Hanbury Brown–Twiss Interference of Anyons

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Hanbury Brown–Twiss Interference of Anyons

Gabriele Campagnano, 1 Oded Zilberberg, 1 Igor V. Gornyi, 2,3 Dmitri E. Feldman, 4 Andrew C. Potter, 5 and Yuval Gefen 1

1 Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot 76100, Israel
2 Institut für Nanotechnologie, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany
3 A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia
4 Department of Physics, Brown University, Providence, Rhode Island 02912, USA
5 Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

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We present a study of a Hanbury Brown–Twiss interferometer realized with anyons. Such a device can directly probe entanglement and fractional statistics of initially uncorrelated particles. We calculate Hanbury Brown–Twiss cross correlations of Abelian Laughlin anyons. The correlations we calculate exhibit partial bunching similar to bosons, indicating a substantial statistical transmutation from the underlying electronic degrees of freedom. We also find qualitative differences between the anyonic signal and the corresponding bosonic or fermionic signals, indicating that anyons cannot be simply thought of as intermediate between bosons and fermions.

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Two-particle interference is a major pillar of quantum mechanics, very much like the phenomenon of single particle interference. Such interference has been observed with photons in the historical Hanbury Brown–Twiss (HBT) experiment [1,2], and much later with electrons [3]. Quantum Hall systems can exhibit emergent particles (dubbed anyons) with fractional statistics [4,5]. Despite intensive study, direct signatures of anyonic statistics remain elusive. Here we study an HBT interferometer with anyons, which can directly probe entanglement and fractional statistics of initially uncorrelated particles. Specifically, we calculate HBT cross correlations of Abelian Laughlin anyons. The correlations exhibit partial bunching similar to bosons, indicating a substantial statistical transmutation from the underlying electronic degrees of freedom [6]. Furthermore, we find qualitative differences between the anyonic signal and the corresponding bosonic or fermionic signals, indicating that anyons cannot be simply thought of as intermediate between bosons and fermions.

Edge channels of a fractional quantum Hall system offer a natural framework to study transport properties of anyons. Earlier attempts to consider entanglement of such quasiparticles (QPs) either addressed time-resolved correlation functions [7] (which may be very hard to measure) or relied on a single source geometry setup [8–10] (which may introduce superficial interaction-induced correlations). Here we study zero-frequency current-current correlations in a truly HBT interferometer setup, whose physics is governed by QPs dynamics. Because of their fractional charge and fractional statistics, scattering of these QPs results in non trivial correlations. Below, we consider the case $\nu = 1/3$ for concreteness, but our analysis can be generalized to other Laughlin fractions.

Consider first a heuristic estimate of these correlations, outlined in Fig. 1. Two particles are emitted, respectively, from two sources $S_1$ and $S_2$ and scattered toward two detectors $D_1$ and $D_2$ by a beam splitter, e.g., a quantum point contact (QPC) for electrons and QPs, or a half-silvered mirror for photons. We evaluate the probability $P(m, 2m)$, $m = 0, 1, 2$, that $m$ particles are collected at the drain $D_1$ while $(2 - m)$ are collected at the drain $D_2$. Consider, e.g., the diagrams contributing to $P(1, 1)$ [cf. Figure 1(a)]. Each diagram represents an amplitude contributing to $P(1, 1)$. Their weights are $tt'\exp[i(1/2)\pi \nu]$ and $rr'\exp[i(3/2)\pi \nu]$, with $\nu = 0, 1, 1/3$ for bosons, fermions, and $\nu = 1/3$ anyons, respectively.

FIG. 1 (color online). Two-particle amplitudes contributing to (a) $P(1, 1)$, the two particles are emitted from $S_1$ and $S_2$, and collected at $D_1$ and $D_2$; (b) $P(0, 2)$, both particles are collected at $D_1$, $t, r, t', r'$ are the single particle scattering amplitudes, $|t|^2 + |r|^2 = 1$, $|t'|^2 + |r'|^2 = 1$. Note the statistical factors, reflected by the winding of one particle around the other. For (a) they are $\exp[i(1/2)\pi \nu]$ and $\exp[i(3/2)\pi \nu]$, respectively.
Note that we have included quantum statistics factors which reflect the extent by which one-particle winds around the other. It follows that \( P(1, 1) = |r^2 e^{i(1/2)\pi \nu} + r r' e^{i(3/2)\pi \nu}|^2 \). For simplicity we consider symmetric scatterers, \( |r| = 1 \), in which case \( P(1, 1) = (1/2)(1 - \cos \pi \nu) \). Similarly, \( P(2, 0) = P(0, 2) = (1/4) \times (1 + \cos \pi \nu) \). For classical particles one sums up probabilities, rather than amplitudes, leading to \( P_{\text{class}}(1, 1) = 1/2 \), \( P_{\text{class}}(2, 0) = P_{\text{class}}(0, 2) = 1/4 \). The results for fermions and bosons coincide with calculations based on second quantization [11]. For \( \nu = 1/3 \) this results in bosonlike bunching [7] \( |P_{\nu=1/3}(2, 0) > P_{\text{class}}(2, 0) \).

Our main analysis, outlined below, reinforces the observation that the scattering of two Laughlin anyons is bosoniclike. At the same time, it also reveals the nonanalytic structure of the interferometry signal of such anyons, implying that the latter are not simple interpolation between fermions and bosons. The schematic setup is depicted in Fig. 2. What replaces optical beams in the solid state device are edge states of the quantum Hall effect, formed due to the Aharonov-Bohm (AB) flux (\( \Phi_{\text{AB}} \)) connected to the four QPCs along the edges), \( \Phi_{\text{tot}} \) is relevant for our analysis.

A QP in a quantum Hall liquid at Laughlin filling factor \( \nu \) can be described as a composite object, consisting of a point charge \( q = e \) with a single quantum magnetic flux solenoid, \( \Phi_0 = hc/e \), attached to it. When a QP encircles another QP it will pick up an AB phase \( \theta = 2\pi \nu \) which accounts for their mutual fractional statistics [14]. When a QP tunnels from the external to the internal edges, its flux is trapped inside the interferometer [15,16]. The magnetic flux enclosed in the active area of the interferometer (depicted in blue, and defined by the line connecting the four QPCs along the edges, in Fig. 2) is \( \Phi_{\text{tot}}(n) = \Phi_{\text{AB}} + \Phi_{\text{stat}}(n) \), where \( \Phi_{\text{stat}} \) is the statistical flux and is given by \( \Phi_0 \) times the number \( n \) of trapped QPs. The dynamics of QPs moving along the edges of the interferometer is then entirely determined by \( n \) modulo \( 3 \), i.e., for a given value of \( \Phi_{\text{AB}}, \) the system can be found in three possible states characterized by \( n \) = 0, 1, 2.

For the study of the nonequilibrium dynamics of our strongly interacting HBT interferometer, we address the Markovian evolution of the system among the three possible values of the statistical flux. Our microscopic Keldysh analysis simplifies, and can be cast in terms of rate equations for a certain parameter range [17]. The rate equations (whose coefficients are obtained by a microscopic analysis) carry information on interference effects of current cross correlations. Below we treat the QP tunneling current at each QPC perturbatively.

Let us define the quantities needed in the ensuing analysis: \( \langle I_i \rangle \) is the average tunneling current measured in drain \( i \) and \( S_{i,f} = \int_{-\infty}^{\infty} dt \langle I_i (t) - \langle I_i \rangle \rangle \langle I_f (t) - \langle I_f \rangle \rangle \rangle \) is the zero-frequency current-current correlations between drains \( i \) and \( f \). The latter is the main object of this Letter. Next, we define \( P(f, t | j) \), the probability to find the system with statistical flux \( j \) [indices are defined modulo(3)] at time \( t \) given that it had statistical flux \( j \) at time zero. The system’s dynamics is governed by a standard master equation:

\[
\frac{d}{dt} P(f, t | j) = \sum_{k=0,1,2} [P(k, t | j) W_{k,f} - P(f, t | j) W_{f,k}].
\]  

Here \( W_{f,k} \) is the total transition rate from the state \( j \) to the state \( f \). In order to study the magnetic flux-dependent part of the current-current correlations, we need to consistently include at least single-QP processes and two-QP processes, i.e., second and fourth order in the tunneling amplitudes \( \Gamma \), respectively. In the high voltage bias limit, \( eV \gg k_B T \), considered here, only processes that transfer QPs from the outer to the inner edges are relevant.
Several microscopic processes, labeled by $\zeta$, contribute to each $W_{j,\ell}$ such that $W_{j,\ell} = \sum_\zeta W^{(\zeta)}_{j,\ell}$. The processes allowed are either $W^{(1)}_{j,j}$, $W^{(1,1)}_{j,j+1}$, or $W^{(1,2)}_{j,j+2}$. The former renormalizes the vacuum current and does not affect any quantity calculated below. $W^{(1,2)}_{j,j+1}$ has contributions from single-QP processes (independent of flux, hence, independent of $j$), as well as from two-QPs processes (dependent of flux). $W^{(1,2)}_{j,j+2}$ consists of two-QPs processes, and may or may not be flux dependent. Each of the rates can be written as $W^{(\zeta)}_{j,\ell} = \bar{W}^{(\zeta)}_{j,\ell} \kappa^{(\zeta)}_j$ with $\kappa^{(\zeta)}_j$ discussed in the caption of Table I, which depicts all relevant processes.

Consider, first, the current collected at any drains. Assuming short-range interactions [18], which is reasonable in the presence of a metallic top gate, this current is flux-independent (similarly to the $\nu = 1$ case [13]), hence, is not of interest for us here. The following argument can be used to show this: consider for instance the current at drain $D_1$: owing to the chiral propagation along the edges this tunneling current does not depend on the scattering at QPC $D$. A gauge transformation can then ascribe the total magnetic flux to QPC $D$—hence the current in drain 3 is AB independent. A similar argument holds for the tunneling currents collected at the other drains.

We, next, consider the AB-dependent component of the cross-current correlations. It is sufficient to express the following rates: the single-QP rates $W^{(1,1)}_{j,j+1} = \gamma|\Gamma_A|^2$ (and similar expressions for rates involving the processes $(1, B, 0), (1, C, 0),$ and $(1, D, 0)$); and the two-QPs rate

$$W^{(1,2)}_{j,j+2} \Phi_{\text{tot}}(\ell)) = \Omega|\Gamma_A \Gamma_B \Gamma_C \Gamma_D|$$

$$\times \cos\left[\frac{2\pi}{3}(\Phi_{AB} + j\Phi_D)/\Phi_0\right].$$

Here the $\Gamma$’s are the QPs tunneling amplitudes at the four QPCs, $\gamma$ and $\Omega$ are coefficients to be calculated below. Using the method developed in Refs. [19,20], we are able to calculate the AB-dependent component of the cross-current correlator $S_{\ell,4}$:

$$S^{\text{AB}}_{\ell,4} = S_{\ell,4} - \langle S_{\ell,4}\rangle_{AB}$$

$$= \frac{e^2|\Gamma_A \Gamma_B \Gamma_C \Gamma_D|^3 \Omega^3 \cos[2\pi(\Phi_{AB} + j\Phi_D)/\Phi_0]}{6(|\Gamma_A|^2 + |\Gamma_B|^2 + |\Gamma_C|^2 + |\Gamma_D|^2)^2 \gamma^2},$$

where $\langle \rangle_{AB}$ refers to averaging over $\Phi_{AB}$.

**Model and Methods.**—The low energy physics of the system is well described by the effective bosonic Hamiltonian [18]

$$H_0 = \frac{\hbar v}{4\pi} \sum_{\ell=1}^4 \int dx (\partial_x \phi_{\ell})^2,$$

**TABLE 1.** Elementary QP transfer processes. Each process, $(\zeta) = (m, N, \phi)$, is characterized according to the change $m$ in the number of QPs trapped in the interferometer, $N$ the QPCs at which QP tunneling takes place, and the flux $\phi$ entering the flux factor $\kappa^{(m,N,\phi)} = \cos(2\pi\phi/(3\Phi_0))$. Note that $\phi = 0$ depicts a flux-independent process and $\phi = \Phi_{\text{tot}}(\ell) = \Phi_{AB} + j\Phi_D$, a process that depends on the total trapped flux. The order of the process (second or fourth in the tunneling amplitude $\Gamma$, the initial and final fluxon states $|j, f\rangle$, where $f-j$ is the added number of statistical fluxons), and the charge added at each drain $[ + 1 \rightarrow$ to the absorption of one QP or charge $q = -(1/3)|e|$ at the drain], are indicated. For example (cf. Figure 3), the process $(\zeta) = (1, A, 0)$ corresponds to the emission of a QP from source $S_A$, its tunneling across QPC $A$, and its trapping at $D_1$. Following the tunneling event, a quasihole is created at edge $S_A$ and a charge $-q$ is consequently absorbed in $D_1$. The flux-dependent processes [the two-QPs trapping process $[2, A B C D, \Phi_{\text{tot}}(\ell)]$ and the single-QP trapping $[1, A B C D, \Phi_{\text{tot}}(\ell)]$] are illustrated in Fig. 3.

**Elementary processes**

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<th>Process $\zeta$</th>
<th>Order</th>
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<th>$D_3$</th>
<th>$D_4$</th>
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<td>$0$</td>
<td>$1$</td>
<td>$-1$</td>
</tr>
<tr>
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<td>$(j, j+1)$</td>
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<td>$1$</td>
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<td>$-1$</td>
</tr>
<tr>
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<td>$\Gamma^2$</td>
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<td>$0$</td>
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<tr>
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<td>$(j, j+2)$</td>
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<td>$0$</td>
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**FIG. 3** (color online). (a) In process $[2, A B C D, \Phi_{\text{tot}}(\ell)]$ two QPs are transferred from edges 1 and 4 to edges 2 and 3, the process is AB-sensitive due to the interference between two amplitudes $A_1$ and $A_2$. In $A_1$, a QP tunnels from edge 1 to edge 3 and a second QP tunnels from edge 4 to edge 2 (red dotted line). In $A_2$, a QP tunnels from edge 1 to edge 2 and a second QP tunnels from edge 4 to edge 3 (blue dashed line). This process changes the statistical flux by two. (b) Process $[1, A B C D, \Phi_{\text{tot}}(\ell)]_1$ and similarly, process $[1, A B C D, \Phi_{\text{tot}}(\ell)]_2$ is also AB-sensitive but in this case only one QP is trapped inside the interferometer changing the statistical flux by one.
describing chiral plasmonic excitations on the four edges (e.g., $S_i D_j$) of the interferometer. Here, $v$ is the plasmonic velocity at the edge. The bosonic fields $\phi_i$ satisfy the commutation relations $[\phi_i(x, t), \phi_j(x', t)]=i\pi \delta_{ik}\text{sgn}(x-x')$. The operators $\exp(i\phi_i/\sqrt{\nu})$ and $\exp(i\phi_i)$ are respectively, proportional to the electron and the quasiparticle creation operator on the edge $i$.

To fully account for the quantum statistics of such particles one needs to multiply these bosonic operators by “string operators,” known as Klein factors [7,15,16,21]. In our analysis, this procedure is replaced by carefully accounting for the dynamics of the statistical flux, which is attached to the tunneling QPs.

The total Hamiltonian, $H=H_0+H_T$, includes a tunneling part, $H_T=(H_T^A+H_T^B+H_T^C+H_T^D)+H_{\text{c.c.}}$, which accounts for the most relevant tunneling operators at the QPCs. We assume that the external (internal) arms are tuned at voltage $V(0)$, select a gauge whereby the flux dependence is attached to $H_T^A$, and redefine the vacuum value of the fields at the external edges $\phi_i(x)\rightarrow \phi_i(x)-eV\sqrt{\text{Fr}}/(\nu h)\ (l=1,4)$. With these manipulations the tunneling operators read

$$H_T^A(t,n) = \Gamma_A e^{ie\nu t/\hbar}e^{2\pi it\phi_{\nu}+\phi_t}/\phi_0 e^{i\phi_0(t,0)}.$$

Note that the magnetic flux attached to $H_T^A$ comprises of both the AB-flux and the statistical flux due to $n \mod(3)$ QPs.

We next calculate the transition rates. The above model facilitates the calculation of the rates of the processes appearing in Table I. Rates are computed using generalized Fermi’s golden rule (see, e.g., [22]) in order to evaluate single and two particles transfer between the edges. Generally, we can write the transition rate between any initial state $|\psi_i\rangle$ with thermal occupation $\rho_i$ to any final state $|\psi_f\rangle$ obtained from the initial one by transferring one or two QPs as $W_{i\rightarrow f}^{(1)} = (2\pi/\hbar)^{1/2}\delta(E_f-E_i)$, where $\tilde{T} = H_T + H_{\text{c.c.}}(E_i-H_0-i\nu)\rightarrow H_T + \ldots$. For example,

$$W_{j,j+2}^{[2,ABC\Phi_{\nu}(j)]} = |\Gamma_A \Gamma_B \Gamma_C \Gamma_D| \times \cos \left[ \frac{2\pi}{3}(\Phi_{AB} + \Phi_j) / \Phi_0 \right] \times \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} G_A^\gamma(\epsilon,-L_4) G_B^\gamma(\epsilon + \nu eV, L_3) \times G_C^\gamma(\epsilon + \nu eV, L_2) G_D^\gamma(\epsilon,-L_1).$$

Here we have introduced $G_i^\gamma(\epsilon,x)$ and $G_i^\gamma(\epsilon,x)$, the Green’s functions in energy-space representation. In time-space representation they are given by $G_i^\gamma(\epsilon,x) = \langle e^{-i\sqrt{\nu t}\phi_i(x,\epsilon)}e^{-i\sqrt{\nu t}\phi_i(0,0)} \rangle$ and $G_i^\gamma(\epsilon,x) = \langle e^{i\sqrt{\nu t}\phi_i(0,0)}e^{i\sqrt{\nu t}\phi_i(x,\epsilon)} \rangle$.

We find for $\gamma$ and $\Omega$ [cf. Equation (2)]

$$\gamma = \frac{\beta^{3/3} e^{\pi/2}}{\sqrt{2}}$$

and

$$\Omega = \frac{32\pi^{1/3} |\nu|^{3/3} \beta^{3/3}}{2^{1/3}(3/2)!^2 \nu^{1/3} \nu^{4/3}}.$$
eventually the generalizations to QPs satisfying non-Abelian statistics.

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[17] For example, at finite temperature we require the thermal length $L_T = \frac{\hbar}{g^\beta_T}$ to be small compared to the interferometer arms. A detailed discussion will appear elsewhere, G. Campagnano et al. (to be published).