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Regularizing GRAPPA using simultaneous sparsity to recover de-noised images

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ABSTRACT

To enable further acceleration of magnetic resonance (MR) imaging, compressed sensing (CS) is combined with GRAPPA, a parallel imaging method, to reconstruct images from highly undersampled data with significantly improved RMSE compared to reconstructions using GRAPPA alone. This novel combination of GRAPPA and CS regularizes the GRAPPA kernel computation step using a simultaneous sparsity penalty function of the coil images. This approach can be implemented by formulating the problem as the joint optimization of the least squares fit of the kernel to the ACS lines and the sparsity of the images generated using GRAPPA with the kernel.

Keywords: Magnetic resonance imaging, parallel imaging, compressed sensing, GRAPPA

1. INTRODUCTION

Cartesian magnetic resonance imaging (MRI) consists of sampling the 3-D spatial Fourier transform (k-space) by encoding trajectories corresponding to lines parallel to the “readout” direction in a uniformly spaced 2-D grid on the transverse plane, containing the orthogonal “phase-encode” directions. The acquisition time is proportional to the number and extent of the encoded k-space lines; the time is not affected by the sample spacing in the readout direction, since the sample spacing along the lines does not affect the total k-space trajectory length. Conventional imaging is limited by the tradeoff between spatial resolution, image size (field of view), signal-to-noise ratio (SNR), and acquisition time. For a $X \times Y \times Z$ mm$^3$ volume divided into voxels of $\Delta x \times \Delta y \times \Delta z$ mm$^3$, with a $k_x$ readout direction, the time $T$ is proportional to

$$T \propto N_{\text{avg}} \frac{YZ}{\Delta x \Delta y \Delta z},$$

and the SNR is proportional to

$$\text{SNR} \propto \sqrt{N_{\text{avg}} \Delta x \Delta y \Delta z}.$$ (2)

The image size is related to the spacing between k-space samples; e.g. $X = 1/\Delta k_x$. The spatial resolution is inversely proportional to the sampled k-space extent; e.g. $\Delta x \approx 1/(2k_{x,\text{max}})$. To reduce the acquisition time, several strategies exist: (1) reduce the extent of k-space that is sampled, resulting in lower spatial resolution; (2) increase the spacing between phase-encode lines, resulting in a smaller field of view and aliasing in the image domain if the object is too large; or (3) averaging fewer repetitions, decreasing the SNR of the result. Since a specific image resolution and noise level are required to resolve features of interest in most applications, we can accelerate the acquisition by reducing the field of view in the phase-encode directions (i.e. sampling k-space below the Nyquist spacing $\Delta k_x \times \Delta k_y \times \Delta k_z$ for the imaged object) and utilize accelerated parallel imaging$^1$ to recover un-aliased images. In this work, the GRAPPA$^2$ accelerated parallel imaging method is improved using a sparsity-regularized calibration step.

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1.1 Accelerated Parallel Imaging with GRAPPA

Rather than use a single uniform receiver coil to measure k-space, we leverage the nonuniform nature of a receive coil array\(^{3}\) to acquire a set of spatially weighted images simultaneously. The spatial variation of the sensitivities of these coils acts like an additional source of spatial encoding. If these sensitivities are known exactly, and the number of coils \(P\) is not less than the total undersampling factor \(R\), SENSE\(^{3}\) can recover the un-aliased image via inversion of a linear system. Alternatively, the SMASH theoretical model\(^{6}\) suggests linear combinations of coil sensitivities approximate complex exponentials, which modulate the image to form frequency-shifted k-space. A missing k-space frequency can be estimated from a linear combination of samples of a neighboring k-space frequency across coils. When coil sensitivity measurements are unavailable, both AUTO- SMASH\(^{6}\) and GRAPPA compute the coefficients used for forming these complex exponentials by fitting a set of Nyquist-sampled calibration data, called the ACS lines. GRAPPA goes further by estimating a missing k-space frequency from multiple neighboring k-space locations. Also unlike SENSE and SMASH, which reconstruct an un-aliased image (or k-space) for the object, GRAPPA fills the missing k-space locations in each coil separately, leaving the final combination of fully sampled k-space as a post-processing step. This combination can be achieved using a root-sum-squares combination of the coils’ images or another technique.

GRAPPA calibrates the set of weights used for estimating target (missing) k-space frequencies from neighboring source (acquired) k-space points by fitting those weights to a set of ACS lines. For 2-D accelerated 3-D GRAPPA, various strategies exist for calibrating the kernel;\(^{7}\) in this work, the 3-D volume is inverse transformed along the readout (fully-sampled) direction into slices of 2-D k-space, and each slice is calibrated and reconstructed separately. One limitation of this approach is the small set of ACS lines available for calibration, since the ACS lines occupy only a small region of the transverse plane, to limit the additional overhead of encoding these lines. To derive the kernel fit equations, we first investigate the 2-D GRAPPA reconstruction equation:

\[
y_q[k_y + r_y, k_z + r_z] = \sum_{p=1}^{P} \sum_{b_y=1}^{B_y} \sum_{b_z=1}^{B_z} g_{p,q,r_y,r_z}[b_y, b_z] y_p[k_y + \left( b_y - \left\lfloor \frac{B_y}{2} \right\rfloor \right) R_y, k_z + \left( b_z - \left\lfloor \frac{B_z}{2} \right\rfloor \right) R_z],
\]

where \(y_q[k_y, k_z]\) is the k-space value at frequency \([k_y, k_z]\) in the \(p\)th coil, the GRAPPA kernel \(g_{p,q,r_y,r_z}\) maps \(B_y \times B_z\) source points uniformly spaced \(R_y \times R_z\) apart from the \(p\)th coil to a target point frequency offset of \([r_y, r_z]\) in the \(q\)th coil’s k-space. Since this reconstruction operation can be expressed as a convolution over acquired data \([R_q k_y, R_z k_z]\), the GRAPPA reconstruction operation has the advantage of being computationally efficient even for large datasets.

Substituting points from ACS lines \(y^{ACS}\) for the data on both sides of Eq. (3), we can form a fit equation for calibrating the GRAPPA kernel \(g_{p,q,r_y,r_z}\):

\[
y_q^{ACS}[k_y + r_y, k_z + r_z] = \sum_{p=1}^{P} \sum_{b_y=1}^{B_y} \sum_{b_z=1}^{B_z} g_{p,q,r_y,r_z}[b_y, b_z] y_p^{ACS} \left[ k_y + \left( b_y - \left\lfloor \frac{B_y}{2} \right\rfloor \right) R_y, k_z + \left( b_z - \left\lfloor \frac{B_z}{2} \right\rfloor \right) R_z \right].
\]

Each kernel has \(B_y B_z\) weights, so the above equation has \(B_y B_z P\) unknowns; thus, assuming enough ACS lines, a least-squares fit can be performed for the kernel weights from at least \(B_y B_z P\) fit equations. The total area of source and target points for a single ACS fit is \(max((B_y - 1) R_y + 1, R_y) \times max((B_z - 1) R_z + 1, R_z)\), so if the ACS lines cover a region of size \(N_{ACS,k_y} \times N_{ACS,k_z}\), the total number of fits is

\[
N_{fits} = (N_{ACS,k_y} - max(B_y - 1, 1) R_y)(N_{ACS,k_z} - max(B_z - 1, 1) R_z).
\]

Collecting the ACS line points on the left and right sides of Eq. (4) for all \(N_{fits}\) fit equations yields the least-squares optimization problem

\[
G = \arg \min_{G} \| Y_{src}^{ACS} G - Y_{trg}^{ACS} \|^2_F,
\]

where each column of \(G\) is the set of GRAPPA kernels \(\{g_{p,q,r_y,r_z} : p \in \{1, \ldots, P\}\}\) for the target offset of \([r_y, r_z]\) in the \(q\)th coil, each row of \(Y_{trg}^{ACS}\) and \(Y_{src}^{ACS}\) represent the target points and source points for a fit from

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a different subset of the ACS lines, and $\|\cdot\|_F$ is the Frobenius norm. The solution of Eq. (6) is the calibrated set of GRAPPA kernels 

$$G = \left( (Y_{\text{src}}^{\text{ACS}})^H (Y_{\text{src}}^{\text{ACS}}) \right)^{-1} (Y_{\text{src}}^{\text{ACS}})^H Y_{\text{trg}}^{\text{ACS}},$$

where $[\cdot]^H$ is the complex conjugate transpose. When there are insufficient fit equations ($N_{\text{fits}} < B_y B_z P$), the matrix $(Y_{\text{src}}^{\text{ACS}})^H (Y_{\text{src}}^{\text{ACS}})$ is rank-deficient, and the calibrated kernel may not be correct.

### 1.2 Improving GRAPPA Kernel Calibration

Assuming the measurement noise in the ACS lines obeys the central limit theorem, the matrices $(Y_{\text{src}}^{\text{ACS}})^H (Y_{\text{src}}^{\text{ACS}})$ and $(Y_{\text{src}}^{\text{ACS}})^H (Y_{\text{trg}}^{\text{ACS}})$ are influenced less by that noise as the number of fits increases, and the calibrated kernel more accurately represents actual frequency shifts. Conversely, as the number of fits decreases, the calibrated kernel fits the noise more, and the resulting reconstruction quality will degrade. The improperly calibrated kernel can increase both noise amplification and coherent aliasing in the GRAPPA result. To counteract the poor fit when $N_{\text{fits}}$ is small, several regularization methods have been developed.

The first approach, known as Tikhonov regularization, has been applied successfully to SENSE. Similarly, this type of regularization can be applied to the GRAPPA kernel computation in Eq. (6):

$$G = \arg \min_G \frac{1}{N_{\text{Fro}}} \|Y_{\text{src}}^{\text{ACS}} G - Y_{\text{trg}}^{\text{ACS}}\|_F^2 + \alpha^2 \|\Gamma \text{vec}(G - G_0)\|_2^2,$$

where $N_{\text{Fro}} = \min(R_y R_z - 1, B_y B_z) P$ is a normalization constant for the Frobenius norm, $\Gamma$ and $G_0$ together represent prior information about the kernel, vec$(\cdot)$ stacks the columns of a matrix into a vector, and the tuning parameter $\alpha > 0$ balances the fidelity to the observations with the prior information. In this work, we impose a minimum energy prior on the kernel, using $\Gamma = I$, and $G_0 = 0$. The solution to this optimization problem is

$$G = \left( \frac{(Y_{\text{src}}^{\text{ACS}})^H (Y_{\text{src}}^{\text{ACS}})}{N_{\text{Fro}}} + \alpha^2 I \right)^{-1} (Y_{\text{src}}^{\text{ACS}})^H Y_{\text{trg}}^{\text{ACS}}.$$  

Another approach leverages the interpretation of a GRAPPA kernel as performing a frequency shift and constrains the kernel with the notion that applying the kernel multiple times should result in output matching shifted original data. In this work, we regularize the GRAPPA kernel calibration problem by promoting the sparsity of the resulting GRAPPA reconstruction, in the appropriate transform domain. For comparison with our proposed method, we mainly consider the first regularization approach, as it has a closed-form solution.

### 1.3 Outline

In Section 2, the proposed method regularizing the GRAPPA kernel calibration using sparsity is described. The implementation of the method with half-quadratic minimization and an iterative least-squares solver is detailed. Using real data, this proposed method is compared against GRAPPA alone and GRAPPA with Tikhonov regularization in Section 3. The proposed method and the experimental results are discussed in Section 4.

### 2. GRAPPA KERNEL REGULARIZATION USING SPARSITY

At high accelerations, the GRAPPA result displays significant noise amplification, which can be mitigated via post-processing the GRAPPA result for de-noising. Since a wide variety of MRI data is sparse in generic transform domains like the discrete wavelet transform (DWT), and the noise amplified by GRAPPA retains its incoherence in the image and wavelet domains, sparsity-based regularization is a prime candidate for regularization. Compressed sensing (CS) theory suggests that a signal with sufficient transform sparsity can be recovered from noisy undersampled data given a suitable measurement matrix and nonlinear reconstruction algorithm like the lasso. However, this work differs from conventional applications of CS in that the encoding is not random; rather it is uniformly undersampled Fourier encoding, which can introduce aliasing. Therefore, we rely on the ACS lines to constrain our GRAPPA kernel appropriately and mitigate aliasing. In addition, MRI datasets are not guaranteed to be strictly sparse, although most datasets are compressible; thus, we must take care not to over-sparsify our result, which would sacrifice image resolution.
The SpRING method\textsuperscript{17–19} utilizes a simultaneous sparsity penalty on the magnitude of the DWT of the images across the coils in conjunction with a GRAPPA fidelity term to de-noise the final result. However, due to the post-processing nature of the SpRING algorithm, the approach does not confront the source of the noise amplification, the GRAPPA kernel. In this work, the sparsity regularization is incorporated into the GRAPPA kernel calibration step, to produce a kernel that produces limited noise amplification.

Given the success of using a convex $\ell_1$ norm to promote sparsity, the hybrid $\ell_{1,2}$ norm is used to promote simultaneous sparsity:

$$\|W\|_{1,2} = \sum_{n=1}^{N} \|W_{n,1}, \ldots, W_{n,P}\|_2,$$

(10)

where the $p$th column of $W$ is the sparse transform representation of the image from the $p$th coil. For convenience, arrange $G$ so that each column contains all the weights for a given output coil ($G$ is now a $B_y B_z P (R_y R_z - 1) \times P$ matrix), and rearrange $Y\text{src}^\text{ACS}$ and $Y\text{trg}^\text{ACS}$ appropriately. Using the sparsity of the GRAPPA reconstruction as a regularizer, Eq. (6) becomes

$$G = \arg \min_{G} \frac{1}{N_{\text{Fro}}} \|Y\text{src}^\text{ACS} G - Y\text{trg}^\text{ACS}\|_F^2 + \lambda \|\Psi F^{-1} f(G, Y\text{acq})\|_{1,2},$$

(11)

where $\lambda$ is a tuning parameter, $\Psi$ is the sparsifying transform, $F^{-1}$ is the inverse Fourier transform, $f(G, Y\text{acq})$ is the GRAPPA reconstruction given the set of GRAPPA kernels $G$ and acquired data (including ACS lines) $Y\text{acq}$. Since the GRAPPA reconstruction operation is a convolution between the kernel and the data, the function $f(G, Y\text{acq})$ is an affine function of $G$; let $f^*\left(\cdot, Y\text{acq}\right)$ be its adjoint with respect to $G$. Since the adjoint of a convolution operation is convolution by the time-reversed conjugate of the convolution kernel, the adjoint is similarly easy to compute.

2.1 Solution using Half-Quadratic Minimization

To solve the convex optimization problem in Eq. (11), we turn to half-quadratic minimization.\textsuperscript{20,21} This iterative approach to solving certain types of optimization problems replaces the $\ell_{1,2}$ sparsity term with a weighted least-squares term, with the weights determined by the derivative of the penalty function. Since the $\ell_{1,2}$ function is not differentiable at the origin, we replace the $\ell_{1,2}$ norm with a smoothed relaxation $\|W\|_{1,2} \approx \sum_{n=1}^{N} \|W_{n,1}, \ldots, W_{n,P}, \epsilon\|_2$, for small $\epsilon > 0$. Each iteration of half-quadratic minimization requires solving the least-squares problem

$$G^t = \arg \min_{G} \frac{1}{N_{\text{Fro}}} \|Y\text{src}^\text{ACS} G - Y\text{trg}^\text{ACS}\|_F^2 + \lambda \frac{1}{2} \|\Delta^t\|_{1/2}^2 \Psi F^{-1} f(G, Y\text{acq})\|_F^2,$$

(12)

where $\Delta^t_{n,n} = 1/\|W^{t-1}_{n,1}, \ldots, W^{t-1}_{n,P}, \epsilon\|_2$ is a diagonal matrix, and $W^{t-1} = \Psi F^{-1} f(G^{t-1}, Y\text{acq})$ is the sparse transform of the coil GRAPPA reconstructed images using the previous iteration’s GRAPPA kernel set.

Differentiating Eq. (12) yields

$$\frac{2}{N_{\text{Fro}}} (Y\text{src}^\text{ACS})^H (Y\text{src}^\text{ACS} G - Y\text{trg}^\text{ACS}) + \lambda \frac{1}{N} f^* (F \Psi^H \Delta^t \Psi F^{-1} f(G, Y\text{acq}), Y\text{acq}) = 0.$$

(13)

For typical MRI datasets, this least-squares problem is too large to solve directly, so an iterative method called LSMR\textsuperscript{22} is used. This program improves upon LSQR\textsuperscript{23,24} by ensuring the residual norm of the system of normal equations for the least squares problem decreases monotonically and not just the residual norm of the original least-squares system. As for LSQR, to solve the linear system $A x = b$, LSMR requires only that we be able to compute $A$ and $A^H$ times a vector. For this example, using a vectorized version of the GRAPPA kernel set $G$ as $x$,

$$A = \sqrt{\frac{2}{N_{\text{Fro}}}} (I_P \otimes \frac{1}{\sqrt{\lambda}} (I_P \otimes ((\Delta^t)^{1/2} \Psi F^{-1} Y\text{conv}^\text{acs}))),$$

(14)

$$A^H = \sqrt{\frac{2}{N_{\text{Fro}}}} (I_P \otimes (Y\text{acs}^\text{src})^H) \frac{\sqrt{\lambda}}{N} (I_P \otimes ((Y\text{conv}^\text{acs})^H F \Psi^H (\Delta^t)^{1/2}))),$$

(15)
Figure 1. The (a) combined magnitude 2-D reference image is shown alongside the four-level ‘9-7’ DWT of the reference coil images. The $\ell_2$ norm across the coils of the sparse transform coefficients for the coil images is shown in (b). The image intensity in (b) is scaled up so that the sparse coefficient pattern is readily visible.

$$b = \left[ \sqrt{\frac{2}{N_{\text{pro}}} \text{vec}(I_P \otimes Y_{\text{ACS}}^{\text{trg}})} \right] - \sqrt{\lambda} (I_P \otimes ((\Delta')^{1/2} \Psi F^{-1})) \text{vec}(Y_{\text{fixed}}^{\text{acs}}),$$

where $f(G, Y^{\text{acs}}) = Y_{\text{conv}}^{\text{acs}} G + Y_{\text{acs}}^{\text{fixed}}$ is the affine representation of the $N \times P$ GRAPPA reconstruction for all the coils, where $Y_{\text{conv}}^{\text{acs}}$ is the convolution matrix and $Y_{\text{acs}}^{\text{fixed}}$ passes the acquired data through for the GRAPPA reconstruction.

### 2.2 Implementation for the Case of Many ACS Lines

When the set of ACS lines is very large (admittedly not the intended operating regime for the proposed method), the matrices $Y_{\text{src}}^{\text{acs}}$ and $Y_{\text{trg}}^{\text{acs}}$ may be prohibitive to store in memory. Alternatively, for any $A$, $B$, and $X$, note that

$$\|AX - B\|_F^2 = \text{tr}(X^H A^H A X - B^H AX - X^H A^H B + B^H B),$$

and

$$\|(A^H A)^{1/2} X - (A^H A)^{-1/2} A^H B\|_F^2 = \text{tr}(X^H A^H A X - B^H AX - X^H A^H B + B^H A (A^H A)^{-1} A^H B).$$

Since Eqs. (17)-(18) are equal up to a constant for all $X$, the Frobenius norm in Eq. (11) can be replaced:

$$G = \arg \min_G \frac{1}{N_{\text{pro}}} \|\tilde{Y}_{\text{acs}}^{\text{src}} G - \tilde{Y}_{\text{acs}}^{\text{trg}}\|_F^2 + \lambda \|\Psi F^{-1} f(G, Y^{\text{acs}})\|_1, \text{tr},$$

where $\tilde{Y}_{\text{acs}}^{\text{src}} = ((Y_{\text{acs}}^{\text{src}})^H Y_{\text{acs}}^{\text{src}})^{1/2}$, and $\tilde{Y}_{\text{acs}}^{\text{trg}} = (Y_{\text{acs}}^{\text{trg}})^{-1} (Y_{\text{acs}}^{\text{src}})^H Y_{\text{acs}}^{\text{trg}}$. Then, $\tilde{Y}_{\text{acs}}^{\text{src}}$ and $\tilde{Y}_{\text{acs}}^{\text{trg}}$ replace $Y_{\text{acs}}^{\text{src}}$ and $Y_{\text{acs}}^{\text{trg}}$ in Eqs. (14)-(16). For $N_{\text{fits}} \gg B_y B_z P$ or $N_{\text{fits}} \gg (R_y R_z - 1) P$, these matrices are much more space efficient. However, this modification only applies when $Y_{\text{acs}}^{\text{src}}$ has full column rank, so $Y_{\text{acs}}^{\text{src}}$ is invertible.

### 3. METHODS AND RESULTS

To compare the performance of the proposed method to GRAPPA alone and to GRAPPA kernel calibration using Tikhonov regularization, a 3-D dataset is acquired using an un-accelerated $T_1$-weighted MPRAGE sequence ($256 \times 256 \times 176$ sagittal; 1.0 mm isotropic voxels) on a Siemens Tim Trio 3 T system (Siemens Healthcare, Erlangen, Germany) and the vendor-supplied 32-channel head-coil receive array, for a total acquisition time of 8 minutes. In addition, a noise-only acquisition (no RF excitation) was performed to estimate the coil noise covariance matrix. This noise covariance matrix is used in conjunction with low-resolution coil sensitivity estimates generated from the block of ACS lines in the center of k-space, to form SNR-optimal coil combination weights, analogous to using unaccelerated SENSE. The volume was inverse Fourier transformed in the axial (readout) direction, and a slice in the transverse axial plane was extracted, cropped, and undersampled in both phase-encode directions (the fully-sampled slice is retained as a gold standard for comparison). As shown in Fig. 1,
the four-level ‘9-7’ DWT is selected as an appropriate sparsifying transform for the selected slice. The different reconstruction algorithms are implemented and run on this data in MATLAB (Natick, MA). The combined magnitude images from the reconstructions are compared visually using difference images and quantitatively using PSNR, although PSNR does not effectively capture localized errors, such as the misrepresentation of a tumor.

Several different comparisons are performed using different choices for the size of the ACS lines and the source kernel size, to demonstrate how regularization affects the reconstruction when the number of ACS fits and the number of kernel source points varies. Hypothetically, we expect regularization to become more important when the number of ACS fits is not much bigger than the number of GRAPPA source weights requiring calibration.

In the first experiment, we operate in a regime with an ACS fit of decent quality without regularization: the undersampled data is spaced four k-space lines apart in both directions ($R_y = R_z = 4$), and a GRAPPA kernel with source neighborhood $3 \times 3$ blocks in size is calibrated from a $36 \times 36$ Nyquist-sampled block of ACS lines. In this case, 784 ACS fits are used to calibrate 288 weights for each target set of GRAPPA kernels. The GRAPPA reconstructions using no regularization, Tikhonov regularization, and sparsity are shown in Fig. 2.

In this experiment, both Tikhonov regularization and promoting sparsity effectively reduce the noise amplification of the GRAPPA result, achieving over 2 dB improvement in PSNR and portraying noticeably less noise in the difference images. However, the difference between regularization types is not significant in this experiment. To achieve greater differentiation, we repeat the experiment with a larger kernel size or fewer ACS lines, to achieve a poorer initial fit.

In the second experiment, the kernel size is increased to cover a $4 \times 4$ block source neighborhood, increasing the number of calibrated weights per target set to 512, and decreasing the number of ACS fits to 576. The undersampling factor and number of ACS lines is kept the same as in the previous experiment. The results for GRAPPA with and without regularization are shown in Fig. 3.

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We first observe that the GRAPPA reconstruction without any regularization has greatly diminished in quality relative to the first experiment, displaying significantly more noise amplification than previously. Evidently, the addition of either type of regularization is sufficient for mitigating this increase in noise amplification, producing results of better quality than in the previous experiment (due to a larger kernel). The difference between both regularizations, however, remains nearly indistinguishable.

In the third experiment, the number of ACS lines is reduced to a $20 \times 20$ block of ACS lines, providing only 64 ACS fit equations to calibrate 512 source points. Since there are fewer fit equations than source points, the un-regularized GRAPPA kernel calibration in Eq. (7) would involve inverting a rank-deficient matrix. Thus we compare only the Tikhonov and sparsity-promoting regularization techniques in Fig. 4.

Unlike the second experiment, Tikhonov regularization cannot produce a GRAPPA kernel that properly un-aliases the image. However, the proposed method of sparsity-promoting regularization calibrates a GRAPPA
4. CONCLUSIONS

By regularizing the GRAPPA kernel calibration step, we are able to greatly improve the visual quality and noise level of the GRAPPA reconstruction. When the number of ACS lines suffices to calibrate the set of GRAPPA kernels, both Tikhonov and sparsity-promoting regularization effectively reduce the noise amplification due to the GRAPPA reconstruction method. Furthermore, when the number of ACS lines is smaller than the number of source weights to be calibrated, the proposed sparsity-promoting method yields a kernel that reconstructs in an un-aliased image. Thus, in addition to de-noising, the proposed method is effective at un-aliasing with fewer calibration lines than GRAPPA with Tikhonov or no regularization requires. In general, when designing an acquisition scheme for use with GRAPPA, we must balance the amount of undersampling with the number of ACS lines to achieve the greatest total acceleration while maintaining the desired contrast and effective resolution. Using Tikhonov regularization, we are able to slightly reduce the amount of ACS lines necessary via de-noising, and with the proposed method, we decrease the required number of ACS lines further. In all, the proposed sparsity-promoting GRAPPA kernel calibration method will enable faster parallel imaging acquisition, increasing the affordability, comfort level, and capabilities of MRI.

While this method is designed for the most common conventional acquisition scheme, namely uniform Cartesian undersampling of k-space, we would like to extend this method to nonuniform, including random, Cartesian and non-Cartesian undersampling, to leverage the desirable properties of those sampling patterns to improve
the reconstruction quality. For the nonuniform case, we would like to investigate the joint estimation problem of the GRAPPA kernel coefficients and the un-acquired k-space. It may be possible to alternatingly construct a GRAPPA kernel and estimate the k-space consistent with that kernel and the acquired data. In addition, a re-gridding operator could be introduced to accommodate non-Cartesian sampling patterns like radial and spiral trajectories.

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