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Deep spin-glass hysteresis-area collapse and scaling in the three-dimensional $\pm J$ Ising model

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We investigate the dissipative loss in the $\pm J$ Ising spin glass in three dimensions through the scaling of the hysteresis area, for a maximum magnetic field that is equal to the saturation field. We perform a systematic analysis for the whole range of the bond randomness as a function of the sweep rate by means of frustration-preserving hard-spin mean-field theory. Data collapse within the entirety of the spin-glass phase driven adiabatically (i.e., infinitely slow field variation) is found, revealing a power-law scaling of the hysteresis area as a function of the antiferromagnetic bond fraction and the temperature. Two dynamic regimes separated by a threshold frequency $\omega_c$ characterize the dependence on the sweep rate of the oscillating field. For $\omega < \omega_c$, the hysteresis area is equal to its value in the adiabatic limit $\omega = 0$, while for $\omega > \omega_c$, it increases with the frequency through another randomness-dependent power law.

Hysteresis in magnetic materials has been a subject of interest for quite some time due to its applications in magnetic memory devices and as a testing ground for theories of nonequilibrium phenomena [1–4]. The hysteresis area which measures the magnetic energy loss in the material is connected with the Barkhausen noise [5,6] due to irreversible avalanche dynamics [7–12]. The existing literature on hysteresis in random magnets focuses mostly on random-field models [12–15] while numerical studies on random-bond models are mostly at zero temperature [16–22]. To our knowledge, there has been no finite-temperature study of the hysteresis loss, especially in the spin-glass phase where large avalanches are expected to be severely prohibited. We here investigate the adiabatic and dynamic hysteresis in the the $\pm J$ random-bond Ising spin glass [23] on a finite, three-dimensional simple cubic lattice with periodic boundary conditions. We show that the hysteresis area obeys a scaling relation in the whole spin-glass phase, in accord with earlier theoretical studies which observed scale invariance over the whole range about the critical disorder for various disorder-driven systems [15–17]. Moreover, this scaling data collapse is also observed for experimental systems over wide ranges of the temperature and the magnetic field: Gingras et al. observed a universal data collapse over four decades in a geometrically frustrated antiferromagnet $Y_2MnO_4$ [24], while Gunnarsson et al. observed such a data collapse for the short-range Ising spin glass Fe$_{0.5}$Mn$_{0.5}$TiO$_3$ [25].

The $\pm J$ Ising spin-glass model is defined by the dimensionless Hamiltonian

$$\beta \mathcal{H} = \sum_{\langle ij \rangle} J_{ij} s_i s_j + H \sum_i s_i , \quad (1)$$

where $\beta = \frac{1}{k_B T}$ is the inverse temperature. The first sum in Eq. (1) is over the pairs of nearest-neighbor sites $(i,j)$, where $J_{ij}$ is the quenched-random local interaction between the classical Ising spins $s_i = \pm 1$. The probability distribution function for $J_{ij}$ is given by

$$P(J_{ij}) = p \delta(J_{ij} + J) + (1 - p) \delta(J_{ij} - J) . \quad (2)$$

$H$ in the second term in Eq. (1) is the uniform external magnetic field. With a proper choice of units, the temperature for the system may be defined as $T \equiv 1/J$. A random distribution of the ferromagnetic and antiferromagnetic bonds gives rise to frustration and yields a spin-glass phase for a range of $p$ values. Ising spin-glass models are widely used as a tool for understanding the properties of experimental spin glasses such as Pt$_{0.5}$Co$_{0.5}$Mn$_{0.96}$Ga$_{0.04}$O$_3$ [11], Fe$_{0.5}$Mn$_{0.5}$TiO$_3$ [25–27], LiHo$_{0.167}$Y$_{0.833}$F$_4$ [28], and Cu$_{1-x}$AlMn$_{1}$ [29]. Without loss of generality we set $p \leq 0.5$ since the partition function is invariant under the transformation $p, \{s^A\}, \{-s^B\} \rightarrow (1-p), \{s^A\}, \{-s^B\}$, where $A$ and $B$ signify the two sublattices.

For small values of $p$ and $H = 0$, the orientational (up-down) symmetry is spontaneously broken below a critical temperature $T_c(p)$ and long-range ferromagnetic order sets in. This phase is well understood within the Landau picture where the free energy landscape is described by two minima at magnetizations $\pm m(T,p)$. Beyond a critical fraction $p_c$ of the antiferromagnetic bonds, reducing temperature drives the system into a glassy phase. The low-temperature phase now retains its orientational symmetry and a new, randomness-dominated phase which has a broken replica symmetry appears [30,31]. In this phase, the free energy landscape is rough, with many local minima at significantly nonoverlapping configurations. Meanwhile, the dynamics slows down to the extent that the relaxation time diverges [32]. At high temperatures $T > T_c(p)$, both ordered phases give way to a paramagnetic state where the entropic contribution to the free energy is dominant. While the critical temperature strongly depends on $p$ along the ferromagnet-to-paramagnet phase boundary, only a weak dependence of $T_c(p)$ on $p$ is observed for the spin-glass phase [32,33]. In this study, we investigate the hysteretic behavior of a spin glass under the uniform magnetic field $H$ that is swept at a constant rate $\omega$. A past computational study similar to ours [34] considered a time-dependent quenched-random magnetic field that was conjugate to the spin-glass order parameter.

We use hard-spin mean-field theory (HSMFT), a self-consistent field theoretical approach [34–50] that preserves the effects due to the frustration (crucial for the spin-glass
phase) generated by the randomly scattered antiferromagnetic bonds. HSMFT is defined by the refined set of self-consistent equations

\[ m_i = \sum_{\{s_j\}} \left( \prod_j P(m_j, s_j) \right) \tanh \left( \sum_j J_{ij} s_j + H \right) \]  \tag{3}

for the local magnetization \( m_i \) at each site \( i \), whose nearest neighbors are labeled by \( j \). The single-site probability distribution is

\[ P(m_j, s_j) = \frac{1 + m_j s_j}{2} \]  \tag{4}

The local magnetization \( m_i \) at site \( i \) satisfies \(-1 \leq m_i \leq 1\). The hard-spin mean-field theory Eq. (3) has been discussed in detail by the authors of Refs. [34–50].

HSMFT has been successfully applied to spin glasses [34,43]. In this paper we make use of the method to investigate the scaling of the hysteresis area under a uniform, time-dependent magnetic field. To this end, we consider a \( 20 \times 20 \times 20 \) cubic lattice with periodic boundary conditions. We have checked in this study and in a previous study [34] that our hard-spin mean-field theory results are independent of size for an \( L \times L \times L \) system for \( L \gtrsim 15 \). A particular realization at a given \((T, p)\) is generated by the assignment of the quenched-random coupling constants \( J_{ij} \) according to the probability distribution of Eq. (2) and, initially, a random and unbiased choice of spins \( s_i = \pm 1 \). To determine the hysteresis field, the system is first saturated by a sufficiently large external field \( H_s \), the minimum value of \( H \) for which Eq. (3) yields an average magnetization \( m = (1/L^3) \sum_i m_i = 1 \) within an accuracy \( \epsilon_m \equiv 10^{-6} \). Then, the path \( H_s \to -H_s \to H_s \) is traversed with steps \( \Delta H = H_s/100 \) or smaller. For each incremental change of the field, the system is allowed to relax a number of time steps \( \tau = \omega_0 \tau_R \). A time step corresponds to successive iterations of Eq. (3) on \( L^3 \) arbitrarily chosen sites. An infinitely slow sweep is obtained as the limit \( \tau \to \tau_R \), where the HSMF equations converge to a self-consistent solution within the tolerance interval \( \epsilon_m \). Thus, \( \tau_R \) is the relaxation time of the system.

The infinitely slow-sweep hysteresis curves obtained in the ferromagnetic and spin-glass phases are shown in Fig. 1. The usual jump in the magnetization at a coercive field \( H_c \), observed for small \( p \), is associated with a system-wide avalanche in the ferromagnetic phase. For \( p \) larger than a critical value \( p_c \), this picture is replaced by a slanted hysteresis curve and a smaller hysteresis area, typical of spin-glass materials [3,11,29]. This converse hysteretic behavior, associated with the Barkhausen noise [5,6], is a consequence of the power-law distribution of avalanches which is well established [6,7,10–12,14–18,20,21,29,51] for several frustrated systems with quenched disorder. The hysteresis area disappears in the paramagnetic phase.

In Fig. 2, we present the infinitely slow-sweep hysteresis area globally, for all temperatures and antiferromagnetic bond probabilities, on a logarithmic color-contour plot. The hysteresis area \( A_0 \) vanishes in the region shown in dark blue, which corresponds to the paramagnetic phase, while it is nonzero in the ferromagnetic and spin-glass phases, respectively, on the left and right of the lower half of Fig. 2. The para-ferro and para-spin-glass phase boundaries are easily determined by locating the temperature at which \( A_0 \) vanishes (i.e., falls below \( \epsilon_m \)). A set of \( p \) scans for different temperatures and a set of temperature scans for various \( p \) values are given in Fig. 3. The low-temperature ferro-spin-glass boundary is located at \( p_c \approx 0.22 \) and is calculated as the inflection point for the maximum slope of the hysteresis curve as a function of antiferromagnetic bond probability [16]. The phase boundaries are consistent with the well-known phase diagram for the three-dimensional \( \pm J \) model [33] and in fair comparison with...
the experimental temperature-concentration phase diagrams
of the various Eu₅Sr₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋ₓ₁₋xDD

We here focus on the scaling form of the hysteresis area in
the spin-glass phase and show that a unique scaling-function
governs the whole range of
\[ A \equiv \mu(\tilde{\rho}, \tilde{J}) = \nu \lambda^{-\mu} \tilde{A}_0(\tilde{\rho}, \tilde{J}), \]
which by letting \( \mu = -\frac{b}{a} \) reduces to
\[ A_0(\tilde{\rho}, \tilde{J}) = \tilde{\rho}^{-\nu} A_0(1, \tilde{J}). \]

The sought collapse is obtained by the choice of scaling
exponents \( \mu = 1 \) and \( \nu = 2 \). The data shown in Fig. 3 collapse
onto a single curve shown in Fig. 4, where the left-hand
side (LHS) of Eq. (7) is plotted against the argument on the
right-hand side (RHS) for 28 evenly spaced values of \( p \) above
\( p_c \). The origin corresponds to the phase boundary between
the spin-glass and paramagnetic phases. The log-log plot
of the same collapse shown in the inset of Fig. 4 suggests
that the scaling function has the form \( f(x) \propto x^{-1/2} \),
yielding a collapse area \( A_0 \propto \tilde{\rho}^\beta \tilde{J}^\alpha \) with \( \alpha \simeq -0.28 \) and \( \beta \simeq 1.72 \).
Interestingly, unlike the case of the usual critical phenomena,
the scale-invariance applies to the entire spin-glass phase
and not just to the vicinity of the critical phase boundary.

Having analyzed the limit with infinitely slow-sweep rate,
we next consider the dynamic hysteretic response as a function
of the magnetic field frequency. One can simulate the finite
oscillation frequency by iterating Eq. (3) for a predetermined
number of steps \( t \), instead of waiting until a steady state
is reached. The sweep rate \( \omega = 1/t \) is proportional to the
frequency of the applied field up to a material-dependent spin
relaxation time. The hysteresis area \( A(\omega, p, J) \) deviates from
the value at infinitely slow sweep \( A_0 = A(\omega = 0, p, J) \) and
increases with increasing sweep rate \( \omega \). This can be understood
by observing that the slow response of the magnetization to a
time-varying field inflates the hysteresis curve along the
field direction. The typical behavior observed in various
experimental and theoretical magnets (typically pure magnets
or random-field systems) [52–56] is
\[ A(\omega, p, J) = A_0 + g(p, J) \omega^b, \]
where \( b \) is the sweep-rate exponent. We investigate whether
the random-bond Ising spin glass obeys a similar scaling
relation.

A typical scan of the hysteresis area as a function of \( \omega \)
displays two dynamic regimes, separated by a critical sweep
by an incremental increase in the field decay within a period $1/\omega$ or smaller. For faster sweeps ($\omega > \omega_c$), the increase in the area follows the power law in Eq. (8), with a $p$-dependent exponent $b$. In the ferromagnetic phase with weak disorder, the two dynamic regimes are separated by a sharp increase in the hysteresis area. This transition gets significantly smoother in the spin-glass phase, especially far from the ferromagnetic-spin-glass boundary. For larger systems, one expects $\omega_c$ to recede and the power-law behavior to dominate.

Figure 6 shows the sweep-rate exponent $b$ calculated as a function of the antiferromagnetic bond fraction $p$, at fixed temperatures $T = 1/J = 2.0, 1.0, \text{and} 0.5$. The hysteresis area is calculated for the sweep rates $\omega = 1, 0.5, 0.3, 0.2, 10^{-1}, \ldots, 10^{-4}$ at each $p$ value, after averaging over ten realizations. The exponent values are obtained through fits to the data in the regime $\omega > \omega_c$ (typically two decades or more), using the functional form of Eq. (8). The error bars reflect only the scatter of the data relative to the fit. In the ferromagnetic phase $p < p_c$, we note that the calculated sweep-rate exponents lie in an interval of fairly good agreement with the various values obtained previously at $p = 0$, namely $b = 2/3$ [52–55] and $b = 0.52 \pm 0.04$ [53] from mean-field theory, $b = 0.61$ [53] from Glauber dynamics simulations, $b = 0.495 \pm 0.005$ [54] and $b = 0.45$ [56] from Monte Carlo simulations.

In conclusion, we have considered here the $\pm J$ Ising model under a uniform external field and investigated the scaling behavior of the saturation hysteresis area (i.e., far from the weak-field limit). We observed that the phase diagram can be derived from the hysteresis area alone and the ferromagnetic-spin-glass phase boundary corresponds to the inflection point with regard to bond-randomness strength $p$. When adiabatically driven, the area displays a data collapse within the entire spin-glass phase for all temperatures and $p$. The scaling function itself has a power-law form and the scale invariance extends far from the phase boundary, deep into the spin-glass phase.
The dynamical response under a fluctuating external field is also interesting. We find that, beyond a threshold value \( \omega_c \), the hysteresis area increases as a function of the field-sweep rate \( \omega \) with a nonuniversal power law. This behavior is not limited to the vicinity of the phase transition. The associated exponent is found to be a function the randomness strength \( p \). Moreover, this function is independent of temperature. In the limit of a pure magnet (\( p \to 0 \)), we observe good agreement with the existing literature, despite the fact that the earlier theoretical work applied to a weak driving field, while we here consider sweeps across saturation limits. Figure 6 suggests that, relative to the ferromagnetic phase, the spin glass displays an amplified sensitivity to the field-sweep rate, again running in apparent contrast to the general wisdom that the hysteretic effects are suppressed within a spin glass. In fact, we note that the increase in the hysteresis area with \( \omega \) is due to the magnet’s delayed response to the changing field, and a signature of the spin-glass phase is the slowing down of precisely such relaxation phenomena.

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