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Analysis of Electromechanical Interactions in a Flywheel System With a Doubly Fed Induction Machine

Li Ran, Senior Member, IEEE, Dawei Xiang, and James L. Kirtley, Jr., Fellow, IEEE

Abstract—This paper analyzes the electromechanical interaction in a flywheel system with a doubly fed induction machine, used for wind farm power smoothing or grid frequency response control. The grid-connected electrical machine is subject to power control, and this can cause it to produce negative damping to the shaft torsional vibration. Resonance must be prevented, and this paper proposes a solution by coordinating the design of the electrical controller and mechanical shaft. Computer simulations are used to demonstrate the problem and the proposed solution.

Index Terms—Damping, doubly fed induction machine, flywheel, power control, torsional vibration, wind power.

I. INTRODUCTION

WITH wind power comes the need of power smoothing and appropriate power response depending on the grid frequency. Flywheels as an energy storage mechanism are capable of power cycles from seconds to a few minutes and are cost effective as compared to some other technologies [1]. High-speed devices running in a vacuum have a superior energy density but low power rating which is limited by cost and the difficulty of cooling [2]. Low-speed flywheels, rated at hundreds of megawatts, have been used in high-energy physics facilities, and they operate in air [3], [4].

Cost is a concern for the power electronic converter used to interface the flywheel-coupled machine to the grid. It is realized that a flywheel can operate in a somewhat restricted speed range without significantly reducing the usable energy due to the quadratic relationship of kinetic energy to speed. This makes it possible to save on the converter cost if a doubly fed induction machine is to be adopted. The stator windings of the machine are directly connected to the grid, while the rotor windings are fed from the converter which only handles the rotor power; the four-quadrant converter also recovers power between the machine and the grid [5]. Theoretically, a speed range of ±30% around the synchronous speed allows 71% of the maximum kinetic energy stored in the flywheel to be used for dynamic control, while the rotor converter is only rated at 23% of the total peak power including the stator and rotor power. This configuration was previously exploited in industry for standby supplies [6]. Reference [7] reports a study on the control of a 20-MW 200-MJ system to smooth the pulsating demand of a synchrotron. Clearly, the configuration can also be used for wind farm power smoothing or to achieve a grid-frequency-dependent power response that the wind farm may be required to provide.

The flywheel has a much larger moment of inertia than that of the machine rotor, which is continually subject to pulsating torque for the power-smoothing objective. Torsional resonance can be excited if the electrical machine provides negative damping at the shaft vibrating frequency, similar to what has occurred with direct-on-line starting electric submersible pumps in the petroleum industry [8] where a solution was to attach a mechanical damper to the drive train shaft [9]. For the reason to be expanded in this paper, negative damping can be produced when the machine operates as a generator in the power control mode; the damping becomes negative if the response of electrical power control is set too fast, which is easily the case when the machine is grid connected. To avoid resonance, the natural frequency of the shaft and the bandwidth of power control must be coordinated. This paper contributes a model to analyze the electromechanical interactions in the flywheel system and to coordinate the design. The study assumes a system configuration incorporating a doubly fed induction machine, but this can be extended to other situations.

Section II describes the configuration of the system, and a typical control strategy is outlined in Section III. Section IV presents a torsional dynamic model and explains the risk of negative damping from the machine. Simulation shows the effect of control parameters. Section V develops a frequency-domain model to calculate the damping response, which is used to coordinate the design of the shaft and controller.

II. SYSTEM CONFIGURATION AND BASIC PARAMETERS

The main objective of this study is to analyze the electromechanical interactions in the form of torsional vibration in the flywheel system, using the configuration with a doubly fed induction machine as an example to facilitate the modeling work which is presented in this paper. Since such a configuration may not be very familiar to some readers, this preparatory section...
describes the basic design considerations and, in particular, the selection of the main parameters for an example system which will be used in simulation to demonstrate the electromechanical interactions.

Fig. 1 shows a flywheel system incorporating a doubly fed induction machine for wind farm power smoothing or power response control. For instance, the flywheel can absorb or release kinetic energy, via the machine and converter, to smooth the output power of the wind farm. During severe grid frequency reduction, e.g., following the tripping of a major power plant, flywheel systems may be required to increase power output to prevent frequency runaway so that boilers in the remaining plants can pick up more load in up to 3 min [10]. This is regarded as necessary by some as the wind penetration in power plants can pick up more load in up to 3 min [10]. This is regarded as necessary by some as the wind penetration in power systems increases. Installing flywheels near wind farm gives the additional benefit of mitigating the power quality problems due to the intermittency of wind power. In both cases, the machine is under power control, with the power level set independent of the flywheel speed.

The basic specifications of the flywheel system are its power and energy capabilities. The current rating of electrical equipment dictates the maximum power, while the energy capability is determined by the total moment of inertia \( J_1 + J_2 \) and the range of speed variation. With a doubly fed induction machine, increasing the speed range will reduce the moment of inertia needed for certain energy capability but increase the size of the power electronic converter for the same peak power. These design variables are related as shown as follows:

\[
P_{\text{conv}} = P_0 \frac{s_{\text{max}}}{1 + s_{\text{max}}} \quad (1)
\]

\[
\Delta E = 2(J_1 + J_2)\omega_{r0}^2 s_{\text{max}} \quad (2)
\]

where \( P_{\text{conv}} \) is the needed converter power rating, \( P_0 \) is the targeted peak power, and \( \Delta E \) is the energy capabilities of the system that can be utilized for dynamic control. \( \omega_{r0} \) is the mechanical synchronous speed (in radians per second), and \( s_{\text{max}} \) is the value of the maximum slip. Equation (1) is derived from the fact that the rotor-side power \( P_r \) equals the stator power \( P_s \) multiplied by the slip [5]. The speed may change between the minimum \( \omega_{r0}(1 - s_{\text{max}}) \) and maximum \( \omega_{r0}(1 + s_{\text{max}}) \), while the stored kinetic energy may vary correspondingly by up to \( \Delta E \). The total power capability of the system in either direction (generating or motoring) varies from \( P_0[1 - 2s_{\text{max}}/(1 + s_{\text{max}})] \) to \( P_0 \). Given the converter rating, the power capability of the whole flywheel system is therefore dependent on the speed. Note that this is also seen in a system with a fully rated converter connected directly on the stator side as the machine torque, in rough proportion to the converter current, will affect the power that can be achieved.

Given \( P_0 \) and \( \Delta E \), the required converter rating changes with the speed range, but the required flywheel moment of inertia changes more dramatically. This paper considers a system with \( P_0 = 5 \) MW and an energy capability of \( \Delta E = 150 \) MJ. The relatively large energy capability, as compared to that in the system of [7], has been set owing to the very low frequency power fluctuation from the wind farm that the flywheel system aims to compensate. For different maximum slip chosen, the required converter power rating and the flywheel moment of inertia \( J_1 \) change as shown in Fig. 2 based on the aforementioned relationship. It is assumed that the doubly fed induction machine is of a four-pole type and 50 Hz, given that \( \omega_{r0} = 157.08 \) rad/s. The rotor of the machine itself has a small moment of inertia of \( J_2 = 851 \) kg · m².

Both the converter rating and the moment of inertia of the flywheel are associated with costs. Fig. 2 shows that the needed moment of inertia of the flywheel increases rapidly as the range of the slip variation is more restricted. For the 5-MW system, the maximum slip is selected to be 0.3 p.u. The converter rating will then be 1.16 MW, while the moment of inertia of the flywheel needs to be 9281 kg · m². Table I contrasts these with the corresponding values in a flywheel system employing a 5-MW four-pole cage 50-Hz induction machine and a fully rated converter assuming that the maximum speed is the same in both cases, i.e., 130% of the synchronous speed. The speed can be reduced to zero in the system with the fully rated converter, but it only drops to 70% of the synchronous speed when the doubly fed induction machine is used. To justify the latter, the cost associated with the increased moment of inertia of the flywheel should be offset by the saving on the converter rating.

The flywheel with \( J_1 = 9281 \) kg · m² is considered to be a steel cylinder with a horizontal axis. Given the radius \( R \) (in meters) of the cylinder, the weight \( W \) (in kilograms) can be
calculated using (3). The circumferential stress \( \sigma_\theta (\text{N/m}^2) \) and radial stress \( \sigma_r (\text{N/m}^2) \) are equal and maximum at the center. This can be calculated using (4) \[ 11 \]

\[
W = \frac{2J_1}{R^2} \quad \text{(3)}
\]
\[
\sigma_\theta = \sigma_r = \frac{3 + \nu}{8} \rho [\omega r_0 (1 + s_{\text{max}})]^2 R^2 \quad \text{(4)}
\]

where \( \rho = 7750 \text{ kg/m}^3 \) is the density of steel and \( \nu = 0.3 \) is the Poisson’s ratio. For \( \omega r_0 = 157.08 \text{ rad/s} \) and \( s_{\text{max}} = 0.3 \), the stresses are plotted in Fig. 3 as a function of \( R \). With a yield stress of strength steel of 690 MN/m² and a safety margin of 200\%, it is decided that \( R = 1.25 \text{ m} \), resulting in a flywheel weight of 11 880 kg and an axial length of 0.312 m. The weight will affect the bearing loss depending on the shaft diameter.

### III. System Control Structure

The objective of control is to regulate the electrical power exchange between the doubly fed induction machine and the grid. The real power can be used to smooth the fluctuation of the wind farm output or provide a proper frequency response. The controller needs to recognize the following constraints: 1) the total power capability which varies with the speed; and 2) the slip range of the rotor which determines the voltage of the rotor-side converter. The control target regarding the reactive power output can be set in different ways, but this paper focuses on the real power behavior and assumes that the power exchange with the grid is at a unity power factor.

The stator power is controlled via the rotor current, and the control algorithm is established in a reference frame orientated to the stator flux linkage space vector \[ 5, 7, 12 \]. Suppose that the \( d \)-axis is aligned with the stator flux linkage, while the \( q \)-axis leads the \( d \)-axis by 90\°. In such a rotating \( d-q \) reference frame, the relationship between the rotor voltage and current is

\[
u_{rd} = R_e i_{rd} + \sigma \frac{d}{dt} i_{rd} - s \sigma i_{rq}
\]
\[
u_{rq} = R_e i_{rq} + \sigma \frac{d}{dt} i_{rq} + s \left( \sigma i_{rd} + \frac{L_m}{L_s} \psi_s \right).
\]

Motor convention is adopted for the voltage and current when viewed into the rotor from the converter. The symbols used earlier are of usual convention. It is further clarified that \( \psi_s \) is the amplitude of the stator flux linkage, and \( \sigma = (L_s L_r - L_r^2) / L_s \) is the equivalent leakage inductance. \( s \) is the slip of the rotor with respect to the synchronous speed, being positive for sub-synchronous speed and negative for supersynchronous speed.

The stator real power is

\[
P_s = T_e \omega r_0 = \omega r_0 K_{Te} i_{rq}
\]

where \( K_{Te} = 3/2 \) \( P_p (L_m / L_s) \psi_s \) is the air-gap torque coefficient \[ 13 \].

The stator real power is controlled via the \( q \)-axis component of the rotor current. Since the rotor power is deterministically related to the slip and stator power, this controlled \( q \)-axis rotor current determines the total real power of the doubly fed induction machine at a given speed. In other words, control of total real power is equivalent to the control of the stator power \( P_s \). This is used to establish the control algorithm of the flywheel system. The reactive power on the stator side of the doubly fed induction machine can be controlled via the \( d \)-axis component of the rotor current, and this is set to zero for unity power factor.

Fig. 4 shows the block diagram to determine the total real power demand \( P^* \) (in generator convention) for the doubly fed induction machine for wind farm power smoothing. A low-pass filtering algorithm is applied to the real power output of the wind farm to derive the local average value. The deficit of the actual power to the extracted average value is the total power demand from the doubly fed induction machine, or a group of such devices sharing the demand. Divided by \( 1-s \), this becomes the stator power demand. Fig. 5 shows the block diagram for rotor current control, where controlling the total real power has been translated into controlling the stator power which, in turn, is controlled through \( i_{rq} \). \( S \) is the Laplace transform operator.

The control block diagram shown in Fig. 5 includes two proportional–integral control loops corresponding to the power and current demands and feedbacks, respectively. The control gains will determine the speed of the power response. The power loop is necessary in addition to the current loop to achieve decoupled control of the real and reactive power in a doubly fed induction machine \[ 14 \]. It will be shown in Section V that the responsiveness of the power/current control is important from the viewpoint of electromechanical interaction in the flywheel system.
Using the notation shown in Fig. 1 and ignoring the friction and windage losses in the system, the dynamic equation governing the relative torsional vibration between the machine rotor and the flywheel can be derived as follows where $T_e$ is the air-gap torque of the machine, being positive when generating [15]

\[
\frac{d^2(\delta_2 - \delta_1)}{dt^2} = -K \left( \frac{1}{J_1} + \frac{1}{J_2} \right) (\delta_2 - \delta_1) - \frac{T_e}{J_2}. \tag{8}
\]

Assume that the speed of the flywheel with a large moment of inertia is approximately constant in a torsional vibration cycle. Speed of the machine rotor changes with respect to the flywheel, corresponding to a change of the machine slip. Let $\delta_{21} = \delta_2 - \delta_1$. The change of the machine rotor speed can be approximated as $\Delta \omega_r = d\delta_{21}/dt$. Considering that $T_e$ will depend on the speed, (8) can be linearized as (9) where $D_e = dT_e/d(\Delta \omega_r)$ is the mechanical damping from the electrical machine to the shaft torsional vibration

\[
\frac{d^2 \Delta \delta_{21}}{dt^2} + \frac{D_e}{J_2} \frac{d \Delta \delta_{21}}{dt} + K \left( \frac{1}{J_1} + \frac{1}{J_2} \right) \Delta \delta_{21} = 0. \tag{9}
\]

It is clear from (9) that $D_e$ needs to be positive for the torsional vibration to be damped. A negative $D_e$ implies that the vibration is unstable; the amplitudes of $\Delta \delta_{21}$ and the corresponding twist torque, $\Delta T_e = K \Delta \delta_{21}$, are to grow with time.

For a given power demand from the machine, Fig. 6(a) shows the steady-state torque–speed relationship: $T_e \cdot \omega_r = P$. In the generating mode, the slope of the curve is negative implying negative values of $D_e$. Torsional vibration would be unstable and gradually amplified in such a case. However, this steady-state torque–speed relationship requires that the power response of the doubly fed induction machine is infinitely fast, i.e., $P$ constant. In practice, the electrical power response does not need to be faster than a torsional vibration cycle as long as the power fluctuation or response that the flywheel system intends to produce is much slower than this. It is therefore hoped that the torque–speed relationship for the generating mode can be modified as sketched in Fig. 6(b). The overall torque–speed curve is still that for a constant power, but the minor loops corresponding to cycles of the torsional vibration are orientated toward “northeast” implying that the phase angles of the torque and speed variations at the vibration frequency are close to each other. The damping at the torsional vibration frequency is then positive. At that frequency, the damping $D_e$ may be a complex number, and the requirement for a stable scenario is that the real part of $D_e$ be positive at the concerned frequency

\[
\text{Re}[D_e(j\omega_{vib})] = \text{Re} \left[ \frac{\Delta T_e(j\omega_{vib})}{\Delta \Omega_r(j\omega_{vib})} \right] > 0 \tag{10}
\]

where $\omega_{vib}$ is the angular torsional vibration frequency (in radians per second) and $\Delta T_e(S)$ and $\Delta \Omega_r(S)$ are Laplace transforms of increments of the air-gap torque and machine rotor speed, respectively. Such a frequency-domain relationship will be derived in the next section to establish a design criterion.

Damping will not significantly affect the torsional vibration frequency of the shaft [16], which can be calculated as

\[
\omega_{vib} = \sqrt{\frac{K(J_1 + J_2)}{J_1J_2}}. \tag{11}
\]

The aforementioned analysis indicates that the choice of electrical control parameters, which determine the responsiveness of power control, and the mechanical design, which determines the shaft torsional vibration frequency, should be coordinated. In terms of the frequency response of $D_e(j\omega)$ in the generating mode, the real part will generally start from negative values when the frequency $\omega$ is low because this will be similar to the steady-state scenario without torsional vibration. $\text{Re}[D_e(j\omega)]$ gradually becomes positive as $\omega$ increases, and the torsional vibration begins to modify the transient torque response of the machine. Suppose that the zero-crossover frequency is $\omega_{cr}$, it may be advisable to set the natural torsional vibration frequency of the shaft, $\omega_{vib}$, three to five times above $\omega_{cr}$, as illustrated in Fig. 7. However, the actual margin that such a design approach can provide depends on the shape of the frequency response; hence, the control gains are adopted as to be shown later, necessitating a detailed model to evaluate $D_e(j\omega)$.

A way to increase the natural frequency of the shaft is to increase the stiffness, as shown hereinafter and demonstrated in the next section. This means either reducing the length $L$ (in meters) or increasing the diameter $D$ (in meters) of the shaft. While there is a minimum requirement on the shaft length to accommodate bearing support, increasing diameter will result in increased bearing losses. The shear stress on the shaft must be checked. The shaft stiffness, $K (\text{N} \cdot \text{m}/(\text{rad}/\text{s}))$, is related to the dimensions for a solid cylindrical shaft as follows [11]:

\[
K = \frac{\pi GD^4}{32L} \tag{12}
\]

where $G$ (in newtons per square meter) is the shear modulus of the material, e.g., 79.3 GN/m² for steel. The shear stress (in newtons per square meter) is

\[
\sigma_s = \frac{16T}{\pi D^3} \tag{13}
\]
where $T$ is the torque (in newton meters) applied on the shaft. When the flywheel system is used for wind farm power smoothing, it is important to check against the fatigue stress under cyclic loading corresponding to the power-smoothing mission profile.

Fig. 8 shows, in simulation results, the phenomenon that this study is trying to prevent by means of coordinated electromechanical design. The parameters of the 5-MW machine are given in Appendix A. The shaft stiffness is $K = 1.010\,000\,N\cdot m/\text{rad}$. It is assumed that the total machine electrical power, $P = P_s - P_r$, is demanded to step change between 0.8 p.u. generating and 0.8 p.u. motoring; the power level is within the power capability limit of the system even at the minimum speed. The transient of power response from the generating mode to motoring mode takes about 100 ms, which is very fast indeed. It is clear that the response of the twist coupling torque $T_c$ is unstable during the generating mode but becomes damped upon returning to the motoring mode. The base values for the per-unit power and torque, as well as current, voltage, speed, and flux linkage, are listed in the Appendix. The control gains given in the figure caption are scaled to the same bases.

Fig. 9 shows the results in the same scenario, but the control gains of the power loop are reduced so that the response is slowed down; the transient of power response now takes about 400 ms instead of 100 ms. In this case, the response of the shaft twist torque $T_c$ is always damped. The damping is clearly greater in the motoring mode. Comparison shows that the characteristic of the power controller in the doubly fed induction machine can indeed affect the damping to the torsional vibration on the shaft. The next section will describe an analytical method to predict the damping and analyze the selection of the shaft stiffness.

Fig. 10. Frequency response of electrical damping.

V. FREQUENCY-DOMAIN ANALYSIS AND DESIGN COORDINATION

According to Fig. 5, the machine air-gap torque, which is proportional to $i_{rq}$ as $T_e = K_{Te} i_{rq}$, is affected by the slip or speed through two mechanisms. First, as the speed changes, the demand to the stator power (the input in Fig. 5) will change to keep the total power at the specified level. This is shown in Fig. 4 as $P_s^* = P^*/(1 - s)$. Second, the speed change will also result in a change of the motion electromotive force (EMF) in Fig. 5, directly acting on the machine model to affect $i_{rq}$.

Derivation of the damping effect $D_e(j\omega)$ needs to recognize these mechanisms. Following Fig. 5

$$P_s(S) = F_1(S)P_s^*(S) + F_2(S)E_{\text{motion}}(S)$$

where the transfer functions $F_1(S) = P_s(S)/P_s^*(S)|_{E_{\text{motion}}(s)=0}$ and $F_2(S) = P_s(S)/E_{\text{motion}}(S)|_{P_s^*(S)=0}$ are dependent on the
Fig. 11. Effect of control gains on electrical damping ($P^* = 1.0$ p.u. and $\omega_r = 1.3$ p.u.).

machine parameters and control gains, as well as the operating point. $F_1(S)$ and $F_2(S)$ are expanded in Appendix B where all quantities are in per unit.

Because $P_s = T_e \omega_r$, then $T_e = P_s$ in per unit using synchronous speed as the base speed. Further, because $P_s^* = P^*/(1 - s) = P^*/\omega_r$ in terms of per unit and $E_{\text{motion}} = -s(\sigma i_{rd} + L_m/L_s \psi_s)$, the following incremental relationships in per unit can be obtained:

$$\Delta P^* = -\frac{\Delta \omega_r}{\omega_r^2} \Delta P^*$$  

$$\Delta E_{\text{motion}} = -\Delta s \left( \sigma i_{rd} + \frac{L_m}{L_s} \psi_s \right) = \Delta \omega_r \left( \sigma i_{rd} + \frac{L_m}{L_s} \psi_s \right).$$  

The damping can then be derived hereinafter by taking the increment of (14) and combining with the Laplace transforms of (15) and (16). The two terms are denoted as $D_{e1}(S)$ and $D_{e2}(S)$, respectively, to distinguish the contribution of the aforementioned two mechanisms. $P^*$, $i_{rd}$, $\psi_s$, and $\omega_r$ are all values at the operating point.

$$\frac{\Delta T_e(S)}{\Delta \Omega_r(S)} = F_1(S) \left( -\frac{P^*}{\omega_r^2} \right) + F_2(S) \left( \sigma i_{rd} + \frac{L_m}{L_s} \psi_s \right).$$  

The damping defined previously in (10) can then be calculated by letting $S = j\omega$. The frequency-domain calculation is used to evaluate the effects of the control gains and operating point. Fig. 10 shows the frequency response of the total and two components of the electrical damping, with 1.0 p.u. power, 1.3 p.u. speed, and control gains which are the same as those used for Fig. 9. The damping component due to the dependence of the stator power on the speed [Re[$D_{e1}(j\omega)$], corresponding to the first term of (17)] always contributes negative damping, while the component associated with the motion EMF [Re[$D_{e2}(j\omega)$], corresponding to the second term of (17)] becomes positive at higher frequencies due to the phase shift at the frequency, eventually resulting in a positive overall damping (Re[$D_e(j\omega)$]). A critical frequency $\omega_{cr}$ is identified as the overall damping crosses zero. A low critical frequency is desirable from the shaft design point of view to avoid having to provide high stiffness. This is usually associated with more sluggish power response. For wind farm power smoothing, the required bandwidth of power response is usually below 1 Hz [17], [18].

Using the control gains for Fig. 9 as the base case, Fig. 11 shows the effect of changing one gain each time on the critical frequency and the amplitude of the electrical damping. Roughly speaking, damping from the machine is reduced, and critical frequency increased when the power or current control loop is tuned faster by increasing the gains.

Fig. 12 shows the effect of the power demand and speed of the initial equilibrium point around which the system model is linearized. As the demanded reference power level of the generating mode increases, the damping reduces and the critical frequency increases. Similarly, as the initial speed reduces, the
negative damping contribution due to the dependence of the stator power on the speed becomes more significant, resulting in a lower overall damping and higher critical frequency for the shaft mechanical design to avoid.

For the shaft stiffness previously used in the simulation for Figs. 8 and 9, i.e., $K = 1010000 \text{ N} \cdot \text{m} / \text{rad}$, the natural frequency of torsional vibration is 36 rad/s or 5.73 Hz. This is just above the critical frequency at maximum power. Consider that the stiffness is increased to $K = 10\,000\,000 \text{ N} \cdot \text{m} / \text{rad}$ with the shaft diameter increased by about 78% according to (12). Then, the natural frequency is increased to 113.8 rad/s or 18.1 Hz. The simulated response to step power changes is shown in Fig. 13, which is to be compared with Fig. 9. The torsional vibration is now well attenuated although the control gains remain the same.

Given a shaft length, for example, $L = 1 \text{ m}$, for the ease of mounting and ventilation, the required stiffness determines the diameter of the shaft. When $K = 1010000 \text{ N} \cdot \text{m} / \text{rad}$, the diameter needed is $D = 0.1067 \text{ m}$. When $K = 10\,000\,000 \text{ N} \cdot \text{m} / \text{rad}$, the diameter is $D = 0.1898 \text{ m}$, which will result in a 216% increase of bearing loss because of the quadratic relationship of the loss to diameter for the same loading and angular speed [19]. Table II lists the diameters, shear stresses at nominal torque [using (13)], and bearing loss for the two stiffness values. Since the bearing loss depends on the actual bearing chosen and the lubrication method, only the ratio between the two cases is indicated. The shear stress is acceptable for $D = 0.1898 \text{ m}$.

The analysis showed that it is possible to obtain a working system; nevertheless, there is also the risk of mismatch between electrical control and shaft mechanical designs. For instance, Fig. 11 implied that if the control gains are set too high for fast power response, the damping from the machine to the torsional vibration can remain negative even the natural frequency is set very high by increasing the shaft stiffness. It is hoped that this paper provides useful knowledge which can be applied in future designs of flywheel systems.

The risk of negative damping to torsional vibration is not only associated with a flywheel system incorporating a doubly fed induction machine which has been modeled in this paper. Such electromechanical interactions can also exist in any other configuration of a flywheel system which has grid connection and is subject to power control. Indeed, flywheels of different topologies have been considered to provide the frequency regulation capability for wind farms [20]. It is also necessary to consider the risk of negative damping to the shaft torsional vibration in these cases in order to achieve a robust design [21]. However, the problem has not happened during the normal operation of a grid-connected wind turbine which is also under power control, and the controlled power response is normally tuned to be very fast. This is because, in a wind turbine, the generator power reference is set according to the machine speed with a positive torque–speed slope, which inherently provides positive damping [22]. The first term of (17) is therefore irrelevant in the wind turbine case. In a flywheel system for wind farm power smoothing or for participating in

<table>
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<tr>
<th>$K$ (Nm/rad)</th>
<th>$D$ (m)</th>
<th>$\sigma_c$ (MN/m$^2$)</th>
<th>bearing loss (W)</th>
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<td>1010000</td>
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<td>10100000</td>
<td>0.1898</td>
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grid frequency regulation, the power reference will need to be independent of its speed. It is this difference that has caused the present concern of negative damping to the shaft torsional vibration in the flywheel system [23].

VI. CONCLUSION

This paper analyzed the electromechanical interactions in a flywheel system incorporating a doubly fed induction machine. There is the benefit of reducing the converter size without significantly increasing the moment of inertia of the flywheel which operates in a more restricted speed range. When the system is used for power smoothing or frequency response control for a wind farm with grid connection, there is the risk that the machine provides negative damping to the shaft torsional vibration in the generating mode. Resonance should and can be avoided by coordinating the design of the electrical machine controller and the mechanical shaft. This paper has described a control strategy, based on which a frequency-domain model is established to predict the dynamic behavior of the flywheel shaft system. The model can be used in the design of the electromechanical device.

A 5-MW 150-MJ flywheel system has been simulated to demonstrate the phenomenon and the proposed solution. It is expected that this paper could provide a case for experimental verification and could be extended to other configurations of flywheel systems targeting similar applications.

APPENDIX A

MACHINE PARAMETERS

The following are the parameters of the per-unit system:

1) base capacity: $S_b = 5 \text{ MW}$;
2) base frequency: $f_b = 50 \text{ Hz}$;
3) base stator voltage (phase, peak value): $V_{sb} = 563.4 \text{ V}$;
4) base stator current (peak value): $I_{sb} = 5.9166 \text{ kA}$;
5) base stator flux linkage: $\Psi_{sb} = 1.7933 \text{ V}$ - s;
6) base torque: $T_b = 31.831 \text{ kN}$ - m;
7) base speed: $\eta_{sb} = 1500 \text{ r/min}$.

The following are the parameters of the machine:

1) $S_n = 5.0 \text{ MVA}$, $f_n = 50 \text{ Hz}$, and $U_n = 690 \text{ V}$ (line–line, rms);
2) winding connection (stator/rotor): star/star;
3) number of pole pairs: $P_p = 2$;
4) stator resistance: $R_s = 0.098841 \text{ p.u.}$;
5) stator leakage inductance: $L_s = 0.1248 \text{ p.u.}$;
6) rotor resistance: $R_r = 0.00549 \text{ p.u.}$;
7) rotor leakage inductance: $L_l = 0.09955 \text{ p.u.}$;
8) magnetizing inductance: $L_m = 3.9527 \text{ p.u.}$.

APPENDIX B

$F_1(S)$ and $F_2(S)$ in Equation (14)

See the equations for $F_1(S)$ and $F_2(S)$, shown at the top of this page.

REFERENCES


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