \( 1/t_p \) will not be refocused by the reversed drive pulse, since the rotary-echo sequence has similar filtering properties as the Hahn echo \([22]\). At the highest drive amplitudes \((f_{\text{Rabi}} > 100 \) MHz\)), we observe a strong increase in decoherence, probably due to heating. The indirect driving can also be reduced by driving the qubit with two antennas with different amplitudes and phases \([35]\), or by directly applying a phase-shifted microwave signal to the SQUID bias line.
To conclude, we have investigated interference effects occurring when driving a qubit that is tunably coupled to a harmonic oscillator. Although the addition of a coupling control parameter opens up an extra channel for dephasing, we show that its influence is reversible with dynamical decoupling techniques. The results are relevant for any type of qubit that is tunably coupled to a resonator, and they show that despite engineering limitations, imperfections can be reversed by applying proper decoupling protocols. In analogy with multipulse Hahn-echo experiments, we expect the incorporation of additional rotary echoes to further improve the coherence times [33].

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