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An active negative index metamaterial that derives its gain from an electron beam is introduced. The metamaterial consists of a stack of equidistant parallel metal plates perforated by a periodic array of holes shaped as complementary split-ring resonators. It is shown that this structure supports a negative-index transverse magnetic electromagnetic mode that can resonantly interact with a relativistic electron beam. Such a metamaterial can be used as a coherent radiation source or a particle accelerator.

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I. INTRODUCTION

Artificially structured metamaterials (MTMs) possess exotic macroscopic electromagnetic properties that cannot be achieved in natural materials. Constructed from simple planar elements such as split-ring resonators and thin wires,1 MTMs enable a variety of applications such as “perfect” lenses, compact transmission lines and antennas, electromagnetic cloaks, and many others.2–5 The negative refractive index1,6–8 is one of the most surprising and thoroughly studied properties enabled by MTMs. In this article we describe a new class of negative-index MTMs that can strongly interact with an electron beam, thereby opening new opportunities for vacuum electronics devices such as coherent radiation sources and particle accelerators. The specific implementation of such a negative-index metawaveguide (NIMW) analyzed in this paper and schematically shown in Fig. 1 is obtained by patterning an array of split-ring resonator cutouts on the plates of a stack of planar metallic waveguides.

The NIMW belongs to the category of complementary metamaterials (C-MTMs).9 C-MTMs utilize the complements of the traditional split-ring resonators (SRR) in order to achieve a complementary electromagnetic response: an SRR exhibits a strong magnetic response while a C-SRR has a strong electric response. Narrow waveguides patterned with C-SRRs have been used10 to demonstrate enhanced tunneling of transverse electromagnetic (TEM-like) waves, as well as for making a negative index metamaterial for wave propagation normal to the metal screens.11 In this paper, we demonstrate that this structure supports a negative-index transverse magnetic (TM) mode: an electromagnetic mode propagating in the x direction, with \( E_x \) being the only nonvanishing component in the waveguide’s mid-plane at \( z = 0 \). As demonstrated below, the negative effective permittivity of the NIMW \( \varepsilon_{\text{eff}} < 0 \) is imparted to it by resonant C-SRRs,9,10 while the negative effective permeability \( \mu_{\text{eff}} < 0 \) is due to the transverse confinement of the TM modes12 supported by the narrow (width \( d \) in the \( z \) direction) waves as \( \lambda \equiv 2\pi c/\omega \) waveguides formed by the neighboring plates. The importance of utilizing TM modes lies in their ability to resonantly interact via finite \( E_x \) with relativistic electron beams when their phase velocity \( v_{\text{ph}} \equiv c/\kappa_x \) is equal to the beam’s velocity \( v_b \). Such an interaction can be exploited to either transfer the electromagnetic energy to the beam (particle accelerator) or to extract energy from the beam (coherent radiation source).

The attraction of the NIMW for coherent high-frequency radiation generation is threefold. First, the opposite sign of the group velocity and the beam velocity can result in an instability utilized in backward-wave oscillators (BWO) or (for lower beam currents) amplifiers (BWA).11 The subwavelength nature (lateral period \( b \ll \lambda \)) of the NIMW supported by its resonant C-SRRs distinguishes it from the traditional BWOs, which rely on the interaction between an electron beam and a spatial harmonic of the electromagnetic field in a periodic structure. Second, the low group velocity \( v_g \ll c \) of the negative-index waves due to the NIMW’s resonant C-SRRs increases spatial gain, reduces the starting current requirement of a BWO, and enables shorter structures. Third, NIMW’s constitutive elements (C-SRRs) can be produced using standard planar fabrication techniques. This is particularly advantageous for the generation of THz and millimeter waves (corresponding to frequencies between 0.3 and 3 THz) because the fabrication of conventional BWOs14 relies on high-precision machining that becomes challenging for shorter wavelengths.

Because the metallic planes of the NIMW are parallel to the direction of the beam’s velocity, they do not obstruct the beam’s propagation in the same way as, for example, multilayer fishnet metamaterials7,15–17 containing small openings through which the beam must propagate normally to the metal surface. The interaction between the beam and the NIMW is further facilitated by the spatial distribution of the longitudinal electric field \( E_z \), which is maximized at the beam’s location as shown in Figs. 1(a) and 1(c). This favorably compares with multilayer fishnet structures which have a very small and highly nonuniform15–17 longitudinal electric field inside the hole. Also, the output radiation frequency of a NIMW can be accurately and rapidly controlled by electric or optical tuning of the resonant frequency of the C-SRRs.18 Finally, unlike the negative index multilayer fishnet structures which have subwavelength dimensions only along one direction, all dimensions of a NIMW (see caption of Fig. 1) can be made strongly subwavelength, thereby making it a genuine metamaterial. We note that the absolute instability of electron beams inside a negative-index medium has been suggested...
A metamaterial composed of an array of metallic plates may be rigorously homogenized by studying the modes excited between any two neighboring plates. Here we apply this homogenization model to the parallel-plate metamaterial of Fig. 1(a). A homogenized representation of the metamaterial is shown in Fig. 2. The effective metamaterial properties are determined by the dominant TM mode, with wave number $k_z$ and frequency $\omega$. We note that the longitudinal component of the electric field, enabled by the parallel plates, can resonantly interact with an electron beam propagating in the $x$-direction. The TM$_1$ mode is symmetric with respect to the $z = 0$ midplane and possesses nonvanishing fields $E_z$ (even function of $z$), $E_x$, and $H_y$ (both odd functions of $z$). The subscript 1 refers to the number of nodes of the transverse field components $E_x$ and $H_y$ along the $z$ direction.

Note that while the PEC assumption is valid for THz and millimeter waves, it starts breaking down for frequencies exceeding tens of THz (mid-IR part of the spectrum) because of the increasing penetration of the field into the metal. Such high frequencies are outside of the scope of this paper. We also note that similar structures have been proposed for negative index propagation perpendicular to C-SRR patterned metal screens. Such propagation geometry does not lend itself to effective beam-structure interaction because the beam would have to propagate through very narrow slits.

In anticipation of the need to emulate the effects of C-SRRs and the electron beam, the waveguide is assumed to be filled with a material characterized by a permittivity tensor with nontrivial components $\tilde{\epsilon}_{xx}$ and $\tilde{\epsilon}_{zz}$ and a single nonvanishing component $\tilde{k}$ defined as in Eq. (1). Finite $\tilde{k}$ emulates magneto-optical coupling introduced by the C-SRR, which $\tilde{\epsilon}_{xx} \neq 1$ emulates resonant electric response of the C-SRR, while $\tilde{\epsilon}_{zz} \neq 1$ emulates the wave’s interaction with an electron beam when the resonance condition $\omega = k_z v_b$ is satisfied. The metamaterial becomes active when $\text{Im}(\tilde{\epsilon}_{zz}) < 0$. This anisotropic gain due to the beam-structure interaction should be contrasted with the isotropic gain obtained from more standard gain media, such as quantum dots or organic dyes, used for loss compensation.

The effective constitutive parameters may be computed by analyzing the propagation properties of the dominant TM$_1$ mode, using the transmission-line characteristic impedances for forward- and backward-propagating TM waves according to $Z_{ch}^\pm = \pm V/I$, where the transmission line’s voltage $V$ and current $I$ are used for extracting the MTM’s constitutive parameters.
current \( I \) are defined according to

\[
I = -\int_{-b/2}^{b/2} dx \bar{H}_x(x = \mp b/2, z = t/2),
\]

\[
V = \frac{1}{b} \int_{-b/2}^{b/2} dx \int_{d/2}^{d/2} dz E_z(x = \mp b/2),
\]

and the top and bottom signs correspond to the forward and backward waves, respectively. While \( Z_{ch}^+ = Z_{ch}^- \) for an air filled transmission line shown in Fig. 2, that would no longer be the case when magneto-electric coupling is present in the filling medium, as would be the case in the more general bianisotropic structure shown in Fig. 1. Effective material parameters can then be obtained from the transmission-line model through

\[
\epsilon_{\text{eff}} = \frac{ck_x}{\omega} \frac{2}{Z_{ch}^+ + Z_{ch}^-} \frac{Z_0}{\rho},
\]

\[
\mu_{\text{eff}} = \frac{ck_x}{\omega} \frac{2}{Y_{ch}^+ + Y_{ch}^-} \frac{1}{\rho} Z_0,
\]

\[
\kappa_{\text{eff}} = \frac{i \epsilon_{zz}(\omega - k_x v_b)}{2} \frac{Z_0}{\mu_{\text{eff}}},
\]

where \( Z_0 = 377 \, \Omega \) is the free-space impedance and \( Y_{ch} \equiv 1/Z_{ch} \) is the characteristic admittance of the transmission line.

Applying the above definitions of effective parameters and characteristic impedances to the TM\(_1\) mode of the conventional parallel-plate metamaterial in Fig. 2 made of smooth metallic plates and suitable filler medium, we obtain \( \kappa_{\text{eff}} = \vec{k} \) and \( \epsilon_{\text{eff}} = \vec{\epsilon}_{zz} \frac{\pi b}{d} \), \( \mu_{\text{eff}} = \left[ \mu_{yy} - \left( \frac{\pi d}{\omega \epsilon_{zz} c} \right)^2 \right] \frac{1}{\mu_{xx}} \frac{d}{\beta b} \), resulting in the dispersion relation for the TM\(_1\) wave,

\[
\frac{ck_x}{\omega} = \pm \sqrt{\vec{\epsilon}_{zz} \mu_{yy} - \vec{k}^2 - \left( \frac{\pi d}{\omega \epsilon_{zz} c} \right)^2 \frac{\vec{\epsilon}_{zz}}{\mu_{xx}}}. \tag{6}
\]

Several insights can be gained from Eq. (5). First, the effective magnetic permeability turns negative for \( \omega < \omega_c \), where \( \omega_c = c \pi / d \) is the cutoff frequency of the considered TM\(_1\) mode. Therefore, one approach to achieving negative-index propagation at \( \omega < \omega_c \) is to pattern the waveguide’s wall in such a way as to ensure that \( \vec{\epsilon}_{zz}(\omega) < 0 \). Second, if \( \text{Im}(\epsilon_{zz}) \neq 0 \) (as is the case for a beam resonantly interacting with the \( E_x \) component of the mode), then \( \text{Im}(\mu_{\text{eff}}) \neq 0 \), resulting in an active (gain) metamaterial. That the longitudinal component of the electric field \( E_x \) (and, therefore, \( \vec{\epsilon}_{xx} \)) contributes to the effective magnetic permeability \( \mu_{\text{eff}} \) of confined TM modes has been known\(^{12}\) from theoretical and experimental studies, but the possibility of employing an electron beam for controlling the imaginary part of \( \mu_{\text{eff}} \) and realizing gain in metamaterials has not been recognized. Finally, Eq. (6) can be recast in the conventional form for the theory of traveling wave tubes (TWTs)\(^{13}\) by assuming that the waveguide is filled with an active medium with permittivity \( \epsilon_{xx}^{(b)} = 1 - \omega_0^2 / (\omega - k_x v_b)^2 \), where \( \omega_0 \) is the electron beam plasma frequency. The resulting dispersion relation for the active NIMW can now be rewritten as

\[
\left[ k_x^2 - \frac{\omega^2}{c^2} (\epsilon_{\text{eff}} \mu_{\text{eff}} - \kappa_{\text{eff}}^2) \right] (\omega - k_x v_b)^2 = \frac{\omega^2}{c^2} (\mu_{\text{eff}} - 1) \epsilon_{\text{eff}} \omega_0^2 \kappa_b^2, \tag{7}
\]

where, because of the wave-beam interaction, the frequency \( \omega \) is a complex number for a real propagation constant \( k_x \). Analogous to the linear theory of the TWT, Eq. (7) is quartic in \( \omega \) having four complex roots that represent three forward waves (with positive, negative, and zero gain) and one backward wave (not affected by the beam). The maximum gain \( \gamma_{\text{max}} = \sqrt{3}/2 \rho \) is achieved at the beam-wave synchronism \( \omega_{\text{NIMW}}(k_x) \equiv k_x v_b \) (zero detuning) condition, where \( \omega_{\text{NIMW}}(k_x) \) is the dispersion relation without the beam, and the Pierce parameter\(^{14}\) of the NIMW is given by

\[
\rho = \left[ \frac{1}{2} \frac{\epsilon_{\text{eff}}}{\epsilon_{\text{eff}} \omega_{\text{NIMW}}^2} \right]. \tag{8}
\]

III. NUMERICAL SIMULATIONS

After gaining significant physical insights from analytic modeling of a smooth-walled structure, we proceed to extract the constitutive parameters of the NIMW with unit cell shown in Fig. 1 through first-principles electromagnetic simulations using COMSOL MULTIPHYSICS.\(^{20}\) Periodic boundary conditions along the \( y \) and \( z \) directions are used, and a finite per-cell phase shift \( \Phi_1 \equiv k_x b \) is assumed in the \( x \) direction. While the present design is for microwave frequencies \( (f \approx f_0 = 5 \, \text{GHz}; \text{physical dimensions are given in Fig. 1}) \), it can be scaled down to mm-wave/THz frequencies. The dispersion relations of the lowest-order modes are shown in Fig. 3(a). A narrow-band negative index (NI) TM\(_1\)-like mode is found in the 5.33 GHz \( < f < 5.65 \, \text{GHz} \) frequency range located below the cutoff frequency \( f_c \equiv \omega_c / 2 \pi \approx 11.7 \, \text{GHz} \). As was remarked earlier\(^{16}\) in the context of multilayer fishnet structures, complex three-dimensional metamaterials rarely support pure TE or TM eigenmodes. The exact eigenmode of the NIMW is a mixture of the two, with the TM component dominating.

Note that a second subcutoff TM\(_1\)-like mode with positive refractive index is also supported by the structure. The positive index (PI) mode’s propagation is due to a higher-order magnetic resonance of the C-SRR around 11 GHz. This resonance strongly affects \( \mu_{yy} \) that enters Eq. (5) \((\mu_{yy} \approx 1 \) is assumed for the NI mode\) and enables \( \mu_{\text{eff}} > 0 \) for \( f > 8.5 \, \text{GHz} \). Detailed discussion of the PI mode is outside of the scope of this paper, and we concentrate below on the NI mode.

The mode-specific effective parameters of the NI mode were extracted by applying Eqs. (3) and (4) to COMSOL-produced electromagnetic field profiles and plotted in Fig. 3(b) for moderate phase advances. We note that \( \mu_{\text{eff}} \) remains relatively flat, consistent with our original conjecture that the transverse confinement of the mode is responsible for its effective negative permeability. On the other hand, \( \epsilon_{\text{eff}} \) displays strongly dispersive behavior, consistent with its origin stemming from the resonant C-SRR element. We further
observe that the bianisotropy coefficient $\kappa_{\text{eff}}$ is rather large and, consistent with Eq. (2), explains why both $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$ are nonvanishing (negative) at the $k_x = 0$ (cutoff) point, where $\varepsilon_{\text{eff}}\mu_{\text{eff}} = k_x^2\varepsilon_0$ is satisfied.

To examine the possibility of creating an active negative index metamaterial using a high-current electron beam coupled into the NIMW and to confirm the analytical predictions of Eqs. (7) and (8), we have carried out COMSOL simulations of the NIMW structure containing an electron beam in the middle of the unit cell. The beam’s presence was modelled by assigning $\varepsilon_{\text{eff}}(b)$ to the region occupied by the beam and by assuming the following beam parameters: $v_b = 0.9c$, beam plasma frequency $\omega_b = 0.01(2\pi f_0)$, and the beam’s radius $R = d/4$. The resulting complex $\omega$, plotted as a function of the phase advance across the cell, is shown in Fig. 4 for phase advances in the vicinity of the beam-mode synchronism condition.

Three distinct complex $\omega$’s are found for each value of $k_x$. Modal degeneracies can be classified according to the value of the detuning parameter $\nu \equiv \omega_{\text{NIMW}} - k_x v_b$. For $\nu > -3\rho/\sqrt{2}$ two “slow” modes with Re[$\omega$]/$k_x < v_b$ degenerate in Re[$\omega$] are found, one of them exponentially growing and the other one decaying. The third, “fast,” mode with Re[$\omega$]/$k_x > v_b$ is neutral (neither growing nor decaying) for $\nu > 0$. For $\nu < -3\rho/\sqrt{2}$ all three modes (two “slow” and one “fast”) become neutral and nondegenerate in Re[$\omega$]. These numerical COMSOL results compare very well with the analytical predictions of Eq. (7) obtained by adjusting the effective beam plasma frequency to $\omega_b^{\text{eff}} = 0.05\omega_b$ to account for only partial overlap between the beam and the negative-index TM mode. This reduction in $\omega_b^{\text{eff}}$ is associated with small shunt impedance of the resonant NIMW, which concentrates the electric energy away from the beam in the vicinity of the C-SRR. While ohmic losses reduce the growth rate, the instability still persists for somewhat higher beam densities corresponding to $\omega_b = 0.03(2\pi f_0)$.

IV. CONCLUSIONS

In conclusion, we have demonstrated a geometry to realize an active beam-driven negative index metawaveguide (NIMW) that supports transverse magnetic (TM) waves capable of resonantly interacting with an electron beam. A number of novel vacuum electronics devices that require backward waves and small group velocity, such as backward-wave oscillators and amplifiers, can be envisioned based on this concept. The subwavelength nature of the unit cell enables strong interaction with electron beams at the fundamental harmonic of the structure, while the resonant nature of the constitutive elements (complementary split-ring resonators) enables low group velocity and, potentially, agile frequency tuning.

One of the important issues not addressed in this paper is the ability of the C-SRR structure to withstand high radiation power. If radio-frequency breakdown can be avoided, then the narrow bandwidth and small group velocity of NIMW could make it a potentially attractive structure for advanced
accelerator application. Breakdown in extremely narrow radio-frequency structures is presently the subject of active research (see, for example, Ref. 27 and references therein) and will be addressed in future publications.

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