Dual baseline search for muon antineutrino disappearance at $0.1\text{eV}^2 < m^2 < 100\text{eV}^2$
Dual baseline search for muon antineutrino disappearance at $0.1\,\text{eV}^2 < \Delta m^2 < 100\,\text{eV}^2$
I. INTRODUCTION

Recently there has been increasing evidence in support of neutrino oscillations in the $\Delta m^2 \approx 1$ eV$^2$ region. The LSND [1] experiment observed an excess of $\bar{\nu}_e$-like events in a $\bar{\nu}_\mu$ beam. MiniBooNE [2–4] has observed an excess of $\nu_e$-like and $\bar{\nu}_e$-like events, in a $\nu_\mu$ beam and $\bar{\nu}_\mu$ beam, respectively. Additional evidence for short baseline anomalies with $L/E = 1$, where $L$ is the neutrino path length in km and $E$ the neutrino energy in GeV, includes the deficit of events observed in reactor antineutrino experiments [5] and radioactive source neutrino measurements [6]. If these anomalies are due to neutrino oscillations in the $\Delta m^2 \approx 1$ eV$^2$ range, then they could imply the existence of one or more new sterile neutrino species that do not participate in standard weak interactions but mix with the known neutrino flavors through additional mass eigenstates. Observation of $\nu_\mu$ ($\bar{\nu}_\mu$) disappearance in conjunction with $\nu_e$ ($\bar{\nu}_e$) appearance in this $\Delta m^2$ range would be a smoking gun for the presence of these sterile neutrinos. Alternatively, constraining $\nu_\mu$ ($\bar{\nu}_\mu$) disappearance can, along with global $\nu_e$ ($\bar{\nu}_e$) disappearance data, constrain the oscillation interpretation of the $\nu_e$ ($\bar{\nu}_e$) appearance signals in LSND and MiniBooNE [7].

Searches for $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance in MiniBooNE were performed in 2009 [8]. No evidence for disappearance was found. The search for $\nu_\mu$ disappearance was recently repeated in MiniBooNE with the inclusion of data from the SciBooNE detector in a joint analysis [9]. Once again, the results were consistent with no $\nu_\mu$ disappearance. The analysis presented here is an improved search for $\bar{\nu}_\mu$ disappearance using data from MiniBooNE and SciBooNE taken while the Booster Neutrino Beamline (BNB) operated in antineutrino mode.

The Monte Carlo (MC) predictions for both MiniBooNE and SciBooNE were updated to account for recent neutrino flux and cross section measurements made with both experiments. The data from both detectors were then simultaneously fit to a simple two-antineutrino oscillation model. Improved constraints on MC predictions, the inclusion of SciBooNE data, and a MiniBooNE antineutrino data set nearly 3 times larger than what was available for the original $\bar{\nu}_\mu$ disappearance analysis, have allowed a 90% confidence level upper limit to be set that dramatically improves upon prior limits in the $\Delta m^2 = 0.1–100$ eV$^2$ region.
collimator and decay in flight along a 50 m long tunnel. A schematic view of the BNB from the beryllium target to both detectors is shown in Fig. 1.

The resulting neutrino beam will have an enhanced flux of either muon neutrinos (neutrino mode) or muon antineutrinos (antineutrino mode). In antineutrino mode beam running, the flux of antineutrinos in the beam will be referred to as the right-sign (RS) flux and the flux of neutrinos in the beam will be referred to as the wrong-sign (WS) flux. These two designations are used because antineutrinos are the signal in this analysis and neutrinos are an intrinsic background. Figure 2 shows the neutrino and antineutrino flux prediction in antineutrino mode at both the MiniBooNE and SciBooNE detectors. Details on the beam line and flux predictions are given in Ref. [10].

The MiniBooNE detector [11] is located 541 m downstream of the antineutrino production target and consists of a spherical 12.2 m diameter tank containing 800 tons of mineral oil (CH$_2$), beneath at least 3 m of earth overburden. The fiducial volume is a sphere 10 m in diameter, with a fiducial mass of 450 tons. The detector is instrumented with 1280 8-inch photomultiplier tubes (PMTs) in the active region, and 240 8-inch PMTs in an outer, veto region. Events are reconstructed based on timing and charge information mostly from Cherenkov radiation. A schematic of the MiniBooNE detector is shown in Fig. 3.

The SciBooNE detector [12] is located 100 m downstream of the target. SciBooNE is a discrete tracking detector comprised of three subdetectors (in order from upstream to downstream): a fully active and finely segmented scintillator tracker (SciBar), an electromagnetic calorimeter (EC), and a muon range detector (MRD). The SciBar subdetector [13] consists of 14336 extruded polystyrene (C$_8$H$_8$) strips arranged vertically and horizontally to construct a $3 \times 3 \times 1.7$ m$^3$ volume. Each scintillator strip is read out by a wavelength shifting fiber attached to a 64-channel multianode PMT (MA-PMT).
The 15 ton SciBar subdetector (10.6 ton fiducial volume) provides the primary interaction target. The EC subdetector is a two plane (vertical and horizontal) “spaghetti”-type calorimeter; 64 modules made of 1 mm scintillating fibers embedded in lead foil are bundled and read out at both ends by PMTs. The MRD subdetector, designed to measure muon momentum, is made from 12 iron plates, each 5 cm thick, sandwiched between 13 alternating horizontal and vertical scintillator planes of thickness 6 mm that are read out via 362 individual 2-inch PMTs. A schematic of the SciBooNE detector is shown in Fig. 4.

### III. MONTE CARLO SIMULATION

Simulation of the neutrino and antineutrino flux, neutrino and antineutrino interactions in the detector, and detector response has been discussed in detail in previous publications for MiniBooNE [14,15] and SciBooNE [16]. Calculation of the neutrino and antineutrino flux at the target (POT) operation in antineutrino mode, from July 2006 through August 2008 are included, comprising a total of $1.53 \times 10^{20}$ POT for the SciBooNE contribution. MiniBooNE event selection and reconstruction is essentially identical to that used for a previous neutrino mode $\nu_\mu$ cross section measurement [14]. Events with only a single $\mu^+$ in the detector are selected. Event selection cuts are based on the beam timing, fiducial volume, observation of two correlated events (the muon and its decay electron), and the likelihood of the fit to the muon hypothesis. These cuts are designed to reject incoming particles (i.e. muons from cosmic rays or from neutrino and antineutrino interactions in the surrounding material), ensure that the event is contained within the detector, and ensure correct event classification as well as accurate muon energy estimation. The capture of $\mu^-$ resulting from initial $\nu_\mu$ charged current quasielastic (CCQE) interaction events is simulated in the MC and these specific events are not selected. In antineutrino mode, a sizable fraction of the events (roughly 20%) are due to $\nu_\mu$ interactions. MiniBooNE cannot distinguish between $\nu_\mu$ and $\bar{\nu}_\mu$ events on an event-by-event basis, so $\mu^-$ from $\nu_\mu$ interactions are an irreducible background.

For SciBooNE, the event selection and reconstruction is nearly identical to the previous inclusive charged current measurement [16]. Two-dimensional SciBar tracks are reconstructed using a cellular automaton algorithm [25] from SciBar hits. Three-dimensional SciBar tracks are then reconstructed based on the timing and end point positions of the two-dimensional SciBar tracks. Two-dimensional tracks in the MRD are independently reconstructed using hits in the MRD that are clustered within a 50 ns timing window. Three-dimensional tracks in the MRD are reconstructed by matching the timing of the two-dimensional projections. If the downstream edge of a SciBar track lies in the last two layers of SciBar, a search for a matching track or hits in the MRD is performed. The upstream edge of the MRD track is required to be on either one of the first two layers of the MRD, and to be within 30 cm of the projected entry point of the SciBar track into the MRD (a more detailed description of the track reconstruction can be found in Ref. [12]).

To select $\mu^\pm$ events, the highest momentum track per event in the beam-on time window is required to have $p_\mu > 0.25$ GeV/c to reduce the number of neutral current (NC) events. The energy loss of the track in SciBar must be consistent with a muon hypothesis, and must originate within the 10.6 ton SciBar fiducial volume. These muon candidate tracks are further categorized as SciBar-stopped or MRD-stopped. SciBar-stopped events have the downstream end point of the muon candidate track contained in the SciBar fiducial volume. MRD-stopped events have the muon candidate track being a SciBar track matched to MRD hits or to an MRD track with a downstream end point that does not exit the back or sides of the MRD. Both SciBar-stopped and MRD-stopped events are used in the analysis. SciBooNE has no overburden so cosmic backgrounds must be subtracted. For cosmic background estimation, the same muon selection criteria are applied to a beam-off time window that is 5 times longer than the beam-on window. This event rate is scaled and subtracted from the beam-on data.

The selected events include $\bar{\nu}_\mu$ and $\nu_\mu$ interactions on carbon and hydrogen in the detectors. The reconstructed antineutrino energy is based on the assumption that the interaction is always a $\bar{\nu}_\mu$ CCQE interaction with a proton at rest in carbon: $\bar{\nu}_\mu + p \rightarrow \mu^- + n$. Hence, it is a function of the measured energy and direction of the outgoing muon. The equation for reconstructed energy is
\[ E_{\nu}^{QE} = \frac{M_n^2 - (M_p - E_B)^2 - M_{\mu}^2 + 2(M_p - E_B)E_{\mu}}{2(M_p - E_B - E_{\mu} + P_{\mu} \cos \theta_{\mu})}, \]  

where \( M_n \) and \( M_p \) are the mass of the neutron and proton, \( M_{\mu}, E_{\mu}, P_{\mu}, \) and \( \theta_{\mu} \) are the mass, energy, momentum, and direction of the outgoing muon, and \( E_B \) is the binding energy (30 MeV for protons in carbon). Equation (1) is applied to all selected events in data and MC, even though a sizable fraction of the events are not CCQE [i.e. charged current single \( \pi \) (CC1 \( \pi \)), charged current multipion (CC multipion), or NC events misidentified as CCQE events]. The impact of the CCQE reconstruction assumption, which leads to reduced accuracy in reconstructed energy for non-CCQE events, is accounted for in MC, which also includes these selected non-CCQE events. MiniBooNE has an estimated resolution for reconstructed energy of 8.3% for CCQE events and 13.9% for all events. SciBooNE has an estimated reconstructed energy resolution of 9.6% for CCQE events and 24.6% for all events.

MiniBooNE and SciBooNE data and MC are put in 21-bin histograms of \( E_{\nu}^{QE} \). The binning goes from 300 MeV to 1.9 GeV, with individual bin widths as follows: bin 1, 100 MeV; bins 2–19, 66.7 MeV; bin 20, 100 MeV; bin 21, 200 MeV. The first and last two bins are wider to ensure adequate event statistics in data and MC.

Figure 5 shows the predicted event distributions in MiniBooNE for reconstructed antineutrino and neutrino energy, for events on hydrogen and carbon nuclei. Figure 6 shows the predictions for MiniBooNE’s reconstructed antineutrino and neutrino energy distributions by interaction type: CCQE, CC1 \( \pi \), and all other interaction types (CC multipion and NC). Table I shows the MC predictions for the selected MiniBooNE events by neutrino and interaction type. \( \nu_e \) and \( \bar{\nu}_e \) contamination is negligible.

The following plots show several properties of the selected SciBooNE events, as predicted by simulation. Figure 7 shows the reconstructed antineutrino and neutrino energy distributions for events on hydrogen and carbon.
nuclei. Figure 8 shows the reconstructed antineutrino and neutrino energy distribution by interaction type: CCQE, CC1$\pi$, and other (CC multipion and NC). Table I shows the MC predictions for the selected SciBooNE events by neutrino and interaction type. The data set is estimated to contain an additional 811 events from cosmic ray muons.

The difference in shape between the SciBooNE RS (WS) and MiniBooNE RS (WS) energy distributions is mainly due to different event selection criteria between MiniBooNE and SciBooNE. MiniBooNE selects for CCQE interaction events and SciBooNE selects for all CC interaction events so the SciBooNE sample has a larger percentage of non-CCQE interaction events. Since the antineutrino energy reconstruction is based on a CCQE interaction assumption, there are more SciBooNE events with a larger discrepancy between true antineutrino energy and reconstructed antineutrino energy than in MiniBooNE, leading to shape differences. Differences in selection efficiency, antineutrino flux at the detector locations, and background rejection between MiniBooNE and SciBooNE also contribute to the shape differences.

Figure 9 shows the distribution of the combined antineutrino and neutrino propagation distances, from production

<table>
<thead>
<tr>
<th>Interaction type</th>
<th>$\bar{\nu}$ events</th>
<th>$\nu$ events</th>
<th>$\bar{\nu}$ events</th>
<th>$\nu$ events</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCQE</td>
<td>37428</td>
<td>9955</td>
<td>4619</td>
<td>1359</td>
</tr>
<tr>
<td>CC1$\pi$</td>
<td>8961</td>
<td>2593</td>
<td>1735</td>
<td>1006</td>
</tr>
<tr>
<td>CC multi-$\pi$ or NC</td>
<td>2364</td>
<td>460</td>
<td>959</td>
<td>610</td>
</tr>
</tbody>
</table>

TABLE I. MC predictions for the number of selected events by neutrino and interaction type in both MiniBooNE and SciBooNE.

FIG. 7. Reconstructed antineutrino and neutrino energy ($E^\nu_{QE}$) distributions for selected RS and WS SciBooNE events on different target types (hydrogen or carbon) from SciBooNE MC. Total events are represented by the solid line, events with interaction on carbon are represented by the dashed line, and events with interaction on hydrogen are represented by the dot-dashed line.

FIG. 8. Reconstructed antineutrino and neutrino energy ($E^\nu_{QE}$) distributions for selected RS and WS SciBooNE events for different interaction types (CCQE, CC1$\pi$, other) from SciBooNE MC. Total events are represented by the solid line, CCQE interaction events are represented by the dashed line, CC1$\pi$ interaction events are represented by the dot-dashed line, and all other interaction (CC multi-$\pi$ or NC) events are represented by the short-dashed line.
in the decay tunnel to interaction in SciBooNE or MiniBooNE.

V. SYSTEMATIC UNCERTAINTIES

Beam and cross section uncertainties are calculated for both MiniBooNE and SciBooNE using the multiregion method [26]. In this procedure, groups of correlated simulation parameters associated with beam production and cross section modeling uncertainties are sampled according to their covariance matrices. The parameters for each source of uncertainty ($\pi^\pm$, $K^\pm$, etc.) are sampled 1000 times to obtain sufficient statistics. Each MC event in MiniBooNE and SciBooNE is reweighted based on these varied parameters forming 1000 new MC predictions of the $E_{QE}$ distribution in both detectors. Covariance matrices, in bins of $E_{QE}$, are then computed for each source of uncertainty by comparing these 1000 new MC predictions to the default MC prediction. The procedure takes care of the correlation of beam production and cross section uncertainties between MiniBooNE and SciBooNE. Cross section and nuclear model uncertainties for $\nu$ and $\bar{\nu}$ events are treated as uncorrelated due to the poor understanding of differences between $\nu$ and $\bar{\nu}$ interactions in nuclear modeling. Some detector specific uncertainties are calculated using the unisim method [26], where uncorrelated detector specific uncertainties are varied up or down by 1 standard deviation.

A. Beam uncertainties

Uncertainties in the delivery of the primary proton beam to the beryllium target, the primary beam optics, secondary hadron production in proton-beryllium interactions, hadronic interactions in the target and horn, and the horn magnetic field, are included in the beam multisims. Uncertainties in the magnetic field horn current, skin effect of the horn, and secondary nucleon and pion interactions in the Be target and Al horn are obtained from previous MiniBooNE analyses [10].

The normalization of the neutrino component in the antineutrino beam was adjusted based on direct measurements in MiniBooNE [15,27]. The beam fraction of $\nu_\mu$ in the antineutrino beam was determined using three methods: a pure data sample of $\nu_\mu$ events from CC1$\pi$ interactions, differences in Michel electron rates between final state $\mu^-$ and $\mu^+$ from $\nu_\mu$ and $\bar{\nu}_\mu$ interactions, respectively, due to $\mu^-$ capture on carbon, and angular distribution differences between final state $\mu^-$ and $\mu^+$ from $\nu_\mu$ and $\bar{\nu}_\mu$ interactions, respectively. Averaging these three methods, the $\pi^+$ production in the beam MC was scaled by a factor of 0.78 and given a 12.8% normalization uncertainty. Uncertainties on the production of $\pi^-$ from the initial $p$-Be interaction are calculated using spline fits to data from the HARP experiment [10]. An updated $K^+$ production simulation with reduced uncertainties for the initial $p$-Be interaction is used. This update is based on a new Feynman scaling fit [28] to recent SciBooNE measurements [29]. The $K^0$ production uncertainties for the initial $p$-Be interaction are from the Sanford-Wang parametrization covariance matrix [10]. $K^-$ production is estimated using the MARS hadronic interaction package [30] due to the scarcity of production measurements in the relevant kinematic regions. $K^-$ production cross section uncertainties from the initial $p$-Be interaction are given a conservative 100% normalization uncertainty.

B. Cross section uncertainties

CCQE cross sections on carbon are calculated assuming an RFG model with parameters $M_A$(axial mass) = 1.35 and $\kappa$(Pauli blocking factor) = 1.007. An additional correction, as a function of $Q^2$, is applied to background CC1$\pi$ interaction events in MC [14]. The uncertainties in $M_A$ and $\kappa$ for CCQE events on carbon are based on the statistical uncertainties of the MiniBooNE neutrino mode measurement [14], to avoid double counting systematic uncertainties accounted for in this analysis as detailed in this section.

Since the purpose of the $Q^2$ correction in the MiniBooNE neutrino mode measurement [14] is to match
the background CC1\(\pi\) interaction events in MC to a selected data sample comprising mainly of CC1\(\pi\) interaction events, there is no uncertainty placed on \(M_A\) for \(\nu_\mu\) CC1\(\pi\) interaction events. However, for \(\bar{\nu}_\mu\) CC1\(\pi\) interaction events, the \(M_A\)-resonant and coherent 1\(\pi\) uncertainties are not constrained by the MiniBooNE neutrino mode measurement and are not reduced. The values and uncertainties of \(M_A\) for CC coherent \(\pi\) interactions, \(M_A\) for multi-\(\pi\) interactions, Fermi surface momentum \((p_F)\), and NC axial vector isoscalar contribution \((\Delta S)\) are identical to previous MiniBooNE and SciBooNE measurements [9,16]. The uncertainties for pion absorption, pion inelastic scattering, and pionless \(\Delta\) decay in the target nucleus \((\pm 25\%, \pm 30\%, \text{and } \pm 100\%\), respectively) are treated in the same way as in a previous measurement [9], however they are treated as uncorrelated between MiniBooNE and SciBooNE (unlike all other cross section uncertainties). Both the \(\nu\) and \(\bar{\nu}\) \(M_A\) values and their uncertainties for quasielastic interactions on hydrogen are based on the latest deuterium measurements [31].

Additional systematic uncertainties are added to account for limitations of the RFG model. Such limitations include the absence of processes such as meson exchange currents and multinucleon knockout events [32–35]. A 10% normalization uncertainty is assigned to both \(\nu\) and \(\bar{\nu}\) CCQE interactions on carbon to cover the difference between data and prediction in the MiniBooNE \(\nu_\mu\) CCQE measurement. An additional 40% normalization uncertainty is placed on \(\bar{\nu}\) CCQE interactions on carbon to cover the discrepancy between the RFG model prediction for \(\bar{\nu}\) and recent nuclear models [32–35]. An additional 10% normalization uncertainty is added to non-CCQE \(\bar{\nu}\) interactions on carbon to account for the limitations of the RFG model for those type of events.

The full list of beam and cross section parameters for MC simulation and its associated systematic uncertainties are shown in Table II.

C. Detector uncertainties

Uncertainties associated with the MiniBooNE detector include light propagation, attenuation, and scattering in the detector as well as PMT response. The optical model for light propagation in the detector [22] uses 35 parameters for properties such as refractive index, attenuation length, scintillation strength, etc. These parameters are tuned to non-MiniBooNE measurements as well as MiniBooNE internal data. Over 100 separate MC data sets were created based on variations in these parameters. In a manner similar to the multism method, these results were used to compute the optical model error matrix in bins of

<p>| TABLE II. Summary of beam and cross section parameters for MC simulation with its associated systematic uncertainties. |</p>
<table>
<thead>
<tr>
<th>Beam</th>
<th>Cross Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^+) production in antineutrino beam (from WS neutrino background)</td>
<td>1.35 ± 0.07 GeV</td>
</tr>
<tr>
<td>(\pi^-) production from (p)-Be interaction</td>
<td>1.007 ± 0.005</td>
</tr>
<tr>
<td>(K^+) production from (p)-Be interaction</td>
<td>1.014 ± 0.014 GeV</td>
</tr>
<tr>
<td>(K^0) production from (p)-Be interaction</td>
<td>1.1 ± 0.275 GeV</td>
</tr>
<tr>
<td>Non-CCQE on carbon ((\bar{\nu}_\mu)) only</td>
<td>1.03 ± 0.275 GeV</td>
</tr>
<tr>
<td>CC1(\pi) on carbon</td>
<td>1.3 ± 0.52 GeV</td>
</tr>
<tr>
<td>CCQ on carbon</td>
<td>(\pm 9) MeV</td>
</tr>
<tr>
<td>CCQE on carbon ((\bar{\nu}_\mu)) only</td>
<td>220 ± 30 MeV/c</td>
</tr>
<tr>
<td>CCQ on carbon ((\bar{\nu}_\mu)) only</td>
<td>(0.0 \pm 0.1)</td>
</tr>
<tr>
<td>NC coherent (\pi) (M_A)</td>
<td>(\pm 1) kA</td>
</tr>
<tr>
<td>Nucleon and pion interaction in Be/Al</td>
<td>Horn skin depth, (\pm 1.4) mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.8% normalization uncertainty [15]</td>
</tr>
<tr>
<td>Spline fit to HARP data</td>
</tr>
<tr>
<td>Table IX in Ref. [28]</td>
</tr>
<tr>
<td>Table IX in Ref. [10]</td>
</tr>
<tr>
<td>100% normalization uncertainty</td>
</tr>
<tr>
<td>Table X in Ref. [10]</td>
</tr>
</tbody>
</table>

\(a\)This uncertainty is not applied to \(\nu_\mu\) CC1\(\pi\) events that are \(Q^2\) corrected.
reconstructed antineutrino energy. To estimate the impact of uncertainties in PMT response, independent MC data sets based on variations in the discriminator threshold, or the PMT charge-time correlations, were created and compared to default MC. Based on comparisons with external data [36–38] and the output of GCalor, an uncertainty of 35% is assigned to pion absorption and 50% is assigned to charge exchange in the detector medium. This is distinct from the uncertainty on pion absorption and charge exchange inside the nucleus.

Uncertainties associated with the SciBooNE detector include uncertainties in the muon energy loss in the scintillator and iron, light attenuation in the wavelength shifting fibers, and PMT response; see Ref. [16]. The crosstalk of the MA-PMT was measured to be 3.15% for adjacent channels with an absolute error of 0.4% [12]. The single photoelectron resolution of the MA-PMT is set to 50% in the simulation, and the absolute error is estimated to be ±20%. Birk’s constant for quenching in the SciBar scintillator was measured to be 0.0208 ± 0.0023 cm/MeV [12]. The conversion factors for analog-to-digital converter counts to photoelectrons were measured for all 14 336 MA-PMT channels in SciBar. The measurement uncertainty was at the 20% level. The threshold for hits to be used in SciBar track reconstruction is 2.5 photoelectrons; this threshold is varied by ±20% to evaluate the systematic error. The time-to-digital converter dead time is set to 55 ns in the MC simulation, with the error estimated to be ±20 ns [39].

The reconstruction uncertainties consist of antineutrino energy reconstruction uncertainties and muon track misidentification uncertainties. For antineutrino energy reconstruction uncertainties, the densities of SciBar, EC, and MRD are varied independently within their measured uncertainties, and the correlations between them are expressed in the total error matrix, $M$, a $42 \times 42$ covariance matrix in MiniBooNE and SciBooNE reconstructed antineutrino energy bins defined as

$$ M_{ij} = \sum_{X=\text{MB-SB, SB-SB}} M_{ij}^X + \sum_{Y=\text{WS-RS, RS-RS}} M_{ij}^Y N_i^Y N_j^Y + \sum_{Z=\text{WS-RS, RS-RS}} M_{ij}^Z N_i^Z N_j^Z + M_{ij}^{\text{stat}} $$

where

$$ M_{ij}^X = \sum_{Y=\text{WS-RS, RS-RS}} M_{i,j}^X N_i^Y N_j^Y + M_{i,j}^X N_i^Y N_j^Z + M_{i,j}^X N_i^Z N_j^Y + M_{i,j}^X N_i^Z N_j^Z $$

FIG. 10. Bin-wise square root of the total (statistical and systematic errors combined) fractional error matrix $\sqrt{M_{ij}} = \sqrt{M_{ij}^{\text{stat}}} / \sqrt{N_i N_j}$ for reconstructed antineutrino energy bin $i$ ($j$). Bins 1 through 21 are MiniBooNE, bins 22 through 42 are SciBooNE.

D. Error matrix

All of the MiniBooNE uncertainties, the SciBooNE uncertainties, and the correlations between them are expressed in the total error matrix, $M$, a $42 \times 42$ covariance matrix in MiniBooNE and SciBooNE reconstructed antineutrino energy bins defined as

$$ M = \begin{pmatrix} M_{\text{MB-SB}} & M_{\text{SB-SB}} \\ M_{\text{MB-SB}} & M_{\text{SB-SB}} \end{pmatrix} $$

where $M_{ij}^{\text{MB-SB}}$, $M_{ij}^{\text{SB-SB}}$, $M_{ij}^{\text{stat}}$ are the covariance elements.

FIG. 11. Correlation coefficients of the total (statistical and systematic errors combined) error matrix $(\rho_{ij} = M_{ij}/(\sigma_i \sigma_j))$. Bins 1 through 21 are MiniBooNE, bins 22 through 42 are SciBooNE. No bins are anticorrelated.
are the bin to bin covariance elements of the full error matrix. X denotes the type of correlation with Y and Z denoting the type of bins (either MiniBooNE or SciBooNE) associated with X. For MiniBooNE to MiniBooNE correlations, X = MB − MB, Y = MB, Z = MB. For SciBooNE to SciBooNE correlations, X = SB − SB, Y = SB, Z = SB. For MiniBooNE to SciBooNE correlations, X = MB − SB, Y = MB, Z = SB. For SciBooNE to MiniBooNE correlations, X = SB − MB, Y = SB, Z = MB. $N_i^{YRS}(N_i^{ZRS})$ and $N_i^{YWS}(N_i^{ZWS})$ are the number of RS and WS events for bin type Y (bin type Z) in reconstructed antineutrino energy bin $i$ (bin $j$), respectively. $\hat{M}_{i,j}^{X} (RS, RS)$ are the elements of the RS to RS correlated fractional error matrix for correlation type X defined as

$$\hat{M}_{i,j}^{X} = \frac{\hat{M}_{i,j}^{X} (RS, RS)}{\sum_{i,j}^{M^{X}}},$$

where $\hat{M}_{i,j}^{X} (RS, RS)$ is the full RS to RS reconstructed antineutrino energy bin covariance for correlation type X. $\hat{M}_{i,j}^{X} (WS, WS)$, $\hat{M}_{i,j}^{X} (RS, WS)$, and $\hat{M}_{i,j}^{X} (WS, RS)$ are similarly defined fractional error matrices for correlation type X with different RS and WS correlations. $M^{X}_{ij}$ is the statistical covariance matrix in reconstructed antineutrino energy bins for correlation type X (only SB − SB and MB − MB have nonzero elements).

The decomposition and reconstruction of the full error matrix $M$ to and from the fractional error matrices allows the error matrix to be updated based on different MC predictions, as a function of the oscillation parameters in the physics parameter space.

For illustrative purposes, Fig. 10 shows the square roots of the elements of the total fractional error matrix, $\sqrt{M_{ij}} = \sqrt{M_{ij}}/\sqrt{N_i^RS N_j^WS}$, where $M_{ij}$ are the elements of the total error matrix and $N_i^RS$ ($N_j^WS$) is the MC prediction for reconstructed antineutrino energy bin $i$ ($j$). Figure 11 shows the correlation coefficients of the total error matrix in reconstructed antineutrino energy bins.

Figure 12 shows the MiniBooNE and SciBooNE default MC $E_p^{QE}$ predictions for RS and WS events with error bars corresponding to the $\sqrt{M_{ij}}$ values of the error matrix diagonal elements.

VI. ANALYSIS METHODOLOGY

Oscillation predictions are based on a two-antineutrino oscillation model, where the oscillation survival probability for a $\bar{\nu}_\mu$ in the beam is given by

$$P(\bar{\nu}_\mu \to \bar{\nu}_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E}.$$  

$L$ is the path length in kilometers, $E$ is the antineutrino energy in GeV, $\theta$ is the mixing angle, and $\Delta m^2$ is the difference in the squares of the masses of two different mass eigenstates.

The $\chi^2$ statistic is formed,

$$\chi^2 = \sum_{i,j=1}^{42} (D_i - N_i)(M^{-1})_{ij}(D_j - N_j),$$

where $(M^{-1})_{ij}$ is the $i$th element of the inverse of the error matrix $M$, the covariance matrix in MiniBooNE and SciBooNE $E_p^{QE}$ bins described in Eq. (2). $D_i$ ($D_j$) is the data count in bin $i$ ($j$) and $N_i$ ($N_j$) is the MC prediction for bin $i$ ($j$), in MiniBooNE and SciBooNE $E_p^{QE}$ bins. $N_i$ is the sum of neutrino (WS) and antineutrino (RS) events in the $i$th bin:

$$N_i = N_i^{RS}(\Delta m^2, \sin^2 2\theta) + N_i^{WS}.$$

As shown in Eq. (7), only the predicted RS event rate depends on the oscillation parameters, $\Delta m^2$ and $\sin^2 2\theta$, for this two-antineutrino oscillation model. The WS flux is assumed to not oscillate. The index runs from 1 to 42 (21 MiniBooNE $E_p^{QE}$ bins and 21 SciBooNE $E_p^{QE}$ bins). For the physics analysis fitting, a $\Delta \chi^2$ test statistic is used as defined by
where $\theta_{\text{BF}}$ refers to the oscillation parameters at the best fit point and $\theta_{\text{phys}}$ refers to the oscillation parameters at a given test point.

The method of Feldman and Cousins [40] is used to determine the $\Delta \chi^2$ value at each point that corresponds to a certain confidence level of acceptance or rejection. To obtain the 90% confidence level exclusion region for disappearance, a $\Delta \chi^2$ distribution is formed for each point $\theta_{\text{phys}}$ in parameter space using many iterations of generated fake data at that $\theta_{\text{phys}}$. The $\Delta \chi^2$ value from actual data at each $\theta_{\text{phys}}$ is then compared to the fake data $\Delta \chi^2$ distribution at each $\theta_{\text{phys}}$. If the $\Delta \chi^2$ value from actual data is larger than 90% of the all the fake data $\Delta \chi^2$ values at a $\theta_{\text{phys}}$ point, then the $\theta_{\text{phys}}$ point in parameter space is excluded at 90% confidence level. The aggregation of all the excluded 90% confidence level $\theta_{\text{phys}}$ points forms the 90% confidence level exclusion region.

### TABLE III. Observed event counts for each MiniBooNE and SciBooNE data bin, MC predictions, and uncertainty.

<table>
<thead>
<tr>
<th>Bin range (MeV)</th>
<th>MB data</th>
<th>MB MC</th>
<th>MB error ±</th>
<th>SB data$^a$</th>
<th>SB cosmic</th>
<th>SB MC</th>
<th>SB error ±</th>
</tr>
</thead>
<tbody>
<tr>
<td>300–400</td>
<td>565</td>
<td>413.5</td>
<td>111.0</td>
<td>1077.0</td>
<td>21.0</td>
<td>997.3</td>
<td>136.8</td>
</tr>
<tr>
<td>400–467</td>
<td>2577</td>
<td>2139.2</td>
<td>464.8</td>
<td>966.8</td>
<td>89.2</td>
<td>915.6</td>
<td>141.9</td>
</tr>
<tr>
<td>467–533</td>
<td>4433</td>
<td>4039.9</td>
<td>802.2</td>
<td>872.8</td>
<td>85.2</td>
<td>834.4</td>
<td>132.1</td>
</tr>
<tr>
<td>533–600</td>
<td>5849</td>
<td>5211.0</td>
<td>1005.7</td>
<td>854.4</td>
<td>72.6</td>
<td>809.4</td>
<td>132.2</td>
</tr>
<tr>
<td>600–667</td>
<td>6411</td>
<td>5725.6</td>
<td>1108.7</td>
<td>856.8</td>
<td>59.2</td>
<td>790.6</td>
<td>137.3</td>
</tr>
<tr>
<td>667–733</td>
<td>6445</td>
<td>5778.3</td>
<td>1130.3</td>
<td>915.0</td>
<td>51.0</td>
<td>781.9</td>
<td>144.3</td>
</tr>
<tr>
<td>733–800</td>
<td>6090</td>
<td>5586.8</td>
<td>1096.9</td>
<td>849.8</td>
<td>52.2</td>
<td>757.3</td>
<td>139.5</td>
</tr>
<tr>
<td>800–867</td>
<td>5678</td>
<td>5268.3</td>
<td>1044.8</td>
<td>876.6</td>
<td>43.4</td>
<td>717.1</td>
<td>138.8</td>
</tr>
<tr>
<td>867–933</td>
<td>5314</td>
<td>4826.2</td>
<td>951.8</td>
<td>787.0</td>
<td>39.0</td>
<td>655.8</td>
<td>138.0</td>
</tr>
<tr>
<td>933–1000</td>
<td>4624</td>
<td>4319.6</td>
<td>865.1</td>
<td>688.0</td>
<td>35.0</td>
<td>639.7</td>
<td>129.6</td>
</tr>
<tr>
<td>1000–1067</td>
<td>4015</td>
<td>3720.3</td>
<td>747.2</td>
<td>628.0</td>
<td>29.0</td>
<td>580.2</td>
<td>125.4</td>
</tr>
<tr>
<td>1067–1133</td>
<td>3349</td>
<td>3163.6</td>
<td>642.1</td>
<td>569.6</td>
<td>28.4</td>
<td>488.7</td>
<td>105.8</td>
</tr>
<tr>
<td>1133–1200</td>
<td>2965</td>
<td>2655.9</td>
<td>554.3</td>
<td>496.6</td>
<td>21.4</td>
<td>403.9</td>
<td>92.2</td>
</tr>
<tr>
<td>1200–1267</td>
<td>2464</td>
<td>2147.2</td>
<td>453.0</td>
<td>377.0</td>
<td>23.0</td>
<td>308.4</td>
<td>74.6</td>
</tr>
<tr>
<td>1267–1333</td>
<td>1937</td>
<td>1726.4</td>
<td>367.8</td>
<td>273.6</td>
<td>22.4</td>
<td>228.4</td>
<td>53.6</td>
</tr>
<tr>
<td>1333–1400</td>
<td>1534</td>
<td>1372.0</td>
<td>297.9</td>
<td>178.6</td>
<td>18.4</td>
<td>150.0</td>
<td>37.2</td>
</tr>
<tr>
<td>1400–1467</td>
<td>1227</td>
<td>1073.3</td>
<td>238.1</td>
<td>111.2</td>
<td>18.8</td>
<td>89.4</td>
<td>23.9</td>
</tr>
<tr>
<td>1467–1533</td>
<td>859</td>
<td>820.5</td>
<td>187.7</td>
<td>65.4</td>
<td>17.6</td>
<td>57.1</td>
<td>15.0</td>
</tr>
<tr>
<td>1533–1600</td>
<td>679</td>
<td>607.2</td>
<td>145.8</td>
<td>39.0</td>
<td>17.0</td>
<td>33.1</td>
<td>10.4</td>
</tr>
<tr>
<td>1600–1700</td>
<td>684</td>
<td>607.2</td>
<td>149.1</td>
<td>40.8</td>
<td>28.2</td>
<td>27.6</td>
<td>9.9</td>
</tr>
<tr>
<td>1700–1900</td>
<td>610</td>
<td>560.1</td>
<td>144.5</td>
<td>37.6</td>
<td>39.4</td>
<td>24.8</td>
<td>7.8</td>
</tr>
</tbody>
</table>

$^a$The SB data has its SB cosmic data background removed.
The full error matrix is used to create the fake data for the Feldman and Cousins tests. First, a Cholesky decomposition is performed on the error matrix \( M \):

\[
M = LL^*,
\]

where \( L \) is a lower triangular matrix and \( L^* \) is the conjugate transpose of \( L \). Then, a vector \( u \) is created, where each of the \( n \) elements, 42 in total, of \( u \) are drawn from a Gaussian distribution with mean 0 and variance 1. A fluctuated fake data histogram is given by

\[
N_{\text{fake}} = N_{\text{default}}(\theta_{\text{phys}}) + Lu,
\]

where \( N_{\text{default}} \) is the default Monte Carlo prediction assuming an oscillation signal with oscillation parameters at point \( \theta_{\text{phys}} \).

### VII. RESULTS

Figure 13 shows the observed event distributions, in reconstructed antineutrino energy, for MiniBooNE and SciBooNE. The systematic uncertainty shown for the MC predictions was computed as just the square roots of the diagonals of the total error matrix without correlations.

Table III lists the event counts in each bin, for data and for MC predictions. The listed uncertainties are based on the square roots of the diagonals of the total error matrix without correlations. (The reported SciBooNE data has fractional counts due to the manner in which the cosmic ray background is subtracted.)

A MiniBooNE-only disappearance analysis is included to give a sense of what the sensitivity would be without the inclusion of SciBooNE data. Figure 14 shows the 90% C.L. exclusion region and best fit point for the MiniBooNE-only \( \bar{\nu}_\mu \) disappearance analysis, completed using the same methodology as the joint disappearance analysis except with the exclusion of SciBooNE data, SciBooNE MC

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**FIG. 14** (color online). 90% C.L. exclusion region (solid line) and best fit point for the MiniBooNE-only \( \bar{\nu}_\mu \) disappearance analysis. Also shown is the 90% C.L. result from the 2009 MiniBooNE disappearance analysis [8] (dashed line) and the Chicago-Columbia-Fermilab-Rochester (CCFR) experiment [41] (dot-dashed line). The expected 90% C.L. sensitivity band from fake data studies is also shown (shaded region); \( 1\sigma \) (68%) of fake data tests, where the fake data had statistical and systematic fluctuations but no oscillation signal, had 90% C.L. limit curves in this shaded region.

**FIG. 15** (color online). 90% C.L. exclusion region (solid line) and best fit point for the joint MiniBooNE, SciBooNE \( \bar{\nu}_\mu \) disappearance analysis. Also shown is the 90% C.L. result from the 2009 MiniBooNE disappearance analysis [8] (dashed line) and the CCFR experiment [41] (dot-dashed line). The expected 90% C.L. sensitivity band from fake data studies is also shown (shaded region); \( 1\sigma \) (68%) of fake data tests, where the fake data had statistical and systematic fluctuations but no oscillation signal, had 90% C.L. limit curves in this shaded region.
prediction, and SciBooNE error matrix uncertainties in the $\chi^2$ statistic. The best fit point is $\Delta m^2 = 5.9$ eV$^2$, $\sin^2 2\theta = 0.076$. At the best fit point, $\chi^2 = 25.7$ (probability 12.4%). For the null oscillation hypothesis, $\chi^2 = 28.3$ (probability 13.7%). With $\Delta \chi^2 = 2.6$, the null oscillation hypothesis is excluded at 52.4% C.L.

Figure 15 shows the $\bar{\nu}_\mu$ disappearance limit for the joint disappearance analysis. For $\Delta m^2 = 1$ eV$^2$ and $\Delta m^2 = 10$ eV$^2$, the 90% C.L. limit for $\sin^2 2\theta$ are at 0.121 and 0.024, respectively. At $\sin^2 2\theta = 1$, the 90% C.L. limit for $\Delta m^2$ is 0.156 eV$^2$. The best fit point from the joint analysis is $\Delta m^2 = 5.9$ eV$^2$, $\sin^2 2\theta = 0.086$. At the best fit point, $\chi^2 = 40.0$ (probability 47.1%). For the null oscillation hypothesis, $\chi^2 = 43.5$ (probability 41.2%). With $\Delta \chi^2 = 3.5$, the null oscillation hypothesis is excluded at 81.9% C.L.

All probabilities in both the MiniBooNE-only and joint disappearance analyses are based on fake data studies.

Figure 16 shows the data to MC ratios for MiniBooNE and SciBooNE, as well as how the best fit signal modifies the MC predictions. From these ratio plots, it can be seen how the best fit signal improves the shape agreement between data and MC. Figure 17 shows the double ratio

$$\frac{\text{MiniBooNE data}}{\text{MiniBooNE default MC}} \times \frac{\text{SciBooNE data}}{\text{SciBooNE default MC}}$$

In Fig. 17, any common normalization difference is removed and the expected result is a value of one. The double ratio result agrees well with the expectation except where statistics are small.

VIII. CONCLUSIONS

An improved search for $\bar{\nu}_\mu$ disappearance using a two-detector combined MiniBooNE/SciBooNE analysis has been performed. Previous flux and cross section measurements, as well as an increased data set, have enabled a substantial improvement in the sensitivity to $\bar{\nu}_\mu$ disappearance. The results are consistent with no short baseline disappearance of $\bar{\nu}_\mu$ and we have dramatically improved on the excluded regions of the oscillation parameter space. MiniBooNE and SciBooNE have pushed the limit on short baseline disappearance of $\bar{\nu}_\mu$ down to roughly 10%, the region of interest for sterile neutrino models.

ACKNOWLEDGMENTS

We wish to acknowledge the support of Fermilab, the U.S. Department of Energy, and the National Science Foundation in the construction, operation, and data analysis for the MiniBooNE and SciBooNE experiments. The SciBooNE detector was mainly constructed and operated...
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PHYSICAL REVIEW D 86, 052009 (2012)

We acknowledge the support of MEXT and JSPS (Japan) with the Japan/U.S. Cooperation Program. We also acknowledge the Los Alamos National Laboratory for LDRD funding. We acknowledge the Physics Department at Chonnam National University, Dongshin University, and Seoul National University for the loan of parts used in SciBar and the help in the assembly of SciBar. We wish to thank the Physics Departments at the University of Rochester and Kansas State University for the loan of Hamamatsu PMTs used in the MRD. We gratefully acknowledge the support of grants and contracts from the INFN (Italy), the Ministry of Science and Innovation and CSIC (Spain), and the STFC (U.K.). We acknowledge the support by MEXT and JSPS with the Grant-in-Aid for Scientific Research A 19204026, Young Scientists S 20674004, Young Scientists B 18740145, Scientific Research on Priority Areas “New Developments of Flavor Physics,” the global COE program “The Next Generation of Physics, Spun from Universality and Emergence,” and the Japan–U.S. Cooperative Science Program between JSPS and NSF.