Search for a Heavy Particle Decaying to a Top Quark and a Light Quark in pp Collisions at $s=1.96$TeV

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<th>Citation</th>
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Detailed Terms
Search for a Heavy Particle Decaying to a Top Quark and a Light Quark in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

(CDF Collaboration)
We present a search for a new heavy particle \(X\) produced in association with a top quark, \(p\bar{p} \rightarrow t(X \rightarrow \bar{t}q)\) or \(p\bar{p} \rightarrow \bar{t}(\bar{X} \rightarrow \bar{t}q)\), where \(q\) stands for up quarks and down quarks. Such a particle may explain the recent anomalous measurements of top-quark forward-backward asymmetry. If the light-flavor quark (\(q\)) is reconstructed as a jet (\(j\)), this gives a \(j + j\) or \(t + j\) resonance in \(t\bar{t} + j\) events, a previously unexplored experimental signature. In a sample of events with exactly one lepton, missing transverse momentum and at least five jets, corresponding to an integrated luminosity of 8.7 \(\text{fb}^{-1}\) collected by the CDF II detector, we find the data to be consistent with the standard model. We set cross-section upper limits on the production (\(p\bar{p} \rightarrow Xt\) or \(X\bar{t}\)) at 95\% confidence level from 0.61 pb to 0.02 pb for \(X\) masses ranging from 200 \(\text{GeV}/c^2\) to 800 \(\text{GeV}/c^2\), respectively.

The standard model (SM) of particle physics has been extensively tested at the Tevatron collider and in initial results from the Large Hadron Collider. A deviation from predictions may provide a clue that reveals the solution to outstanding theoretical concerns with the SM, such as the hierarchy problem or the fermionic mass hierarchy. One area of particular interest is properties of the top quark, whose large mass suggests that it may play a special role in electroweak symmetry breaking [1]. The Tevatron experiments have performed detailed studies of the properties of the top quark. Recently, CDF reported a measurement of the top-quark production forward-backward asymmetry (\(A_{FB}\)) that is significantly larger than predicted by the SM [2], and is especially significant at large mass of the \(t\bar{t}\) system; D0 reported [3] a result consistent with the inclusive CDF measurement, but without a high-mass enhancement. Many models have been built to explain such a discrepancy, most involving the production of a new heavy mediating particle \(X\) that enhances \(A_{FB}\). Many of these models [4] predict significant enhancements of the \(t\bar{t}\) production cross section [5], the single-top production cross section [6], or the same-sign top-quark pair-production cross section [7,8], none of which have been confirmed in experimental tests.

One class of models [9,10] evades the same-sign top-quark limits by prohibiting the particle from acting as its own antiparticle, and can satisfy the \(t\bar{t}\) and single top-quark cross-section constraints for some coupling values. In addition, models in this class predict a new, unexplored experimental signature: the production of a heavy new particle \(X\) in association with a top quark (\(p\bar{p} \rightarrow Xt\) or \(p\bar{p} \rightarrow X\bar{t}\)) which decays via \(X \rightarrow tq\) or \(X \rightarrow \bar{t}q\). Since the light-flavor up quark or down quark \(q\) is reconstructed as a jet \((j)\), the final state is \(t\bar{t} + j\) with a resonance in the
$\bar{t} + j$ or $t + j$ system, which has not been previously examined. This Letter reports the first search for such a resonance.

We consider the production mode $p\bar{p} \rightarrow Xt \rightarrow \bar{t}q \rightarrow W^+bW^-\bar{b}q$ (and its conjugate $\bar{t}\bar{f}$ mode) in which one $W$ boson decays leptonically (including leptonic $\tau$ decays) and the second $W$ boson decays to a quark-antiquark pair. This decay mode features large branching ratios while reducing to a manageable level the backgrounds other than SM $\bar{t}\bar{t}$ production. Such a signal is similar to SM top-quark pair-production and decay, but with an additional jet coming from the $X$ resonance decay.

We analyze a sample of events corresponding to an integrated luminosity of $8.7 \pm 0.5$ fb$^{-1}$ recorded by the CDF II detector [11], a general purpose detector designed to study collisions at the Fermilab Tevatron $p\bar{p}$ collider at $\sqrt{s} = 1.96$ TeV. CDF has a charged-particle tracking system consisting of a silicon microstrip tracker and a drift chamber that are immersed in a 1.4 T magnetic field [12]. Electromagnetic and hadronic calorimeters surrounding the tracking system measure particle energies, and an additional system of drift chambers located outside the calorimeters detects muons.

Events enter this sample by satisfying online selection criteria (trigger), requiring an $e$ or $\mu$ candidate [13] with transverse momentum $p_T > 18$ GeV/c. After trigger selection, events are retained if the electron or muon candidate has a pseudorapidity $|\eta| < 1.1$ [14], $p_T > 20$ GeV/c and satisfies the standard CDF identification and isolation requirements [13]. We reconstruct jets in the calorimeter using the JETCLU [15] algorithm with a clustering radius of 0.4 in $\eta - \phi$ space, and calibrated using the techniques outlined in [16]. Jets are selected if they have transverse energy $E_T > 15$ GeV and $|\eta| < 2.4$. Missing transverse momentum [17] is reconstructed using fully corrected calorimeter and muon information [13].

The signature of $\bar{t}q \rightarrow W^+bW^-\bar{b}q \rightarrow \ell b\bar{q}b\bar{q}b\bar{q}$ (and the conjugate $\bar{t}\bar{f}$ mode) is a charged lepton ($e$ or $\mu$), missing transverse momentum, two jets from $b$-quarks and three jets from light quarks. We select events with exactly one electron or muon, at least five jets, and missing transverse momentum greater than 20 GeV/c. Since a signal would have two jets with $b$-quarks, we require (with minimal loss of efficiency) evidence of decay of a $b$-hadron in at least one jet. This requirement, called $b$-tagging, makes use of the SECVTX algorithm [18].

We model the production of $Xt$ and $X\bar{t}$ with $m_X = 200$–800 GeV/c$^2$ and subsequent decays with MADGRAPH [19]. Additional radiation, hadronization, and showering are described by PYTHIA [20]. The detector response for all simulated samples is modeled by the GEANT-based CDF II detector simulation [21].

The dominant SM background to the $\bar{t}\bar{t} + j$ signature is top-quark pair production with an additional jet due to initial-state or final-state radiation. We model this background using PYTHIA $\bar{t}\bar{t}$ production with a top-quark mass $m_t = 172.5$ GeV/c$^2$, compatible with the best current determination [22]. We normalize the $\bar{t}\bar{t}$ background to the theoretical calculation at next-to-next-to-leading order (NNLO) in $\alpha_s$ [23]. In addition, events generated by a next-to-leading order generator, MC@NLO [24] are also used in estimating an uncertainty in modeling the radiation of an additional jet.

The second largest SM background process is the associated production of a $W$ boson and jets. Samples of $W$ boson +jets events with light- and heavy-flavor jets are generated using the ALPGEN [25] program, interfaced with a parton-shower model from PYTHIA. The $W$ boson +jets samples are normalized to the measured $W$ boson production cross section, with an additional multiplicative factor for the relative contribution of heavy- and light-flavor jets, following the same technique utilized previously in measuring the top-quark pair-production cross section [18]. Multijet background, in which a jet is misreconstructed as a lepton, is modeled using a jet-triggered sample normalized to a background-dominated region at low missing transverse momentum where the multijet background is large.

The SM backgrounds due to single top quark and diboson production are modeled using MADGRAPH interfaced with PYTHIA parton-shower models and PYTHIA, respectively, and normalized to next-to-leading-order cross sections [26].

A signal may be observed as an excess of events above expectations from backgrounds in event distributions versus the mass of the $\bar{t}j$ system ($X \rightarrow \bar{t}j$) or the $\bar{t}j$ system ($X \rightarrow \bar{t}j$). In $\bar{t} + j$ events, we first identify the jets belonging to the $\bar{t}\bar{t}$ system using a kinematic fitter [27] to select from all available jets in the event the four jets most consistent with the $\bar{t}\bar{t}$ topology. In the fit, the top-quark and $W$-boson masses are constrained to be 172.5 GeV/c$^2$ and 80.2 GeV/c$^2$, respectively. All remaining jets are considered candidates for the light-quark jet in the $tj$ or $\bar{t}j$ resonance. These remaining jets each are paired with the

<table>
<thead>
<tr>
<th>Process</th>
<th>$\bar{t}\bar{t}$</th>
<th>$W + $jets</th>
<th>Total Bg.</th>
<th>$Xt + X\bar{t}$</th>
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<tr>
<td>Yield</td>
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<td>79</td>
<td>670</td>
<td>34</td>
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<tr>
<td>JES</td>
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<td>15%</td>
<td>16%</td>
<td>9%</td>
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<tr>
<td>Cross section</td>
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<td>30%</td>
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</tr>
<tr>
<td>$\bar{t}\bar{t}$ generator</td>
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<td>-</td>
<td>5%</td>
<td>-</td>
</tr>
<tr>
<td>ISR/FSR</td>
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<td>-</td>
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<td>4%</td>
</tr>
<tr>
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<td>5%</td>
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</tr>
<tr>
<td>Mult. interactions</td>
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</tr>
<tr>
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<td>2%</td>
<td>-</td>
</tr>
<tr>
<td>Total syst. uncert.</td>
<td>22%</td>
<td>39%</td>
<td>22%</td>
<td>11%</td>
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</tbody>
</table>
reconstructed top quark and anti-top quark, and the largest invariant mass of all such pairings is chosen as the resonance-mass reconstruction, $m_{t\bar{t}}$. Backgrounds, in which there is no resonance, give a broad and low distribution of $m_{t\bar{t}}$, while a signal would be reconstructed near the resonance mass.

We consider several sources of systematic uncertainty on the predicted background rates and distributions, as well as on the expectations for a signal. Each systematic uncertainty affects the expected sensitivity to new physics, expressed as an expected cross section upper limit in the no-signal assumption. The dominant systematic uncertainty is the jet energy scale (JES) [16], followed by theoretical uncertainties on the cross sections of the background processes. To probe the description of the additional jet, we compare our nominal $t\bar{t}$ model to one generated by MC@NLO and take the full difference as a systematic uncertainty. We also consider systematic uncertainties associated with the description of initial- and final-state radiation [27], uncertainties in the efficiency of reconstructing leptons and identifying $b$-quark jets, and uncertainties in the contribution from multiple proton interactions. In addition, we consider a variation of the $Q^2$ scale of $W$ boson + jet events in ALGPEN. In each case, we treat the unknown underlying quantity as a nuisance parameter and measure the distortion of the $m_{t\bar{t}}$ spectrum for positive and negative fluctuations of the underlying quantity. Uncertainties in the theoretical cross-section normalization are also included. Table I lists the contributions of each of these sources of systematic uncertainty to the yields. The dominant systematic shape uncertainty is from the JES.

We validate our modeling of the SM backgrounds in three background-dominated control regions. The $t\bar{t}$ background is validated in events with exactly four jets and at least one $b$-tag. We validate $W$ + jets backgrounds in events with exactly four jets and at least one $b$-tag. We validate $W$ + jets backgrounds in events (a) $W$ boson + jet control region: at least 5 jets, exactly zero $b$-tags.

(b) $t\bar{t}$ plus additional radiated jet control region: at least 5 jets, at least one $b$-tag, $H_T < 225$ GeV.

FIG. 1 (color online). Distribution of events versus reconstructed $ij$ or $ij$ invariant mass ($m_{ij}$) for observed data and expected backgrounds in two control regions. The lower panes give the relative difference between the observed and expected distributions; the hatched areas show the combined statistical and systematic uncertainties of the expected background.

FIG. 2 (color online). Distribution of events versus reconstructed $ij$ or $ij$ invariant mass, $m_{ij}$, for observed data and expected backgrounds in the signal region. Three signal hypotheses are shown, assuming a total cross section of 0.1 pb. The lower pane gives the relative difference between the observed and expected distributions; the hatched area shows the combined statistical and systematic uncertainties of the expected background.
the models that satisfy the observed anomalous excluded regions include some of the parameter space of coupling-mass space \([30]\), as shown in Fig. 4. The colored triplet particle \([9]\), and construct exclusion regions one where 

The observed limits are consistent with expectation in the background-only hypothesis \([29]\). We interpret the limits in terms of specific physics models. These limits constrain a small portion of the model parameter space. Analysis of collisions at the Large Hadron Collider may probe the remaining allowed regions.

In conclusion, we report on the first search for top-quark forward-backward production asymmetry \(A_{FB}\). For each accepted event, we reconstruct the resonance mass \((m_{ij})\), and find the data to be consistent with at least five jets and no \(b\) tags. Finally, modeling of SM \(t\bar{t}\) events with an additional jet is validated by examining a signal-depleted region with at least five jets, at least one \(b\) tag and \(H_{T}\), the scalar sum of lepton and jet transverse momenta, less than 225 GeV. As shown in Fig. 1, we find that the backgrounds are well modeled within systematic uncertainties.

Figure 2 shows the observed distribution of events in the signal region compared to possible signals and estimated backgrounds. We fit the most likely value of the sum of the \(Xt\) and \(Xt\rightarrow t\bar{t}j\) cross sections by performing a binned maximum-likelihood fit in the \(m_{ij}\) variable, allowing for systematic and statistical fluctuations via template morphing \([28]\). There is no evidence for the presence of top-quark + jet resonances in \(t\bar{t}j\) events, so we set upper limits on the combined production \((p\bar{p}\rightarrow Xt\text{ or }X\bar{t})\) at 95\% confidence level using the CLs method \([29]\). The observed limits are consistent with expectation in the background-only hypothesis \((\text{Fig. 3})\). We interpret the observed cross-section limit in terms of specific models, one where \(X\) is a color singlet particle and one where \(X\) is a colored triplet particle \([9]\), and construct exclusion regions in coupling-mass space \([30]\), as shown in Fig. 4. The excluded regions include some of the parameter space of the models that satisfy the observed anomalous \(A_{FB}\) and the production cross sections of the top quarks.

In conclusion, we report on the first search for top-quark + jet resonances in \(t\bar{t}j\) events. Such resonances are predicted by new physics models explaining the anomalous top-quark forward-backward production asymmetry \(A_{FB}\). For each accepted event, we reconstruct the resonance mass \((m_{ij})\), and find the data to be consistent with SM background predictions. We calculate 95\% C.L. upper limits on the cross section of such resonance production from 0.61 pb to 0.02 pb for \(X\) masses ranging from 200 GeV/c\(^2\) to 800 GeV/c\(^2\) and interpret the limits in terms of specific physics models. These limits constrain a small portion of the model parameter space. Analysis of collisions at the Large Hadron Collider may probe the remaining allowed regions.

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FIG. 3 (color online). Upper limits at 95\% C.L. on \(t\bar{t} + j\) production via a heavy new resonance \(X\), as a function of the resonance mass. Also shown are theoretical predictions \([9]\) assuming a unit coupling. The band around the theoretical predictions is a \(\pm 15\%\) PDF uncertainty.

FIG. 4 (color online). Excluded region in the space of resonance mass versus resonance coupling \((g_{FB})\) for two specific models, where the \(X\) particle is part of a new color singlet \((a)\) or color triplet \((b)\) resonance \([9]\), respectively. Also shown are regions \([30]\) which are consistent with the observed anomalous \(A_{FB}\) and constraints from top-quark pair production and single-top production cross-section measurements.
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[30] These regions simultaneously satisfy the observed high —

\[ m_t (> 450 \text{ GeV}) A_{fb}, \text{ low } - m_t (< 450 \text{ GeV}) A_{fb}, \text{ and the} \]

\( t\bar{t} \) cross-section better than the standard model. Mathematically, they are defined as the regions with \( \chi^2 < 2.8 \), where \( \chi^2 \) is defined in Equation 22 in Ref. [4]. \( \chi^2 \) for the standard model is 2.8.