Electrothermal simulation of superconducting nanowire avalanche photodetectors

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Electrothermal simulation of superconducting nanowire avalanche photodetectors

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We developed an electrothermal model of NbN superconducting nanowire avalanche photodetectors (SNAPs) on sapphire substrates. SNAPs are single-photon detectors consisting of the parallel connection of N superconducting nanowires. We extrapolated the physical constants of the model from experimental data and we simulated the time evolution of the device resistance, temperature and current by solving two coupled electrical and thermal differential equations describing the nanowires. The predictions of the model were in good quantitative agreement with the experimental results. © 2011 American Institute of Physics. [doi:10.1063/1.3560458]

We simulated the photoresponse of NbN superconducting nanowire avalanche photodetectors (SNAPs) (Ref. 1) on sapphire substrates. SNAPs are single-photon detectors consisting of the parallel connection of N superconducting nanowires (N-SNAPs, see Fig. 1), which provide a signal-to-noise ratio (SNR) of factor \( N \) higher than ordinary superconducting nanowire single-photon detectors (SNSPDs),1 consisting of a single nanowire. Our group recently demonstrated that this enhancement of the SNR was crucial to reading out the photoresponse of ultra-narrow (20-nm-wide) nanowires.2

The detector is connected in series with an inductor \( L_{\text{S}} \), to ensure the correct operation of the device3 and in parallel with a readout resistor \( R_{\text{load}} \). As all the parallel sections are nominally equal and have the same kinetic inductance \( L_{\text{K}} \), at the steady state they are biased at the same current \( I_{\text{B}}/N \), where \( I_{\text{B}} \) is the total bias current through the device.

When one section absorbs a photon, it switches to the normal state (initiating section) and diverts part of its current to the remaining sections (secondary sections) and \( R_{\text{load}} \). If \( I_{\text{B}} \) is higher than a particular current level (the avalanche threshold current, \( I_{\text{AV}} \)), the current redistributed from the initiating section biases the secondary sections above their critical current \( I_{\text{C}} \). At this point, all the sections become resistive, so most of the current flowing through the device \( I_{\text{B}} \) is diverted to \( R_{\text{load}} \), where a current pulse \( N \) times higher than the current in a single section is formed.

\( I_{\text{AV}} \) is a key parameter for the device design and can be easily measured, providing a convenient validity check for the model of the device physics. As the existing model of SNAP operation failed to fit the experimental data,4 we developed an alternative phenomenological model describing the dynamics of the device after the formation of a normal domain in the initiating section (see Ref. 2 for a direct comparison between the two models).

We solved the two coupled electrical and thermal differential equations reported in Ref. 5 for each section of the SNAP at every instant of the simulation, obtaining the time evolution of the currents in the circuit and of the nanowires resistances and temperature profiles. Prior models for superconducting-nanowire-based detectors were not suitable to accurately describe the cascade switching of the secondary sections in SNAPs because they either (1) disregarded the Joule heating in the photon-induced normal domain;6 (2) could only model single-nanowire detectors;5,7 (3) described parallel-nanowire detectors with a purely electrical model;8 or (4) described SNAPs as equivalent wider-nanowire single-nanowire detectors.9

We assumed the thermal response of the NbN nanowire to be bolometric and described the electron and phonon subsystems with a single reduced temperature (see Refs. 5 and 11 for further details). We did not use a two-temperature \( (2-T) \) description of our system such as the one proposed in Ref. 7 for the following reasons. First, the phonon escape time of few-nanometer-thick NbN on sapphire \( (\tau_{\text{es}}) \) is of the same order as the phonon–electron interaction time \( (\tau_{\text{p-e}}) \),10 which makes assuming thermal equilibrium between the two subsystems a reasonable approximation [which does not apply e.g., to Nb, for which \( \tau_{\text{p-e}} \sim 10 \tau_{\text{es}} \) (Ref. 10)]. Second, the \( 2-T \) model relies on six parameters which are extremely challenging to determine experimentally, while our bolometric heat equation relies on only four parameters, two of which can be easily estimated from independent dc electrical measurements (see Ref. 11, section B). Third, although we

![FIG. 1. (Color) Equivalent electrical circuit of an N-SNAP. The initiating section is in red, the secondary sections are in blue. The value of \( R_{\text{load}} \) was 50 \( \Omega \), and the value of \( L_{\text{S}} \) was chosen to satisfy the condition: \( L_{\text{S}}(N-1)/L_{\text{S}}=10 \) (Ref. 3).](image-url)
over-estimated the cooling of the electrons by the phonons (for assuming the two subsystems to be in thermal equilibrium), the discrepancy between the reduced temperature of our model and the electron temperature predicted by the 2-T model was partially compensated by under-estimating the cooling of the phonons by the substrate (see Ref. 11, section C).

Our model disregards the mechanism of formation and expansion of the photon-induced hotspot, 12 so the absorption of a photon results in the immediate superconducting-to-normal transition of a nanowire slab at the center of the nanowire. We assumed this initial normal slab to be as long as the NbN coherence length at zero temperature (ξ) and at a temperature (the normal-slab temperature, T sub) higher than the substrate temperature (T sub). This last assumption was motivated by the fact that if we simulated the photoresponse of an SNSPD at low bias (I_B < 0.7I_C) and at too small a T sub value (for example, T sub = T sub), the initial normal slab did not expand and no current was diverted to the load, which was in contrast to the experimental data. The normal-slab temperature was assumed to be T sub = 8.5 K, based on the following criterion: we performed preliminary simulations of the photoresponse of an SNSPD (of inductance L = 36 nH) biased at I_B = 0.6 I_C for a variety of values of T sub such that I_C(T sub) < I_B and we adopted the minimum T sub for which more than 50% of I_B was diverted to R load as the value of the normal-slab temperature in later device simulations.

To accurately describe the avalanche formation mechanism in SNAPs, we needed to model the nanowire response as a parameter in the hotspot-plateau regime simulations. The normal-slab temperature, T sub, was assumed to be

\[ T_{sub} = 8.5 \text{ K} \]

based on the following criterion: we performed preliminary simulations of the photoresponse of an SNSPD (of inductance L = 36 nH) biased at I_B = 0.6 I_C for a variety of values of T sub such that I_C(T sub) < I_B and we adopted the minimum T sub for which more than 50% of I_B was diverted to R load as the value of the normal-slab temperature in later device simulations.

We modeled the thermal coupling between NbN and sapphire with a state-independent heat-transfer coefficient with unit area α = A · T 3.3 As the outcome of our electrothermal simulations (e.g., the value of I WL) was strongly dependent on the value of A, we estimated it from experimental data with the following method: we performed several electrothermal simulations of a nanowire voltage-biased in hotspot-plateau regime 14 [Fig. 2(a)] varying the value of A to reproduce the behavior observed experimentally.

The circuit simulations started with the nanowire in the superconducting state, the bias voltage V_B = 0 V, and the current I_B = 0 A. At time = 0 s, we suddenly increased V_B and as a result, I_B increased until it exceeded the nanowire I_C [Fig. 2(b)]. A normal domain then formed at the center of the nanowire (around the weak link), whose size varied in time [see Fig. 2(c)] until it stabilized when I_B reached a constant value (the hotspot current, I_HS), as expected for the hotspot-plateau regime. 14 The nanowire responded to any variation in V_B by changing the size of the normal domain and keeping I_B constant and equal to I_HS, which confirmed that our model correctly describes the hotspot-plateau regime. To find the correct value of A for our nanowires we relied on the fact that the value of I_HS depends on the thermal coupling between the NbN film and the sapphire substrate. 14 We used A as a parameter in the hotspot-plateau regime simulations to reproduce a value of I_HS matching the hot-spot current measured on 30-nm-wide-nanowire SNSPDs (I_HS = 1.6 ± 0.1 μA, extracted from the current-voltage curves measured on 20 devices). We note that we could not use the analytical expression of I_HS as a function of A reported in Ref. 14 to extrapolate A from the experimental value of I_HS. Indeed, the expression in Ref. 14 was derived under the assumption that a stationary normal domain could exist only where T(x) > T_C. We argue that a stationary normal domain could exist under the more general condition; I_C(T(x)) < I_HS. Indeed, our simulations showed that the stationary normal domain of Fig. 2(c) was associated with a temperature profile [Fig. 2(d)] T(x) < T_C everywhere.

To illustrate the capabilities of our model, we simulated the photoresponse of a 2-SNAP. We recorded the time evolution of the temperature and resistivity along the initiating and secondary sections of the device (Fig. 3). We also extracted the total resistance of each section and the current through the different parts of the circuit (Fig. 4).

To simulate a photon being absorbed in the initiating section (at time = 1 ps), a ξ-long slab is switched to the normal state [Figs. 3(a) and 3(b)]. The normal domain expands due to Joule heating, so the resistance in the initiating section increases [Fig. 4(a)] and the current through it starts redistributing to the secondary section [Fig. 4(b)]. When the cur-

![](image)
The series inductance was $L_s = 135 \text{ nH}$. The current through the secondary section becomes overcritical (at time=20 ps) both sections become current-dependent resistors connected in parallel, so their resistance and current fluctuate until they equilibrate, reaching the same value. At this point, the current through the device is redistributed to the read out (which converts it into a proportional voltage signal), until the two sections switch back to the superconducting state [at time=227 ps, Fig. 4(b)].

Performing simulations at different values of the $I_B$, we could estimate the avalanche current of SNAPs with any number of parallel sections. The simulated values of $I_{AV}$ for 2-, 3-, and 4-SNAPs were in close quantitative agreement with the experimental values (see Table 1).

The quality of the agreement between experiment and theory was perhaps surprising given the assumptions in the model (e.g., the fact that we combined the electron and phonon temperatures in the metal). This high-quality agreement may be partially coincidental, or may suggest that the relative dynamics between the electrons and the phonons does not play a significant role in the device operation.

In conclusion, we developed an electrothermal model of N-SNAPs. Our simulations predicted avalanche currents in agreement with the experimental values and clarified the operation mechanism of these devices.

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<th>2-SNAP</th>
<th>3-SNAP</th>
<th>4-SNAP</th>
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<tr>
<td>$I_{AV}/I_{SW}$ (experiment)</td>
<td>0.68±0.015</td>
<td>0.78±0.02</td>
<td>0.84±0.03</td>
</tr>
<tr>
<td>$I_{AV}/I_{SW}$ (simulation)</td>
<td>0.67</td>
<td>0.78</td>
<td>0.82</td>
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![Image of simulated time evolution of the temperature and resistivity along the initiating and secondary sections of a 2-SNAP](image)

![Image of simulated time evolution of the current through the initiating section and the secondary section](image)
Supplementary Information

Francesco Marsili¹, Faraz Najafi¹, Charles Herder¹, Karl K. Berggren¹²

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²Kavli Institute of Nanoscience, Delft University of Technology, Lorentzweg 1, 2628CJ Delft, The Netherlands.

A. Parameters of the electro-thermal simulation.

We present and discuss additional parameters of the electro-thermal simulations in the following (see Table SI - I). The substrate temperature used in the model was $T_{\text{sub}} = 4.7$ K, which was based on the temperature measured by a Si diode sensor glued with cryogenic varnish to a detector chip mounted on our cryogenic-device-measurement setup¹. The nanowires simulated in this paper were 30 nm wide and 4.5 nm thick, like the devices in ref.¹. The nanowire critical temperature was $T_C = 10.8$ K, which was measured on bare NbN films. The nanowire critical current was $I_C = 7.2$ µA, as we measured on 30-nm-wide-nanowire SNSPDs¹. The coherence length of our NbN films was assumed to be $\xi = 7$ nm following ref.², which reports on similar films. The nanowire inductance per square was $L = 80$ pH/□, which was estimated from the fall time of the photoresponse pulse of 30-nm-wide-nanowire SNSPDs. The nanowire resistance per square was $R = 680$ Ω/□, which was estimated from the ohmic branch of the $I$ - $V$ curves of 30-nm-wide-nanowire SNSPDs measured at 4.7 K.

Table SI - I. Parameters of the electro-thermal simulation.

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<th>Quantity</th>
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<td>substrate temperature</td>
<td>4.7 K</td>
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<tr>
<td>$T_C$</td>
<td>critical temperature</td>
<td>10.8 K</td>
</tr>
<tr>
<td>$I_C$</td>
<td>critical current</td>
<td>7.2 µA</td>
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<tr>
<td>$\xi$</td>
<td>Ginzburg-Landau coherence length</td>
<td>7 nm</td>
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<tr>
<td>$L$</td>
<td>kinetic inductance per square</td>
<td>80 pH/□</td>
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<tr>
<td>$R$</td>
<td>resistance per square</td>
<td>680 Ω/□</td>
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<td>$T_n$</td>
<td>normal-slab temperature</td>
<td>8.5 K</td>
</tr>
<tr>
<td>$A$</td>
<td>Temperature coefficient of the heat-transfer coefficient per unit area</td>
<td>300 W/(m²K⁴)</td>
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B. Derivation of the bolometric heat equation

The general form of the one-dimensional two-temperature heat equations for NbN is³:
\[
\frac{\partial C_e(T_e)T_e}{\partial t} = -\frac{C_e(T_e)}{\tau_{ee}(T_e)}(T_e - T_p) + j^2 \rho + \frac{\partial}{\partial x} \left[ \kappa_e(T_e) \frac{\partial T_e}{\partial x} \right]
\]

(1)

\[
\frac{\partial C_p(T_p)T_p}{\partial t} = \frac{C_p(T_p)}{\tau_{ep}(T_p)}(T_e - T_p) - \frac{C_e(T_e)}{\tau_{ne}(T_e)}(T_e - T_{nnb}) + \frac{\partial}{\partial x} \left[ \kappa_e(T_e) \frac{\partial T_p}{\partial x} \right]
\]

(2)

where \(T_e\) and \(T_p\) are the electron and phonon temperatures; \(C_e\) is the electron specific heat: \(C_e \propto T_e\) in the normal state and 
\(C_e \propto \exp[-\Delta(T_e)/k_B T_e]\) in the superconducting state \(^2\); \(C_p \propto T_p^3\) is the phonon specific heat \(^2\); \(\tau_{e-p} \propto T_e^{-1}e\) is the electron-phonon interaction time \(^4\); \(\tau_{esc}\) is the phonon escape time to the substrate; \(\kappa_e\) and \(\kappa_p\) are the temperature-dependent electron and phonon thermal conductivities; \(\rho\) is the NbN resistivity; \(j\) is the nanowire current density.

The bolometric heat equation is obtained adding equation (1) to equation (2) and setting \(T_e = T_p = T_r\):

\[
\frac{\partial}{\partial t} \left[ C(T_r)T_r \right] = j^2 \rho - \frac{C_p(T_r)}{\tau_{ne}}(T_r - T_{nnb}) + \frac{\partial}{\partial x} \left[ \kappa(T_r) \frac{\partial T_r}{\partial x} \right]
\]

(3)

where \(T_r\) is the reduced temperature, \(C = C_e + C_p\) and \(\kappa = \kappa_e + \kappa_p\).

We further simplified equation (3) into:

\[
\frac{\partial}{\partial t} \left[ C(T_r)T_r \right] = j^2 \rho - \frac{\alpha}{d}(T_r - T_{nnb}) + \frac{\kappa(T_r)}{d} \frac{\partial^2 T_r}{\partial x^2}
\]

(4)

neglecting the phonon thermal conductivity and the spatial dependence of \(\kappa_e\) (\(\partial \kappa_e / \partial x \cdot \partial T_r / \partial x \approx 0\)) as in ref. \(^5\). We note that we expressed the phonon-substrate coupling term in equation (3) in terms of the heat-transfer coefficient per unit area \(\alpha = A \cdot \kappa_3\) and the film thickness \(d\). For the value and temperature dependence of \(C\) we followed ref. \(^2\) and we estimated \(\kappa_e\) from the nanowire resistivity as in \(^6\).

C. Comparison between the bolometric and the 2-T heat equations

In this section we evaluate the validity of our choice of using the bolometric model over the 2-T model to describe the thermal response of our nanowires.

The value of \(A / d\) that we estimated from the experimental value of the hotspot current was of the same order of magnitude, but lower than the ratio between the temperature coefficient of \(C_p\) \((C_p)\) and \(\tau_{esc}\) reported in ref. \(^2\) for similar films: \(A / d = 67\) W/(mm\(^3\)K\(^4\)) \(\sim C_p / \tau_{esc} = 125\) W/(mm\(^3\)K\(^4\)). The fact that the phonon escape time estimated from our value of \(A / d\) (146 ps) was a factor \(\sim 2\) larger than in ref. \(^2\) (78 ps) implied that we under-estimated the cooling of the phonons by the
substrate, which in fact partially compensated for over-estimating the cooling of the electrons by the phonons (for assuming thermal equilibrium between the two sub-systems).

To quantitatively support this last claim, we compared the thermal response of a superconducting nanowire to an optical excitation pulse simulated with the bolometric equation and our value of $A$, with the result of the 2-$T$ equations relying on the parameters reported in 2. For simplicity, we assumed the temperature to be homogeneous along the nanowire and then neglected the thermal conduction terms. The excitation pulse had a peak optical power density of 1.5 mW $\mu$m$^{-3}$ and a duration of 300 ps, which reproduced the joule heating produced by a current of 3 $\mu$A flowing through a photon-induced normal domain. The time evolution of the reduced temperature ($T_r$) simulated with the bolometric model was in agreement with the average temperature ($T_{avg}$) between the electron and phonon temperatures simulated with the 2-$T$ model (Figure SI - 1). Considering that the results of the 2-$T$ model were obtained with no free parameters, we concluded that our bolometric model described the nanowire thermal response with an acceptable approximation respect to the more complete model.

![Figure SI - 1. Simulated time evolution of the reduced temperature ($T_r$ in black), the electron temperature ($T_e$, in red), the phonon temperature ($T_p$, in blue), and the average between $T_e$ and $T_p$ ($T_{avg}$, in blue).](image)