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Dynamical Decoupling and Dephasing in Interacting Two-Level Systems

Simon Gustavsson,1 Fei Yan,2 Jonas Bylander,1 Fumiki Yoshihara,3 Yasunobu Nakamura,3,4,* Terry P. Orlando,4 and William D. Oliver1,5

1Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
2Department of Nuclear Science and Engineering, MIT, Cambridge, Massachusetts 02139, USA
3The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan
4Green Innovation Research Laboratories, NEC Corporation, Tsukuba, Ibaraki 305-8501, Japan
5MIT Lincoln Laboratory, 244 Wood Street, Lexington, Massachusetts 02420, USA

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We implement dynamical decoupling techniques to mitigate noise and enhance the lifetime of an entangled state that is formed in a superconducting flux qubit coupled to a microscopic two-level system. By rapidly changing the qubit’s transition frequency relative to the two-level system, we realize a refocusing pulse that reduces dephasing due to fluctuations in the transition frequencies, thereby improving the coherence time of the entangled state. The coupling coherence is further enhanced when applying multiple refocusing pulses, in agreement with our $1/f$ noise model. The results are applicable to any two-qubit system with transverse coupling and they highlight the potential of decoupling techniques for improving two-qubit gate fidelities, an essential prerequisite for implementing fault-tolerant quantum computing.

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A universal set of quantum gates, sufficient for implementing any quantum algorithm, consists of a two-qubit entangling gate together with single-qubit rotations [1]. However, fault-tolerant quantum computing with error-correcting protocols sets strict limits on the allowable error rate of each gate. Initial work focused on perfecting single-qubit gates [2–4], and in recent years there has been progress on characterizing two-qubit gate operations [5–7]. In superconducting systems, two-qubit gates have been implemented in a variety of ways, for example, through geometric couplings [8,9], tunable coupling elements [10,11], microwave resonators [12,13], or with microwave-induced interactions [6,14–17]. Regardless of the nature of the coupling, any variation of the qubit frequencies or in the coupling parameter during the two-qubit interaction leads to dephasing of the entangled state and puts an upper limit on the obtainable gate fidelity [5].

For single qubits, dephasing due to low-frequency fluctuations in the precession frequency is routinely reduced with refocusing techniques [18–20], originally developed in nuclear magnetic resonance [21]. In this Letter, we apply similar techniques to improve the coherence of an entangled state formed between a flux qubit and a microscopic two-level system (TLS). The refocusing pulse is implemented by rapidly changing the qubit frequency relative to the TLS, thereby acquiring a phase shift [22]. When the phase shift equals $\pi$, the pulse refocuses the incoherent evolution of the coupled qubit-TLS system, giving a fourfold improvement of its coherence time. We further prolong the decay times by applying multiple refocusing pulses [23,24], thus extending dynamical decoupling techniques [25] to correct for the dephasing of entangled states. The results are first steps towards implementing error-correcting composite gate pulses [26,27] and optimal control methods [28], schemes with strong potential for improving two-qubit gate operations.

We use a flux qubit [29], consisting of a superconducting loop interrupted by four Josephson junctions (see Ref. [24] for a detailed description of the device). The qubit’s diabatic states correspond to clockwise and counterclockwise persistent currents $\pm I_p$, with $I_p = 180$ nA. The inset of Fig. 1(a) shows a spectrum of the device versus external flux, with $\Phi_{\text{qb}}$ defined as $\Phi_{\text{qb}} = \Phi + \Phi_0/2$ and $\Phi_0 = \hbar/2\epsilon$. The qubit frequency follows $f_{\text{qb}} = \sqrt{\Delta^2 + \epsilon^2}$, where the tunnel coupling $\Delta = 5.4$ GHz is set by the design parameters and the energy detuning $\epsilon = 2\hbar I_p \Phi_{\text{qb}}/\hbar$ is controlled by the applied flux. The device is embedded in a SQUID, which is used as a sensitive magnetometer for qubit readout [18].

At $\Phi_{\text{qb}} = \Phi^* = \pm 4.15$ m$\Phi_0$, the qubit becomes resonant with a TLS [30]. The microscopic nature of the TLS is unknown, but studies of two-level systems in similar qubit designs show that the most likely origin is an electric dipole in one of the tunnel junctions [31]. Figure 1(a) shows a magnification of the region around $-4.15 \ h \Phi_0$, revealing a clear anticrossing with splitting $S = 76$ MHz. We describe the system using the four states $\{|0_g\}, \{|1_g\}, \{|0_e\}, \{|1_e\}\}$, where (0, 1) are the qubit energy eigenstates and (g, e) refer to the ground and excited states of the TLS. On resonance, $\{|1_g\}$ and $\{|0_e\}$ are degenerate and coupled by the coupling energy $hS$. To characterize the coupling, we use the pulse scheme depicted in Fig. 1(b) [32]. Starting with both qubit and TLS in their ground states ($|0_g\rangle$), we rapidly shift the flux to a position $\delta \Phi = \Phi_{\text{qb}} - \Phi^* = 1.2$ m$\Phi_0$, where the qubit frequency $f_{\text{qb}} = 6.3$ GHz is far detuned from the TLS. By applying a
microwave pulse, resonant with $f_{q\beta}$, we perform a $\pi$ rotation on the qubit and put the system in $|1g\rangle$. We then rapidly shift $\delta\Phi$ to a value close to zero, effectively turning on the interaction $S$, whereupon the system will oscillate between $|1g\rangle$ and $|0e\rangle$. After a time $\tau_i$, the interaction is turned off by shifting $\delta\Phi$ away from zero, and we measure the final qubit state by applying a readout pulse to the SQUID. Since the measurement outcome is stochastic, we repeat the sequence a few thousand times to acquire sufficient statistics to estimate the SQUID switching probability $P_{SW}$ and thereby the qubit state.

Figure 1(c) shows the qubit state after the pulse sequence, measured versus interaction time $\tau_i$ and flux detuning $\delta\Phi$. At $\delta\Phi = 0$ and for $\tau_i = 1/(2S) = 6$ ns, the pulse sequence implements an ISWAP gate between qubit and TLS, taking $|0e\rangle \rightarrow i|1g\rangle$ and $|1g\rangle \rightarrow i|0e\rangle$ [32–34]. The characteristic decay time of the oscillations is shown in Fig. 1(d). The oscillations persist the longest at $\delta\Phi = 0$; at this point, the decay time is $\sim 800$ ns. However, as $\delta\Phi$ is moved away from the optimal point, the decay time quickly decreases towards zero. We attribute the reduction in coupling coherence to low-frequency flux noise, present in all superconducting devices [35]. When $\delta\Phi \neq 0$, fluctuations in $\delta\Phi$ induce variations in the effective coupling frequency $f_{osc}$ [Fig. 1(e)], leading to dephasing of the entangled state.

For single qubits, dephasing due to low-frequency fluctuations of the qubit frequency can be reduced in a Hahn-echo experiment [21]. By applying a $\pi$ pulse after a time $t$ of dephasing, the qubit’s noise-induced evolution will reverse directions and refocus at time $2t$, provided that the fluctuations are slow on the time scale $2t$ [24,36]. Here, our goal is to extend such single-qubit refocusing techniques to the mitigation of noise in coupled systems with multiple qubits, which requires implementing refocusing pulses for entangled states. Note that the purpose here is to increase coherence times, as opposed to turning off unwanted couplings [37,38].

We start by describing the system’s dynamics. Following Refs. [30,31,34], we write the total Hamiltonian as $\hat{H} = \hat{H}_{qb} + \hat{H}_{TLS} + \hat{H}_{int}$, with $\hat{H}_{qb} = -(h/2)f_{q\beta}\hat{\sigma}_{q\beta}^\dagger\hat{\sigma}_{q\beta}$ and $\hat{H}_{TLS} = -(h/2)f_{TLS}\hat{\sigma}_{TLS}^\dagger\hat{\sigma}_{TLS}$ and with the interactions described by $\hat{H}_{int} = -(h/2)\hat{\phi}_{q\beta}\hat{\sigma}_{x}^\dagger\hat{\sigma}_{x}$. Here, $\hat{\sigma}_{q\beta}$ and $\hat{\sigma}_{TLS}$ are Pauli operators for the qubit, $\hat{\sigma}_{x}^\dagger\hat{\sigma}_{x}$ are TLS operators, and $f_{TLS}$ is the TLS frequency. To focus on the interactions between the qubit and the TLS, we restrict the discussion to the subspace spanned by the states $\{|1g\rangle, |0e\rangle\}$. The Hamiltonian becomes

$$\hat{H}_{sub} = -\frac{h}{2}(\delta\Phi \hat{\sigma}_{z}^\dagger + S\hat{\sigma}_{sub}),$$

(1)

where $\delta\Phi = -f_{TLS} - f_{q\beta}$ and $\hat{\sigma}_{sub} = \hat{\sigma}_{x}^\dagger\hat{\sigma}_{x}$ are subspace Pauli operators. The dynamics of Eq. (1) can be visualized on a Bloch sphere, with the north and south poles corresponding to $|1g\rangle$ and $|0e\rangle$, respectively, and with $S$ and $\delta\Phi$ representing the length of torque vectors along the $x$ and $z$ axes [see Fig. 2(b)]. The frequency of the coherent oscillations seen in Fig. 1(c) is then given by the effective coupling strength $f_{osc} = \sqrt{\delta f^2 + S^2}$,

(2)

which is plotted together with the data in Fig. 1(e).

With the coupling dynamics described by Eq. (1), we discuss the details of the refocusing sequence, shown in Figs. 2(a) and 2(b). The system is brought into the $\{|1g\rangle, |0e\rangle\}$ subspace by applying a $\pi$ pulse to the qubit [step I in Figs. 2(a) and 2(b)], followed by a nonadiabatic shift in $\delta\Phi$ to bring the qubit and TLS close to resonance. $|1g\rangle$ is not an eigenstate of the coupled system, so the interaction $S$ will cause the system to rotate around the $x$ axis, oscillating between $|1g\rangle$ and $|0e\rangle$. Low-frequency fluctuations in the effective coupling strength will cause the Bloch state vector to fan out (over many realizations of the experiment), and the system loses its phase coherence (step II).

The refocusing pulse is now implemented by applying a flux shift pulse that rapidly detunes the qubit and the TLS to $\delta f = 550$ MHz. With $|\delta f| \gg |S|$, the state vector is effectively rotating around the $z$ axis (step III) [19], and we realize a $\pi$ rotation by setting the pulse duration.
\[ \tau_{\text{refocus}} = 0.5/\delta f. \]

The system is then rapidly brought back into resonance (step IV), and the state vector continues to rotate around the \( x \) axis. The inhomogeneous broadening that caused the state vector to diffuse during the first interval \( \tau_1 \) will now realign them again. The refocusing is complete after a time \( \tau_2 = \tau_1 \) (step V). Figures 2(c) and 2(d) illustrate the result of the refocusing sequence. Without the refocusing pulse [Fig. 2(c)], the coherent oscillations between \( |1g\rangle \) and \( |0e\rangle \) decay almost completely after 100 ns. When inserting a refocusing pulse at \( \tau_1 = 100 \) ns, the oscillations start to revive, eventually forming an echo at \( \tau_2 = \tau_1 \).

The revival of phase coherence seen in Fig. 2(d) requires careful calibration of the refocusing pulse. Figure 3 shows an example of a calibration experiment, where we fix \( \tau_1 = 97 \) ns and \( \delta f = 550 \) MHz and measure refocused oscillations versus the refocusing time \( \tau_{\text{refocus}} \). The data show strong oscillations whenever the refocusing pulse rotates the state vector by an odd integer of \( \pi \), i.e., when \( \tau_{\text{refocus}} = (2n + 1) \times 0.5/\delta f, \) oscillations versus the refocusing discussed in Fig. 2(b).

The exponential decay constant \( \bar{T}_1 \) is due to energy relaxation, while \( T_{\nu,N} \) represents the dephasing with \( N \) refocusing pulses. At \( \Delta \Phi = 0 \), we measure a pure exponential decay with time constant \( \bar{T}_1 = 800 \) ns, which is shorter than the relaxation time of both the qubit (\( 7.45 \) µs) and the TLS (\( T_{1,\text{TLS}} = 1 \) µs). However, to get an expression for \( \bar{T}_1 \), we need to consider all possible absorption or emission rates in the full four-level system [40]. In the relevant situation \( hS \ll k_B T \ll h\nu_{\text{TLS}}, h\nu_{\text{qb}} \), we have

\[ 1/\bar{T}_1 = \frac{1}{2}(1/\nu_{\text{qb}} + 1/T_{1,\text{TLS}}) + \frac{1}{2}(\Gamma_+ + \Gamma_-), \]

where \( \Gamma_\pm (\Gamma_\pm) \) represents relaxation (excitation) between the two energy eigenstates \( |\pm\rangle = (|0g\rangle \pm |1e\rangle)/\sqrt{2} \) of Eq. (1), with energy splitting \( h\nu_{\text{osc}} \). The polarization rate \( \Gamma_+ + \Gamma_- = S_L(f_{\text{osc}})/2 \) depends on the noise power \( S_L \) that couples transversely to the diagonalized subspace Hamiltonian in Eq. (1), which for \( \Delta \Phi = 0 \) corresponds to fluctuations \( S_{\delta\nu}/(f_{\text{osc}}) \) in the frequency detuning \( \delta f \) [20]. Using Eq. (4) and the measured values of \( \bar{T}_1, \nu_{\text{qb}}, \) and \( T_{1,\text{TLS}} \), we get \( S_{\delta\nu}(f = 76 \) MHz\( ) = 2.8 \times 10^6 \) rad/s. We cannot distinguish whether this noise comes from fluctuations in \( f_{\text{qb}}, f_{\text{TLS}}, \) or a combination thereof, but we note that the measured value is a few times larger than fluctuations in \( f_{\text{qb}} \) expected from flux noise. From independent measurements of the flux noise power \( S_{\Phi_{\text{qb}}} \) in the same device, we have \( S_{\Phi_{\text{qb}}} = S_{\Phi_{\text{qb}}} (\delta f_{\text{qb}}/\delta \Phi_{\text{qb}})^2 = 1.1 \times 10^6 \) rad/s at \( f = 76 \) MHz and \( \Phi_{\text{qb}} = -4.15 \) mG. 

Away from \( \Delta \Phi = 0 \), the decay envelope becomes Gaussian, and we extract the dephasing time \( T_{\nu,N} \) by fitting the data to Eq. (3), assuming a constant relaxation time \( \bar{T}_1 = 800 \) ns. The extracted decay times versus flux \( \Delta \Phi \)
FIG. 4 (color online). (a) Decay times of the coherent oscillations, measured with \((N = 1)\) and without \((N = 0)\) a refocusing pulse. At \(\delta \Phi = 0\), the decay is limited by energy relaxation, but the coherence times decrease away from \(\delta \Phi = 0\) due to increased sensitivity to flux noise. For large \(|\delta \Phi|\), the refocusing sequence improves the decay time by more than a factor of 4. The inset shows examples of the decay envelope \(h(t)\), measured at \(\delta \Phi = -60 \mu \Phi_0\). For the \(N = 1\) case, we use \(\tau_1 = \tau_2 = t/2\). (b) Decay times of multipulse refocusing sequences, showing the improvement coherence as the number of pulses increases. (c), (d) Time evolution of the qubit-TLS system, measured with and without refocusing pulses. Note the echo signals appearing after each refocusing pulse, giving a strong enhancement of the coherence time. The data are taken at \(\delta \Phi = -84 \mu \Phi_0\).

are shown in Fig. 4(a). The refocusing sequence gives considerably longer decay times over the full range of the measurement except around \(\delta \Phi = 0\), where the decay is limited by \(T_1\). Note that, to capture both the exponential and the Gaussian decay, we plot the time \(T_e\) for the envelope to decrease by a factor of 1/e. Examples of decay envelopes together with fits are shown in the inset of Fig. 4(a), measured with and without a refocusing pulse at \(\delta \Phi = -60 \mu \Phi_0\).

We model the decreased phase coherence away from \(\delta \Phi = 0\) in terms of flux noise. In analogy with coherence measurements on single flux qubits [42], we assume \(1/f\)-type fluctuations in \(\Phi\), with noise spectrum \(S_\phi(\omega) = A_\phi/|\omega|\). The flux noise couples to the oscillation frequency \(f_{osc}\) through Eq. (2), leading to the dephasing rate

\[
1/T_{\varphi,N} = 2\pi\sqrt{c_n A_\phi} |\partial f_{osc}/\partial \Phi|.
\]

Here, \(c_n = 0 = \ln(1/\omega_{low} t)\) and \(c_{N=1} = \ln(2)\) relate to the filtering properties of the pulse sequence [20,24], with the low-frequency cutoff \(\omega_{low}/2\pi = 1\) Hz fixed by the measurement protocol. The solid lines in Fig. 4(a) are fits to Eq. (5), using a single fitting parameter \(A_\phi = (1.4 \mu \Phi_0)^2\).

This amount of flux noise is consistent with previous results [24,42].

The overall good agreement between Eq. (5) and the data verifies the noise model and further confirms the validity of the refocusing sequence. However, for the range \(\delta \Phi > -30 \mu \Phi_0\), the refocused data show slightly lower coherence times than expected from the model. We attribute this to the finite rise time of our shift pulses. The refocusing sequence requires the frequency sweep rate \(\partial f/\partial t\) to be fast compared to the interaction time scale \(1/\Sigma \approx 10\) ns (to make the shifts nonadiabatic) but slow compared to the qubit precession time \(1/f_{\phi q} \approx 0.2\) ns (to avoid driving the system out of the \{|1g\}, \{0e\}\} subspace). The constraints can be phrased in terms of the probability of undergoing Landau-Zener transitions, giving \(S_\phi \ll \partial f/\partial t \ll f_{\phi q}^2\) [43]. Using pulses with 1.5 ns Gaussian rise time, we have \(\partial f/\partial t = 550\) MHz/1.5 ns = (606 MHz)^2, and on average the constraints are well fulfilled. However, \(\partial f/\partial t\) is lower during the slowest parts of the shift pulse (the beginning and the end), and artifacts due to imperfect nonadiabaticity appear when these parts of the pulse occur where the effective coupling is the strongest (at \(\delta \Phi = 0\)). The limited nonadiabaticity is also the reason for the slight asymmetry around \(\delta \Phi = 0\) in Fig. 1(c) [22].

We now extend the refocusing technique to implement dynamical decoupling protocols with multipulse sequences. For \(1/f\)-type noise, it has been shown that the Carr-Purcell sequence [23], consisting of equally spaced \(\pi\) rotations, improves coherence times by filtering the noise at low frequencies [24,36,44,45]. Figures 4(c) and 4(d) show the coherent evolution of the system when repeatedly applying refocusing pulses. Echo signals form between each pair of \(\pi\) pulses, giving considerable longer coherence times compared to the \(N = 0\) case. The increase in decay time with the number of refocusing pulses \(N\) is plotted in Fig. 4(b), measured for a few different values of \(\delta \Phi\). The improvement is consistent with the filtering properties of the pulse sequence; for larger \(N\), the filter cutoff frequency increases and, since the noise is of the \(1/f\) type, the total noise power leading to dephasing is reduced.

To summarize, we have implemented refocusing and dynamical decoupling techniques to correct for noise and improve the lifetime of entangled two-level systems. Although implemented between a flux qubit and a microscopic two-level system, the method applies to any transversely coupled spin-1/2 systems where the relative frequency detuning can be controlled. We expect the findings to be of importance when developing decoupling techniques to improve two-qubit gate fidelities.

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*Present address: Research Center for Advanced Science and Technology (RCAST), University of Tokyo, Komaba, Meguro-ku, Tokyo 153-8904, Japan.


