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Quasielastic \((e,e')p\) reaction on \(^{12}\text{C}\), \(^{56}\text{Fe}\), and \(^{197}\text{Au}\)

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We report the results from a systematic study of the quasielastic \((e,e')p\) reaction on \(^{12}\text{C}\), \(^{56}\text{Fe}\), and \(^{197}\text{Au}\) performed at Jefferson Lab. We have measured nuclear transparency and extracted spectral functions (corrected for radiation) over a \(Q^2\) range of 0.64–3.25 (GeV/c)^2 for all three nuclei. In addition, we have extracted separated longitudinal and transverse spectral functions at \(Q^2\) of 0.64 and 1.8 (GeV/c)^2 for these three nuclei (except for \(^{197}\text{Au}\) at the higher \(Q^2\)). The spectral functions are compared to a number of theoretical calculations. The measured spectral functions differ in detail but not in overall shape from most of the theoretical models. In all three targets the measured spectral functions show considerable excess transverse strength at \(Q^2 = 0.64\) (GeV/c)^2, which is much reduced at 1.8 (GeV/c)^2.

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INTRODUCTION

The value of studying electronuclear reactions has long been recognized. In such studies the entire nucleus is accessed via a well-understood interaction. A new avenue of investigations has been opened up with the completion of the continuous beam, multi-GeV electron accelerator at the Thomas Jefferson National Accelerator Facility, also known as Jefferson Lab (JLab). The present paper reports results from the first experiment done at this facility, which is a study of \((e,e'p)\) reactions in the quasielastic region. This experiment utilized one of the advantages of electron scattering, namely, that the transferred energy and momentum can be varied separately, and one of the main features of JLab, namely, the high-intensity continuous electron beam of CEBAF which makes it possible to do coincidence measurements orders of magnitude more extensive than could be done previously.

The simplest model of a nucleus is one of independent nucleons populating the lowest available shell-model orbits. In a simple picture of \(e-p\) scattering within a nucleus, the electron scatters from a single proton which is moving due to its Fermi momentum. The struck proton may then interact with the residual \(A-1\) nucleons before leaving the nucleus. Of course, neither the nucleus nor the scattering process are this simple and the deviations from these simple pictures reveal much about nuclei and their constituents, both real and virtual. The present experiment consisted of measuring proton spectra in coincidence with inelastically scattered electrons with the energy of the electrons chosen so as to be in the “quasielastic” region, i.e., at energies corresponding to scattering from single off-mass-shell nucleons. The spectra were taken in an angular region about the “conjugate” angle, i.e., the angle for scattering from stationary nucleons, over an angular range sufficient to cover the smearing of the two-body kinematics caused by the Fermi momentum of the confined protons. Data were taken over the range \(0.64 < Q^2 < 3.25\) (GeV/c)^2 where \(Q^2\) is the square of the four-momentum transferred to the struck proton.

For an electron knocking a proton \(p\) out of a nucleus \(A\) with energy transfer \(\omega\) and (three) momentum transfer \(\vec{q}\) leaving a scattered proton \(p'\) and a residual nucleus \(A-1\), two important kinematic quantities are the missing energy

\[
E_m = \omega - T_{p'} - T_{A-1}
\]

and missing momentum

\[
\vec{p}_m = \vec{p}_{p'} - \vec{q},
\]

where \(T_{p'}\) and \(T_{A-1}\) are the kinetic energies of the knocked-out proton and recoiling nucleus, respectively. The spectral functions were extracted from the \(E_m\) and \(\vec{p}_m\) spectra and compared to a variety of theoretical calculations. The total \((e,e'p)\) yields are obtained by integrating over the spectral functions and the transparencies then determined by comparing these yields with those predicted by plane wave impulse approximation (PWIA) calculations. Because the PWIA does not allow for final-state interactions the ratio of measured to calculated yield should just be the fraction of outgoing protons which do not suffer a final-state interaction and this is what is defined to be the transparency. Determinations of nuclear transparencies using the \((e,e'p)\) reaction have been reported for a range of targets covering the periodic table, at Bates for \(Q^2=0.34\) (GeV/c)^2 [1], at SLAC for \(Q^2\) between 1 and 7 (GeV/c)^2 [2,3], and more recently at JLab between 3 and 8.1 (GeV/c)^2 [4]. The present work maps out regions not previously covered and is of greater statistical accuracy. Longitudinal-transverse (L-T) separations were performed at two values of \(Q^2\) from which the first reported extensive separated spectral functions are obtained. Some transparency results from the present experiment have been previously published [5], as have the separated spectral functions for carbon [6].

The differential cross section for elastic electron-proton scattering is given by the well-known Rosenbluth formula:

\[
\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{Q^2}{|q|^2} \left[G_E^2(Q^2) + \tau \epsilon^{-1} J_M^2(Q^2)\right],
\]

where \(\left(d\sigma/d\Omega\right)_{\text{Mott}}\) is the differential cross section for the scattering of an electron off a unit point charge, \(\epsilon = 1/[1 + 2(1 + \alpha \tan^2(\theta/2))]\) is the virtual polarization parameter, \(\tau = |q|^2/Q^2 - 1\), \(G_E\) is the proton electric form factor, and \(G_M\) is the proton magnetic form factor in units of the nuclear magneton e\(h/2M_p\) where \(M_p\) is the proton mass.

The L-T separation is performed by measuring the cross section at different values of \(\epsilon\) while keeping \(Q^2\) constant, thus permitting the extraction of \(G_E\) and \(G_M\).

In scattering from a nucleus the cross section is expressed in terms of four response functions and in the PWIA the coincidence \((e,e'p)\) cross sections can be written as
\[
\frac{d^2 \sigma}{dE_e d\Omega_e dE_p d\Omega_p} = \rho' E'_p \sigma_{\text{Max}} \left[ \lambda^2 W_L(q, \omega) + \left( \frac{\lambda}{2} + \tan^2 \left( \frac{\theta}{2} \right) \right) W_L(q, \omega) \cos(\phi) \right] dE_e d\Omega_e dE_p d\Omega_p, \]

where \(\lambda = Q^2 / |q|^2\), \(\theta\) is the scattering angle, and \(\phi\) is the azimuthal angle between the scattering plane and the plane containing \(\vec{q}\) and \(\vec{p}'\).

The physics of interest is contained in the four response functions \(W_L, W_T, W_{LT},\) and \(W_{TT}\). Both of the interference terms, \(W_{LT}\) and \(W_{TT}\), are proportional to \(\sin \gamma\), where \(\gamma\) is the angle between the scattered proton and the transferred momentum \(\vec{q}\). Therefore, when measurements are made along \(\vec{q}\), i.e., in “parallel kinematics,” the interference terms are absent. Varying the incident energy makes it possible to vary \(\theta\) at constant \(q\) and \(\omega\) and thus disentangle \(W_L\) and \(W_T\), that is, perform an \(L-T\) separation. Although, the position of the spectrometers allowed measurements only in the scattering plane, the interference term \(W_{LT}\) could be investigated by varying the proton angle about the direction of \(\vec{q}\). Measurements were taken by varying both \(\theta\) and \(\gamma\). This is the first \(L-T\) separation measured for quasielastic \(e,e'p\) scattering that covers a large range in both \(A\) and \(Q^2\).

The free electron-proton elastic cross sections play a central role in interpreting much of the data in the present work. They are used in the determination of nuclear transparency, in the extraction of the separated spectral functions, as well as in the determination of the spectrometers’ acceptances. Unless otherwise noted, the proton electric form factor \(G_E\) is taken to have the dipole form,

\[
G_E = \left( 1 + \frac{Q^2}{0.71} \right)^{-2},
\]

and the magnetic form factor \(G_M\) is taken from the Gari-Krümpelmann [7] parametrization, which, to a good approximation [better than 4% for \(Q^2 < 2.0\) (GeV/c)^2] [8], yields \(G_M = \mu_p G_E\). Rosenbluth separation measurements of \(e-p\) scattering [9,10] support the validity of this relationship.

In recent years a series of experiments have been performed [11,12] in which the ratio \(\mu_p G_E/G_M\) is determined by measuring the direction of the polarization vector of the protons scattered by a longitudinally polarized electron beam. In contrast to the Rosenbluth separation results, these experiments find \(G_E/G_M\) decreasing approximately linearly with \(Q^2\) out to at least \(Q^2 = 5.6\) (GeV/c)^2. The ratio \(\mu_p G_E/G_M\) was found to be approximately 0.79 at 1.8 (GeV/c)^2, where one of the present \(L-T\) separations was performed. At the other momentum transfer where an \(L-T\) separation was performed, 0.64 (GeV/c)^2, this ratio was found to be about 5% less than the \(Q^2 = 0\) value of unity. The implications of the polarization transfer results for the present work are discussed in the section on \(L-T\) separations.

**EXPERIMENT**

**Electron beam**

The experiment was performed in 1995-1996 in Hall C at JLab and was the first experiment performed at the Laboratory. Data were taken at (nominal) electron energies \(E_e = (0.8N + 0.045)\) GeV with \(N = 1-4\) representing the number of “passes” the electrons made around the accelerating track. The absolute beam energy was determined at one-pass by two independent methods. One method (described in detail in Ref. [13]) is to use the inelastic scattering to an excited state whose energy is accurately known to calibrate the dispersion of a spectrometer and then use the calibrated spectrometer to measure the energy of the scattered electron as a function of nuclear target mass. For these measurements a carbon target was used and the dispersion determined by measuring the difference in position of the electrons scattered to the ground and the 4.438 91±0.000 31 MeV [14] first excited state. A BeO target was then substituted and the energy of the beam, \(E\), determined using the formula

\[
\Delta E_{\text{recoil}} = 2E^2 \sin^2 \frac{\theta}{2} \left( \frac{1}{M_1} - \frac{1}{M_2} \right). \tag{6}
\]

One can accurately determine \(\Delta E_{\text{recoil}}\) because once the dispersion has been accurately measured the only unknown in Eq. (6) is the beam energy \(E\). This procedure was repeated for several values of the spectrometer magnetic field. With both targets a small correction was made for the energy loss of the electrons in the target.

The other method is to determine the angle of the diffraction minimum for scattering to a state where the position can be accurately calculated. The minimum for scattering to the \(^{12}\)C ground state is known to be at \(Q^2 = 0.129\) (GeV/c)^2 [15]. The (four) momentum transfer can be written as

\[
Q^2 = 4EE' \sin^2 \frac{\theta}{2} \quad E' = \frac{E}{1 + \frac{2E^2 \sin^2 \frac{\theta}{2}}{M}} \tag{7}
\]

where \(M\) is the mass of the scattering nucleus and \(\theta\) is the electron scattering angle. An improvement in accuracy in the measurement of \(Q\) is obtained by using the ratio of elastic scattering to inelastic scattering. Again, then, the only unknown is the incident electron energy \(E\). The two methods agreed to 1 part in 2000 and the absolute energy determination using these methods is believed to be accurate to 10^{-3}. These methods become less feasible as the energy is increased. The beam energy can also be determined by measuring the energy and angle of the scattered particles in electron-proton elastic scattering. Because of the uncertainties in the angle and momentum measurements this method is less accurate than the other two but has the advantage that it can be used over the entire range.
of incident electron energies. Elastic $e\cdot p$ scattering was used to measure the energy of the three-pass beam with an uncertainty of 1 part in 500. Beam energy was also determined by measuring the magnetic field needed to bend the beam around the Hall C arc. The energy calibration as well as other aspects of the experiment are discussed more completely elsewhere [16].

Beam currents of 10 to 60 $\mu$A were used. The currents were monitored by three microwave cavities that were installed for this purpose in the Hall C beam line [17]. The absolute calibration was performed by comparison with an Unser cavity, which is a parametric dc current transformer with very stable gain but a drifting offset which was determined as part of our daily calibration procedure. The overall accuracy in the beam current measurement was $\pm 1\%$.

**Targets**

Data were taken with $\approx 200$ mg/cm$^2$ C, Fe, and Au targets mounted on a steel ladder in an aluminum scattering chamber. The target thicknesses were determined to about 0.1%. The $e\cdot p$ elastic scattering data used for calibration were taken using the 4.0 cm cell of the Hall C cryogenic target [18]. During the early part of the experiment, before the cryogenic target was available, some data were taken with a solid CH$_2$ target but these data were only used to check some kinematic offsets. The compositions of hydrocarbon targets are subject to change under beam irradiation and therefore all the calibration data were taken with the liquid hydrogen target. The cryogenic targets are also mounted on a ladder with both ladders contained in the aluminum scattering chamber. The 123.0 cm diameter scattering chamber has entrance and exit snouts for the beam and several pumping and viewing ports. The particles that went to the high-momentum spectrometer (HMS) exited through a 0.4 mm aluminum window and those to the short orbit spectrometer (SOS) through a 0.2 mm aluminum window. For both spectrometers the particles had to pass through about 15 cm of air before entering the spectrometer.

**Spectrometers**

Data were taken with the HMS and the SOS in coincidence. This experiment served as the commissioning experiment for these spectrometers. The HMS detected the electrons and the SOS the protons, except at the highest $Q^2$ where the roles of the spectrometers were reversed.

**High-momentum spectrometer**

The HMS is a $25^\circ$ vertical bend spectrometer made up of superconducting magnets in a $QOOQ$ configuration. The dipole field is monitored and regulated with an NMR probe and kept constant at the $10^{-4}$ level. The spectrometer rotates on a pair of rails between 12.5° and 90° with respect to the beam line. The HMS maximum central momentum is 7.3 GeV/c and in preparing for the present experiment the spectrometer was tested up to 4.4 GeV/c, although the highest setting at which data were taken was 2.6 GeV/c. The usable momentum bite of the spectrometer is $\approx 20\%$. A momentum resolution ($\sigma$) of $< 1.4 \times 10^{-3}$, and an in-plane (out-of-plane) angular resolution of 0.8 (1.0) mrad was achieved for the HMS. With no collimator in place the solid angle subtended for a point target is 8.1 msr. A 6.35 cm thick HEAVYMET (machinable Tungsten alloy, 10% Cu and Ni; density=17 g/cm$^3$) collimator with a flared octagonal aperture limited the solid angle to 6.8 msr. The higher momentum particles were usually detected in the HMS and except at the backward (electron) angles these were the electrons. Detailed information about the HMS can be found in Ref. [19].

**Short orbit spectrometer**

The SOS consists of three (normal conducting) magnets in a $QDD$ configuration. The deflection is vertical with the net bend of 18° at the central momentum. The magnetic fields are monitored with Hall probes. With its short path length of 11 m this spectrometer is particularly well suited for detecting short-lived particles, though obviously this attribute was not used in the present experiment. The spectrometer can be moved between 13.1° and 168.4° with respect to the beam line (in this experiment the minimum angle was 14.5°) and can be moved up to 20° out of the horizontal plane, though this was not done in this experiment. The spectrometer maximum central momentum is 1.8 GeV/c with a nominal momentum bite of 40%. A momentum resolution ($\sigma$) of $< 1.0 \times 10^{-3}$, an in-plane (out-of-plane) angular resolution of 4.5 (0.5) mrad was achieved for the SOS. The solid angle subtended is $\approx 9$ msr for a point target, although a collimator similar to that used with the HMS limited the solid angle to 7.5 msr. As with the HMS, further details about the SOS can be found in the spectrometer documentation [19,20].

**Detector stacks**

The detector stacks in the two spectrometers are virtually identical. The particles pass through, in order, a set of drift chambers, a pair of hodoscopes, a gas Čerenkov detector, another pair of hodoscopes, and then a lead-glass calorimeter. The particle velocity is inferred from the time of flight between the two pairs of hodoscopes, though the spectra proved to be so clean that it was not necessary to use time of flight for particle identification. Signals from the hodoscope planes provide the trigger and in the electron arm particle identification can be incorporated into the trigger by requiring a signal from the Čerenkov counter and/or a sufficiently large pulse from the calorimeter. Coincidences between the triggers selected out the $(e,e'p)$ events that make up the physics data.

The drift chambers serve to determine the particles’ position, $x(y)$, and direction, $x'(y')$, in the bend (nonbend) plane of the spectrometer and it is these quantities that are used to reconstruct the events. Each spectrometer has two chambers and each chamber contains six planes of wires. In each HMS chamber one pair measures $x$, one pair measures $y$, and the remaining two planes are rotated $\pm 15^\circ$ with respect to the $x$ plane. The purpose of the third pair of planes is to correlate the $xy$ information when more than one particle traverses a chamber during the readout interval. In the SOS chambers...
one pair is in the $x$ plane and the other two pairs of planes are at $\pm60^\circ$ with respect to the $x$ plane. Position resolution per plane is $<250\ \mu m$ in the HMS chambers and $<200\ \mu m$ in the SOS. The wire chamber data were used to reconstruct the trajectory of the particles and determine the particles momentum fraction relative to the central momentum, $\delta p/p$.

Wire chamber tracking efficiency is an important element in the overall system efficiency and, as such, must be accurately measured. This was done by using the position information in the hodoscopes to tag particles passing through a small central region of the chambers and then see what fraction of such events was reconstructed from the wire chamber signals. In both spectrometers typical tracking efficiency was greater than 97%, which was determined to be better than 1%. The main sources of wire chamber tracking inefficiency are inefficiencies in the chambers themselves (we require that five of the six planes have good hits) and inefficiency in the reconstruction algorithm. The measured inefficiency was the sum of these inefficiencies and no attempt was made to disentangle the two.

**Calibrations**

**Spectrometer optimizations**

Because this was the first experiment performed in Hall C, considerable effort went into first optimizing the performance of the spectrometers and then optimizing the data analysis so as to achieve the highest possible accuracy. The magnetic field of the HMS quadrupoles was mapped to determine its optical axis and its effective field length versus current, with effective field length defined as the line integral of the field divided by the average field. However, the HMS dipole was not mapped but, rather, its magnetic field to current ($B$-to-$I$) relation was calculated using the TOSCA program [21]. The measured field map of the quadrupole and the TOSCA generated map of the dipole were used to build an optics model of the spectrometer with the COSY program [22]. For a desired magnetic field of the dipole (i.e., a desired central momentum) the dipole current was set according to the $B$-to-$I$ relation predicted by the TOSCA program, while the COSY model was used to get the starting value of the quadrupole to dipole ratio ($Q/D$). The $Q/D$ ratio was then varied to get the best focus in the spectrometer and these optimized ratios were used to determine the current settings of the quadrupole for a desired central momentum of the spectrometer. From elastic $e-p$ scattering data it was later determined that the $B$-to-$I$ relation of the dipole predicted by TOSCA was low by about 0.9%. The dipole currents were adjusted accordingly to correct for this difference. A similar procedure was followed for the SOS except that the quadrupole was not mapped and the optics model was formulated using the COSY program assuming the field of the quadrupole magnet to be an ideal quadrupole. The SOS dipole $B$-to-$I$ relation was also found to be slightly low (0.55%) and suitable corrections were made to the setting procedure.

The basic strategy in determining the momentum and direction of the scattered particles is to use the wire chamber data to determine the position $(x,y)$ and the angles $(x',y')$ of the particles at the focal plane which, in turn, specifies the trajectory of the particle through the spectrometer. This, of course, requires knowing the fields of the spectrometer, which are represented by a set of matrix elements that relate the position and direction of the particles as they cross the focal plane, to the particle’s momentum, angles of emission, and starting position along the beam direction. The accuracy of the final results then depends on how well the matrix elements simulate the spectrometers and hence a great deal of effort went into optimizing these matrix elements.

The COSY program was used to calculate an initial set of reconstruction matrix elements using the mapped fields for the HMS magnets and the SOS dipoles and an assumed pure quadrupole field for the SOS quadrupole. The Hall C matrix element optimization package CMOP [23] was used to optimize the reconstruction matrix elements. In this package the dispersion matrix elements are optimized using momentum scans, i.e., varying the central momentum by varying the magnetic fields. For each spectrometer these momentum scans were performed for both elastic $p(e,e'p)$ and elastic $^{12}C(e,e'p)$ scattering. In order to obtain the angular matrix elements sieve slits, which are collimators containing accurately positioned holes, were placed in front of each of the spectrometers so that rays of known initial position and direction could be traced. The angular matrix elements were then fit by the CMOP package (using singular value decomposition method) to accurately reproduce the known positions of the sieve slit holes. Similarly the target $y$ position (projection of the target length along the beam) reconstruction was optimized by utilizing the CMOP package with data from scans along the beam direction. These scans were performed by raising and lowering a slanted carbon target and the continuum portion of the carbon spectrum was used. Most of these calibration data were taken at one-pass, 845 MeV, with a check for reproducibility made with two-pass, 1645 MeV, electrons.

**Acceptances**

The spectrometer’s acceptances were studied with the aid of the simulation code SIMC, which is an adoption to the JLab Hall C spectrometers of the $(e,e'p)$ simulation code written for SLAC experiment NE18 [24]. This simulation package employs models for each of the spectrometers (HMS and SOS). The same models were also used to study the optical properties of the spectrometers. These models use COSY generated sets of matrices to simulate the transport of charged particles through the magnetic field of the spectrometer to each major aperture of the spectrometer. Energy loss and multiple scattering in the intervening material were also included. The events that passed through all apertures were then reconstructed back to the target using another set of matrices generated by COSY. Surviving events were assigned a weight based on the PWIA cross section, radiative corrections, and Coulomb corrections. The PWIA cross section was calculated using the deForest [25] prescription $\sigma_{el}$ for the off-shell $e-p$ cross section and an independent particle shell model (IPSM) spectral function for the target nucleus involved. The PWIA calculations and the IPSM spectral functions are elaborated in the following two sections. The radiative corrections in SIMC were performed according to
the Mo and Tsai [26] formulation adapted for the coincidence $(e, e’p)$ reaction as described in Ref. [27]. Further, a normalization factor was calculated from the experimental luminosity, phase space volume, and the total number of events generated, so that the simulation provided a prediction of the absolute yield.

The reconstructed momentum, scattering angle, out-of-plane angle, and target length distributions generated by the model were compared with the distributions obtained from the $e-p$ elastic scattering data as shown in Fig. 1. These results are an indicator of how well the model acceptance simulated the true acceptance of the spectrometer.

**Corrections**

**Radiative corrections**

A major issue in electron scattering experiments is radiative corrections. The incoming and outgoing electrons can interact with the Coulomb field of the nucleus involved in the scattering which results in the emission and absorption of virtual photons and emission of real, primarily soft, photons. Formulas for correcting for these radiative losses have been worked out by Mo and Tsai [26]. Correcting spectral functions deduced from $(e, e’p)$ coincidence spectra is considerably more complicated because in this case the radiated momentum as well as the lost energy must be allowed for. Although these are real physical processes, they are experiment specific and so most theoretical calculations do not take them into account. The prescription for doing this for coincidence $(e, e’p)$ reactions developed by Ent et al. [27] was used in the present work. Using this prescription, radiated spectra are generated which can be directly compared with the experimentally measured spectra. This point is discussed further in the section on spectral functions.

**Nuclear reactions**

Protons, being hadrons, will undergo strong interactions in traversing the detector stack and valid coincidences will be lost. This loss was measured directly using $e-p$ elastic scattering. Each scattered electron must have an accompanying proton and electrons were selected from a small region at the center of the acceptance, thus ensuring that protons could only be lost through nuclear interactions and other spectrometer inefficiencies. Transmissions of close to 95% were measured for both spectrometers, consistent with calculations of absorption of protons in the traversed material, and believed to be known to 1%. The absorption is virtually constant over the range of proton energies encountered in this experiment and therefore the small uncertainty in the absorption has little effect on any of the results.

**Deadtimes**

There were two data acquisition deadtimes of possible concern: electronic deadtime and computer deadtime. Electronic deadtime occurs when triggers are not counted because the electronics hardware is busy processing previous triggers. Electronic deadtime is dependent on the width of the logic signals, which for nearly all of the gates was 30 ns. This deadtime was measured by recording the rates of multiple copies of the trigger with varying widths and then extrapolating to the rate at zero width. For both spectrometers the electronic deadtime was found to be <0.1%. Computer deadtime is a more serious matter. Most of the earlier data were taken in nonbuffered mode where the processing time was about 400 μs. Later data were taken in the buffered mode with processing times of about 75 μs. Over 80% of the data were taken with deadtimes of <10% but there were a few runs where deadtimes were as great as 60%. Even in these extreme cases the loss of event is known to better than 0.5% from the ratio of the number of triggers generated to the number of triggers recorded by the data acquisition. This method was checked by measuring a large rate run and then varying the fraction of triggers recorded by the data acquisition.

**RESULTS**

**Kinematics**

Table I shows the kinematics settings where data were taken. The protons in the nucleus have finite momentum and therefore the struck protons from quasielastic scattering will emerge in a cone about the three-momentum transfer $\vec{q}$ and measurements must be taken across this cone. The lower the magnitude of $\vec{q}$, the broader the cone but, fortunately, the cross section increases with decreasing $Q^2$. While it is desirable to take data over as large a range of $Q^2$ as possible the cross section falls off so rapidly with increasing $Q^2$ that at the highest $Q^2$ point, 3.25 (GeV/c)$^2$, the cross section is so small that data could only be taken on one side of the conjugate angle. $L-T$ separations were performed at $Q^2$ of...
TABLE I. Table of kinematics for experiment E91-013. The central proton angles in bold represent the conjugate angle.

<table>
<thead>
<tr>
<th>Beam energy (GeV)</th>
<th>Central electron energy (GeV)</th>
<th>Central electron angle (deg)</th>
<th>Central proton energy (MeV)</th>
<th>Central proton angle (deg)</th>
<th>$Q^2$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.445</td>
<td>2.075</td>
<td>20.5</td>
<td>350</td>
<td>51.4</td>
<td>55.4</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>0.93</td>
</tr>
<tr>
<td>0.845</td>
<td>0.475</td>
<td>78.5</td>
<td>350</td>
<td>35.8</td>
<td>39.8</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.38</td>
</tr>
<tr>
<td>3.245</td>
<td>2.255</td>
<td>28.6</td>
<td>970</td>
<td>32.6</td>
<td>36.6</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.83</td>
</tr>
<tr>
<td>1.645</td>
<td>0.675</td>
<td>80.0</td>
<td>970</td>
<td>26.8</td>
<td>30.8</td>
<td>1.83</td>
</tr>
<tr>
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<td></td>
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<td></td>
<td>0.31</td>
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<td>31.5</td>
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</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.81</td>
</tr>
<tr>
<td>3.245</td>
<td>1.40</td>
<td>50.0</td>
<td>1800</td>
<td>25.5</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.54</td>
</tr>
</tbody>
</table>

$0.64 \text{ (GeV/c)}^2$ and $1.8 \text{ (GeV/c)}^2$. In order to get a good separation, data should be taken at as divergent values of $\epsilon$ [Eq. (1)] as possible, which translates into a large $\epsilon$ point at small (electron) angle and large incident energy and a low $\epsilon$ point at large angle and small energy (Table I). The cross section decreases rapidly with increasing angle and so it was only possible to cover one side of the proton cone at $\epsilon$ $=0.31$, $Q^2=1.8 \text{ (GeV/c)}^2$ and even at $Q^2=0.64 \text{ (GeV/c)}^2$ there was time for only one point on the low-angle side of the cone. Furthermore, no gold data were taken at the larger angle and higher $Q^2 \text{ (1.8 (GeV/c)}^2$).

Spectral functions

In the plane wave impulse approximation the cross section can be factorized into a product of an elementary electron-proton cross section $\sigma_{ep}$ and a nuclear spectral function $S(E_p, \vec{p}_m)$, i.e.,

$$\frac{d^8\sigma}{dE_e d\Omega_e d^3\vec{p}} = K \sigma_{ep} S(E_p, \vec{p}_m),$$

where $K$ is just a kinematic factor. The spectral function for protons in a nucleus $S(E_p, \vec{p}_m)$ is defined as the prob-

ability of finding a proton with separation energy $E_s$ and momentum $\vec{p}_m$ inside that nucleus. Obtaining spectral functions was a major objective of the present work and this section details how the spectral functions were deduced from the measured missing energy and missing momentum spectra.

Hydrogen data

A missing energy and a missing momentum spectrum were obtained at each data point. For the hydrogen target this served as a measure of the response of the system while for the other targets these are the spectra from which the spectral functions are determined. Hydrogen missing energy spectra along with the Monte Carlo calculated spectra at the various kinematics are shown in Fig. 2. The fact that the low-energy tail is well reproduced out to the highest missing energy accepted (80 MeV), shows that the radiative corrections are being handled correctly. Missing energy resolution, which is not of primary importance in the present work, is clearly not well incorporated into the code in that the calculated zero missing energy peak is always narrower than that observed. The peaks get broader with increasing energy of the scattered particle (see Table I), as could be expected, and this effect is not adequately accounted for. The effect is most dramatic at the two values of $Q^2$ where data were taken at two different electron angles, and the peak is much broader at the forward angle where the electron energy is higher, while the proton energy remains the same. The missing energy resolution...
TABLE II. Ratio of observed to predicted yield for e-p elastic scattering. Uncertainties are statistical only, except for the (e,e’p) point at 3.25 (GeV/c)^2 where there is an additional systematic uncertainty that is discussed in the text.

| Q^2 (GeV/c)^2 | \( \epsilon \) | Data/simulation | H(e, e’p) | H(e, e’)
|---------------|------------|-----------------|----------|----------
| 0.64          | 0.93       | 1.006±0.005     | 1.015±0.005 |
| 0.64          | 0.38       | 0.986±0.005     | 0.997±0.005 |
| 1.28          | 0.81       | 1.007±0.005     | 1.009±0.005 |
| 1.80          | 0.83       | 0.991±0.005     | 1.003±0.005 |
| 1.83          | 0.31       | 0.987±0.005     | 0.989±0.005 |
| 3.25          | 0.54       | 0.94±0.0120±0.06| 0.991±0.007 |

ranges from \( \alpha = 2.1 \) MeV at \( Q^2 = 0.64 \) (GeV/c)^2 (backward angle) to \( \alpha = 6.0 \) MeV at \( Q^2 = 1.8 \) (GeV/c)^2 (forward angle).

The ratio of the observed to predicted e-p elastic scattering yield is shown in Table II. The typical systematic uncertainty for these measurements was 2.3%. However, the large uncertainty in the (e, e’p) yield at \( Q^2 = 3.25 \) (GeV/c)^2 is due to an uncertainty in the proton efficiency due to malfunctioning wire chambers in the HMS. For all of the other points, including the single-arm electrons at 3.25 (GeV/c)^2, calculated and measured yields agree to within about 1%. The setting for \( Q^2 = 3.25 \) (GeV/c)^2 was the only one at which the protons were detected in the HMS and this efficiency problem was corrected before the data on the complex nuclei were taken.

At \( Q^2 = 0.64 \) (GeV/c)^2, there is little difference between the form factors obtained from Rosenbluth separations [9,10] and those from polarization transfer [11] and the form factors obtained from our measured cross sections at two values of epsilon are in agreement with both. At \( Q^2 = 1.80 \) (GeV/c)^2, where we also took data at two widely separated epsilons, the electric scattering contributes less than 20% of the cross section. The polarization transfer method only measures \( G_E/G_M \) [11,12] and so all that can be compared is the relative cross sections at the two epsilon points and at this \( Q^2 \) these differ by only 5%. Thus, while our measurements are in better agreement with previous Rosenbluth determinations our systematic uncertainties are too large for these results to be considered as evidence against the polarization transfer determinations. The Rosenbluth separation determinations are used in analyzing the present data and, where relevant, the implications of the results of [11] are noted.

**Missing energy spectra for the nuclear targets**

A missing energy and missing momentum spectrum were obtained at each data point for all three nuclear targets. These are the raw spectra from which the spectral functions were extracted after unfolding the radiative effects, the phase space weight, and the e-p cross-section weight. The raw missing energy spectra are shown in Figs. 3, 4, and 5.

Figure 3 shows the missing energy spectra for carbon. At \( Q^2 = 0.64 \) (GeV/c)^2 the spectra show a rather sharp peak corresponding to populating low-lying levels in \(^{11}\)B which can be attributed to removing p-shell protons from \(^{12}\)C and a

FIG. 3. Measured missing energy spectra for carbon at the different \( Q^2 \). The top left and middle right panels refer to the forward and backward electron angle kinematics, respectively, at \( Q^2 = 0.64 \) (GeV/c)^2 and the middle left and bottom left panels refer to the forward and backward electron angle kinematics, respectively, at \( Q^2 = 1.80 \) (GeV/c)^2. These are the L-T separation kinematics.

FIG. 4. Measured missing energy spectra for iron at the different \( Q^2 \). The top left and middle right panels refer to the forward and backward electron angle kinematics, respectively, at \( Q^2 = 0.64 \) (GeV/c)^2 and the middle left and bottom left panels refer to the forward and backward electron angle kinematics, respectively, at \( Q^2 = 1.80 \) (GeV/c)^2. These are the L-T separation kinematics.
broader peak at higher missing energies which is primarily due to removing \( s \)-shell protons. The missing momentum dependence of the spectral functions discussed below supports this identification. The valley between the two groups is increasingly filled in as \( Q^2 \) increases, because the (absolute) energy resolution broadens as the energy of the particles increases, as noted above in discussing the hydrogen spectra of Fig. 2. At the two values of \( Q^2 \) at which \( L-T \) separations were performed the valley between the \( s \)-shell and \( p \)-shell regions is less distinct at the forward electron angle, again reflecting the poorer energy resolution that was also observed in the hydrogen spectra. The missing energy spectra for iron are shown in Fig. 4. The ground-state region peak is more prominent at low \( Q^2 \) and backward angles. The missing energy spectra for gold are shown in Fig. 5. The statistical uncertainties are much poorer for gold than for the other targets and no trends are apparent.

**Radiative and acceptance corrections**

As previously noted, energy and momentum are lost by the electrons radiating photons in the Coulomb field of the target nucleus both before and after the scattering. The electrons can also emit bremsstrahlung radiation in passing through material in the spectrometers. The net result is a distortion of the spectra and the corrections to this distortion are model dependent. The code SIMC was used to generate correction factors for “deradiating” the observed spectral functions. Model spectral functions were used to populate bins in \( p_m \) and \( E_m \) space with both the radiative corrections turned on and turned off and the ratio was applied as a correction factor, bin by bin, to the spectral functions derived from the experimental data. The Monte Carlo was also used to calculate the experimental phase space for each \( (E_m, p_m) \) bin. The experimental counts in each \( (E_m, p_m) \) bin corrected for radiation and divided by the phase space for that bin were used to obtain the “experimental” spectral function:

\[
S^{\text{rad}}(E_m, p_m) = \frac{1}{\mathcal{L} H(E_m, p_m)} \times \sum_{\text{counts}} \frac{1}{\sigma_{ep} E_e p_p (E_m, p_m)} C^{\text{rad}}(E_m, p_m),
\]

where \( \mathcal{L} \) is the luminosity, \( H(E_m, p_m) \) the phase space for the given \( E_m, p_m \) bin, \( C^{\text{rad}}(E_m, p_m) \) the correction factor for the same bin, and \( \sigma_{ep} E_e p_p (E_m, p_m) \) the off-shell \( e-p \) cross section and kinematic factors averaged over the \( E_m \) and \( p_m \) bin. This “experimental” spectral function is then compared to the input model spectral function and if the two differ by more than a specified amount the experimental spectral functions become the new model spectral functions and the whole process is iterated until a satisfactory convergence is achieved. In order to test the validity of this procedure, nonphysical spectral functions were input as the model spectral functions and it was demonstrated that after several iterations the extracted spectral functions are virtually independent of the initial model function. The consistency of this deradiation procedure was also checked using Monte Carlo generated data. It should be noted that these corrected spectral functions still include distortions due to the effects of final state nuclear interactions, including absorption.
Experimental spectral functions

At each electron angle the above procedure was used for each proton angle to obtain experimental (distorted, as defined above) spectral functions and these were integrated over the proton angles to obtain the experimental spectral functions for that target, electron angle, and $Q^2$. These summed spectral functions are functions of both missing momentum and missing energy and therefore the missing momentum was integrated over in order to obtain the energy spectral functions and the missing energy was integrated over to obtain momentum distributions. The momentum distributions are shown in Figs. 6, 7, and 8.

The carbon momentum distributions are shown in Fig. 6. They have been normalized to the spectral functions at $Q^2$ of 1.8 (GeV/c)$^2$. These spectra show little variation with $Q^2$. The dip at zero missing momentum for missing energy between 10 and 25 MeV is attributable to the fact that the protons in this energy region are primarily $l=1$ while only $l=0$ protons can have zero missing momentum. There is a left-right ($\pm$) asymmetry in the momentum distributions that is discussed below. As with carbon the iron momentum distributions (Fig. 7) and gold momentum distributions (Fig. 8) show little change with $Q^2$.

Independent particle shell model

Model spectral functions were calculated in the IPSM approximation, in which the nucleus is considered a sum of nucleons occupying distinct shells with each proton in the lowest possible shell. The parameters of the spectral function were adjusted to reproduce data from low-$Q^2$ $A(e,e'p)$ and $A(p,2p)$ experiments. For $^{12}$C the removal energy and energy width of the two shells, $s_{1/2}$ and $p_{3/2}$, are based on the Saclay $^{12}$C($e,e'p$) data [28]. The removal energy and energy width for the $^{56}$Fe shells were based on the $^{58}$Ni($e,e'p$) data from Saclay [28], with the removal energy corrected for the 2 MeV difference between $^{56}$Fe and $^{58}$Ni. The removal energy for the shells not resolved in the Saclay experiment.
were obtained from Hartree-Fock calculations [29] and the widths for these shells were calculated according to the Brown and Rho [30] parametrization of data for $A < 58$. Similarly for $^{197}$Au the removal energies and widths are based on those measured for nearby nucleus $^{208}$Pb in $A(e,e'p)$ experiments at NIKHEF [29], with removal energies corrected for the 2.2 MeV difference between $^{208}$Pb and $^{197}$Au. The parameters for the unmeasured shells were obtained from Hartree-Fock calculations [29] and the Brown and Rho parametrization as mentioned above. Further details are given elsewhere [16].

Momentum distributions were obtained for each shell by solving the Schrödinger equation in a Woods-Saxon potential using the code DWEEPY [31]. For $^{12}$C the parameters used in the potential were based on the Saclay $^{12}$C$(e,e'p)$ data [28]. The $^{56}$Fe and $^{197}$Au momentum distributions were based on those measured for the nearby nucleus $^{58}$Ni and $^{208}$Pb, modified to agree with the $^{58}$Fe$(e,e'p)$ and $^{197}$Au$(e,e'p)$ data from SLAC experiment NE-18 [32], respectively. For $^{56}$Fe and $^{197}$Au a Perey factor (with $\beta = 0.85$) [33] was used to correct for the nonlocality or energy dependence of the potential.

The experimental missing energy spectral function for carbon at $Q^2 = 1.28$ (GeV/c)$^2$ is compared to the IPSM spectral function in Fig. 9. The model predicts slightly too much yield in the dip region between the $s_{1/2}$ and the $p_{3/2}$ shells possibly implying that the $s$ shell is more tightly bound than generally thought. The momentum distribution (Fig. 10) in the region of the low missing energy peak, considered to be the $p$-shell region, shows a much shallower minimum at $p_m = 0$ than the IPSM prediction, while for protons from the $s$-shell region the $p_m = 0$ yield is smaller than predicted. Agreement is much better if 8% of the $s$-shell strength is included (the $E_m$ cut allows some $s$-shell strength into the $p$-shell region and vice versa). The spectroscopic factors

![Image](https://example.com/image1.png)

**FIG. 10.** (Color online) Measured momentum distribution for carbon at $Q^2 = 1.28$ (GeV/c)$^2$ in the $p$-removal energy region (top panel, $10 < E_m < 25$ MeV) and $s$-removal energy region (bottom shell, $30 < E_m < 50$ MeV) compared to theoretical predictions. The solid line is the IPSM; dashed line is IPSM with an 8% $s$-shell strength. Dotted line is a DWIA calculation from Zhalov et al. [36] and the dot-dashed line is the same DWIA calculation with color transparency included.

![Image](https://example.com/image2.png)

**FIG. 11.** Measured missing energy spectral function for iron at $Q^2 = 1.28$ (GeV/c)$^2$ compared to theoretical models. The solid line is drawn using the IPSM. The dashed line is a calculation from Benhar et al. [37] and the dot-dashed line is from calculations using the TIMORA code described in Ref. [38] with spreading widths taken from the IPSM.

![Image](https://example.com/image3.png)

**FIG. 12.** Measured missing energy spectral function for gold at $Q^2 = 1.28$ (GeV/c)$^2$ compared to the IPSM.
functions have also been calculated by Benhar [37]. Here single-particle spectral functions are modified by adding terms dependent on the nuclear density. Results are shown in Fig. 13 (iron) and Fig. 14 (gold). Including the density dependence does increase the large $p_m$ tail, though not by enough to reproduce the data. These calculations also underestimate the $p_m=0$ region (it must be remembered that the momentum distribution is weighted by $p_m^2$ in normalizing calculation to experiment). The calculated energy spectral function for iron shows more structure than is observed, reflecting the fact that the IPSM spreading width was also used in the Benhar calculation (Fig. 11).

Energy and momentum distributions for iron have been calculated using the TIMORA code written by Horowitz [38] and based on the $\sigma$-$\omega$ mean-field theory of Walecka [39]. Details of this calculation are given elsewhere [40]. As can be seen in Fig. 11, this calculation gives a better fit to the observed structure, or lack thereof, than does either the IPSM or the Benhar [37] calculation.

**Transparencies**

As noted in the Introduction, the basic strategy used to obtain nuclear transparencies was to compare the measured yield to that calculated under the assumption that the struck proton escapes the nucleus without further interaction, i.e., the transparency is defined as the ratio of the measured yield to that calculated using the plane wave impulse approximation, or PWIA.

**PWIA**

For each target, incident electron energy, outgoing electron angle, and outgoing proton angle, the transparency was

---

**FIG. 13.** (Color online) Measured momentum distribution for iron integrated over an $E_m$ range $0<E_m<80$ MeV at $Q^2=1.28$ (GeV/c)$^2$, compared to theoretical predictions. Solid line is drawn using the IPSM. Dotted line is DWIA calculation from Zhalov et al. [36] without including color transparency and dot-dashed line is the same with color transparency included. Dot-dot-dash line is a calculation from Benhar et al. [37] and dash-dot-dash line is from calculations using the TIMORA code described in Ref. [38].

**FIG. 14.** Measured momentum distribution for gold integrated over an $E_m$ range $0<E_m<80$ MeV at $Q^2=1.28$ (GeV/c)$^2$ compared to theoretical predictions. Solid line is drawn using the IPSM model and dashed line is a calculation from Benhar et al. [37].

---

*Other calculated spectral functions*

Distorted wave impulse approximation (DWIA) calculations of the (distorted) spectral functions using the Hartree-Fock model with Skyrme’s interaction to describe the single particle aspects of the nuclear structure [35] have been performed by Zhalov [36]. These calculations include an estimate of the effects of color transparency, which are negligible for carbon (Fig. 10) and barely discernible in iron (Fig. 13). These calculations overestimate the yield at small missing momentum and fall off too rapidly at large $|p_m|$. Spectral...
FIG. 15. Normalized transparency as a function of angle relative to the conjugate angle for carbon (top), iron (middle), and gold (bottom). Normalization was done at the conjugate angle. The solid circles and squares correspond to forward angle and backward angle kinematics, respectively, at $Q^2=0.64$ (GeV/c)$^2$.

determined as the ratio of the observed $e$-$p$ coincidence yield, integrated over missing momentum ($\pm$300 MeV/c) and missing energy (up to 80 MeV), to that calculated using the PWIA. However, before the expected coincidence $e$-$p$ spectra in the absence of final state interactions can be calculated, a number of complications must be dealt with. As its name implies, the PWIA treats the incoming and outgoing particles as plane waves. There are, of course, the radiative corrections that are discussed above. Additionally, the incident and outgoing waves are distorted by the Coulomb field of the target and residual nucleus, respectively. It has been shown [38] that these distortions can be approximated by attaching a phase factor to the plane wave expansion. The acceleration by the Coulomb field increases the electron momentum $k$ by

$$\delta k = f \frac{Z\alpha}{R},$$

where factor $f$ varies between 1.1 and 1.5 depending on the size of the nucleus and $R$ is the Coulomb radius of the nucleus. This can be used to estimate the effect of Coulomb distortion on the cross section with satisfactory accuracy [41]. This Coulomb acceleration of the electron necessitates using an effective momentum transfer and also alters the missing momentum [27]. All of these effects were incorporated into the PWIA and spectral function calculations.

The PWIA calculations were done using the “traditional” $e$-$p$ free cross sections in which $\mu_pG_V/G_M=1$. The fact that the target proton is moving and is bound to a nucleus (i.e., “off shell”) introduces considerable complications. Off-shell prescriptions for quasifree $e$-$p$ cross sections have been given by deForest [25] and the prescription $\sigma_{cc}$ was used in the present work in calculating the PWIA cross sections. Another complication is the fact that the response function is no longer the incoherent sum of the longitudinal and transverse response functions but there are also the interference terms $W_{LT}$ and $W_{TT}$ [Eq. (2)]. The response function $W_{LT}$ is antisymmetric about the conjugate, or free $e$-$p$ scattering, angle and thus vanishes in this direction, known as “parallel kinematics.” Of course, parallel kinematics is the only kinematics in free $e$-$p$ scattering and the cross section is given by the familiar Rosenbluth formula.

While it is a reasonable first approximation to take complex nuclei as a collection of $A$ nucleons moving in an average potential with orbits filled in order of increasing energy, this is too simplistic a picture to use in extracting transparencies. Short-range nucleon-nucleon correlations are present and one effect of these is to extend some single particle strength up to hundreds of MeV in $E_m$ and well beyond the

<table>
<thead>
<tr>
<th>$Q^2$ (GeV/c)$^2$</th>
<th>Carbon</th>
<th>Iron</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.64 ($\theta$, forward)</td>
<td>0.61(0.02)</td>
<td>0.47(0.01)</td>
<td>0.38(0.01)</td>
</tr>
<tr>
<td>0.64 ($\theta$, backward)</td>
<td>0.64(0.02)</td>
<td>0.54(0.01)</td>
<td>0.43(0.01)</td>
</tr>
<tr>
<td>1.28</td>
<td>0.60(0.02)</td>
<td>0.44(0.01)</td>
<td>0.32(0.01)</td>
</tr>
<tr>
<td>1.80 ($\theta$, forward)</td>
<td>0.57(0.01)</td>
<td>0.40(0.01)</td>
<td>0.29(0.01)</td>
</tr>
<tr>
<td>1.83 ($\theta$, backward)</td>
<td>0.59(0.01)</td>
<td>0.44(0.01)</td>
<td></td>
</tr>
<tr>
<td>3.25</td>
<td>0.58(0.02)</td>
<td>0.42(0.01)</td>
<td>0.28(0.01)</td>
</tr>
</tbody>
</table>
and gold, respectively, are calculated of 1.11±0.03, 1.26±0.08, and 1.32±0.08 for carbon, iron, momentum and the missing energy limit, correlation factors from suppression of the spectral function below the Fermi momentum. Under the assumption that the correlations produce a uniform suppression of the spectral function below the Fermi momentum and the missing energy limit, correlation factors of 1.11±0.03, 1.26±0.08, and 1.32±0.08 for carbon, iron, and gold, respectively, are calculated [42] and these corrections have been applied to the PWIA cross sections in extracting the transparencies. 

Extracted transparencies

The apparent transparencies [i.e., ratio of measured to PWIA calculated (e, e'p) coincidence yield] relative to that at the conjugate angle are shown in Fig. 15 for the carbon (top), iron (middle), and gold (bottom) targets, for the various electron kinematic settings. The transparencies are significantly asymmetric. One possible reason could be the presence of interference terms in the response function, i.e., a $\sigma_{el}$ in excess of that included in the deForest prescription $\sigma_{el}$. This is not unexpected because modern relativistic models predict such asymmetries [43,44]. However, it should be noted that Coulomb distortion of the electron waves can alter the effective scattering angle and therefore induce an asymmetry about the free conjugate angle. While much of the Coulomb distortion can be allowed for by introducing the momentum increase given by Eq. (10), it could well be that this correction is not adequate. Coulomb distortions are known to increase with $Z$ [45].

Fermi momentum in $p_m$. The missing energy spectra are indeed above the IPSM predictions at the high energy end but because of the acceptance cutoff of the spectrometers only a small portion of this “pushed-up” strength could be detected. Under the assumption that the correlations produce a uniform suppression of the spectral function below the Fermi momentum and the missing energy limit, correlation factors of 1.11±0.03, 1.26±0.08, and 1.32±0.08 for carbon, iron, and gold, respectively, are calculated [42] and these corrections have been applied to the PWIA cross sections in extracting the transparencies. 

The angular dependence of the quasifree scattering depends directly on the momentum distribution of the scattering nucleons and the tendency of the transparency to peak at the conjugate angle that is seen in the iron and gold distributions could be due to an underestimate of the number of protons at large angles relative to the conjugate angle. None of these complications appear to be present in the carbon data and so we can conclude that in carbon at least there is evidence of an interference term in the response function that decreases with increasing $Q^2$. At the large proton angles relative to the conjugate angle there is a small contribution from other reaction mechanisms such as proton rescattering or $(e,e'n)$ followed by $(n,p)$ charge exchange, but the yields are weak compared to the dominant quasifree strength [46,47].

The outgoing proton cone was integrated over in order to determine the transparency for that electron kinematic setting. The values thus obtained are shown in Table III and are plotted as a function of $Q^2$ for the various targets along with previous measurements in Fig. 16. There are three types of errors in the transparencies.

(i) Statistical: These are down in the 0.01 region and are never greater than 0.02.

(ii) Systematic: These are about 2.5% overall and about 2% from point to point.

(iii) Model dependence: These include uncertainties in the radiative corrections, the off-shell $e-p$ cross sections, and the correlation corrections. The sum in quadrature of the model dependent uncertainties is about 5% for C and 8% for Fe and Au. The relative uncertainties in comparing different points with the same target are less than 5%.

In addition to the obvious trend of decreasing with increasing $A$, the transparencies also decrease with increasing $Q^2$, at least at the low end of the $Q^2$ range covered here. The
While at the low Q^2 the conjugate angle that the differential cross sections are not symmetric about over the interference terms in the response function. The fact that the nucleons are bound in a nucleus. The separated spectral functions for carbon have already been reported [5]. Separated spectral functions for iron are shown in Fig. 17. Because of the increasing dominance of the magnetic scattering with increasing Q^2 [Eq. (1)] the errors in S_L increase with increasing Q^2 while the errors in S_T decrease somewhat. The transverse strength is clearly smaller at the higher Q^2 and, at 0.64 (GeV/c)^2, S_T is clearly greater than S_L. At Q^2=1.8 (GeV/c)^2, the errors on S_L are too great to allow any conclusions as to whether there are (relative) changes in S_L similar in magnitude to those found in S_T. Similar results were found for carbon [6].

An L-T separation for gold was only done at 0.64 (GeV/c)^2 and the resultant spectral functions are shown in Fig. 18. As with the other two targets at this momentum transfer, there is an excess of transverse strength.

The results described above were all obtained using the proton form factors with \( \mu_p G_E = G_M \) as mentioned in the In-
troduction. Because the spectral functions are close to inversely proportional to the square of the form factors large changes in the form factors lead to large changes in the separated spectral functions. A comparison of the spectral functions obtained using the $L$-$T$ separation [9,10] and the polarization transfer [11] form factors is shown in Fig. 19 (Fig. 20) for carbon at $Q^2=0.64$ (1.8) (GeV/c)$^2$. At 0.64 (GeV/c)$^2$ there is little effect on either spectral function and the substantial decrease in transverse strength at the higher $Q^2$ is still apparent. However, the form factors of Ref. [11] lead to a 60% increase in the longitudinal strength between the two values of momentum transfer. While the systematic errors of the present work are too large to require such a conclusion, mechanisms that would lead to such a $Q^2$ dependence are not obvious and it is clear that the final interpretation of the present (and a great deal of other) data must await a resolution of the question of the free proton electric form factor.

The behavior of the transverse spectral function as a function of $Q^2$ is consistent with a recent calculation of the separation of the longitudinal spectral function at 1.8 (GeV/c)$^2$ and 1.8 (GeV/c)$^2$ with the iron and gold separations being the first such data on medium and heavy nuclei. Considerable excess transverse strength is found at $Q^2=0.64$ (GeV/c)$^2$ which is much reduced at 1.8 (GeV/c)$^2$. This excess strength is attributed to multinucleon effects that have less effect on smaller distance probes. Recently reported determinations of $G_s/G_L$ for the proton which are in substantial disagreement with previously accepted values may, if they are confirmed, substantially alter the magnitude of the longitudinal spectral function at 1.8 (GeV/c)$^2$, but are within the systematic uncertainties of the present measurement. However, because $G_m$ is primarily determined by the absolute cross section the transverse spectral function will be little affected.

CONCLUSIONS

Taking advantage of the high-quality electron beams and associated detection systems that have become available with JLab coming into operation, $(e,e'p)$ coincidence measurements were made on carbon, iron, and gold targets at momentum transfers $Q^2$ of 0.64, 1.28, 1.8, and 3.25 (GeV/c)$^2$. Spectral functions were measured for missing momentum out to 300 (GeV/c) and missing energy up to 80 MeV and these differ in detail, but not in overall shape, from independent particle shell model calculations. Other reported calculations do not give much better fits except those from a code based on a $\sigma-\omega$ mean field theory. By comparing the experimental yields integrated over missing energy and missing momentum with PWIA calculations nuclear transparency for 350–1800 MeV protons were determined with an accuracy that is considerably greater than previously reported transparency determinations.

Longitudinal-transverse separations were performed at 0.64 (GeV/c)$^2$ and 1.8 (GeV/c)$^2$ with the iron and gold separations being the first such data on medium and heavy nuclei. Considerable excess transverse strength is found at $Q^2=0.64$ (GeV/c)$^2$ which is much reduced at 1.8 (GeV/c)$^2$. This excess strength is attributed to multinucleon effects that have less effect on smaller distance probes. Recently reported determinations of $G_s/G_L$ for the proton which are in substantial disagreement with previously accepted values may, if they are confirmed, substantially alter the magnitude of the longitudinal spectral function at 1.8 (GeV/c)$^2$, but are within the systematic uncertainties of the present measurement. However, because $G_m$ is primarily determined by the absolute cross section the transverse spectral function will be little affected.

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