Experimental Characterization of Quantum Dynamics Through Many-Body Interactions

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We report on the implementation of a quantum process tomography technique known as direct characterization of quantum dynamics applied on coherent and incoherent single-qubit processes in a system of trapped $^{40}\text{Ca}^{+}$ ions. Using quantum correlations with an ancilla qubit, direct characterization of quantum dynamics reduces substantially the number of experimental configurations required for a full quantum process tomography and all diagonal elements of the process matrix can be estimated with a single setting. With this technique, the system’s relaxation times $T_1$ and $T_2$ were measured with a single experimental configuration. We further show the first, complete characterization of single-qubit processes using a single generalized measurement realized through multibody correlations with three ancilla qubits.

Characterization of quantum dynamics is an important primitive in quantum physics, chemistry, and quantum information science for determining unknown environmental interactions, estimating Hamiltonian parameters, and verifying the performance of engineered quantum devices. This has led to a major effort in developing tools for the full characterization of quantum processes, known as quantum process tomography (QPT). The standard approach for QPT is resource intensive, requiring $12^N$ experimental configurations for a system of $N$ qubits [1,2], where each experimental configuration consists of the preparation of input probe states and the measurement of process outputs [3]. Using ancilla qubits but only joint separable measurements, the number of experimental configurations is still $12^N$ [4–6]. However, the use of many-body interactions to ancilla qubits in the preparation and/or measurements can significantly decrease this number to anywhere from $4^N$ to a single configuration depending on the nature and complexity of quantum correlations [6]. Using two-body correlations, direct characterization of quantum dynamics (DCQD) requires up to $4^N$ experimental configurations for full quantum process tomography, and in particular only one experimental setting for estimating certain parameters (e.g., relaxation times) [7,8]. Experimental efforts in this direction include a partial and nescalable implementation of DCQD [9,10], an ancilla-assisted process tomography [4,5], and a joint effort efficiently implementing DCQD in a photonic system [11].

Alternatively, efficient gate-fidelity estimation methods such as randomized benchmarking [12], or tomographic methods such as selective and efficient QPT [13,14] and compressed sensing for quantum process tomography [15–17], have recently been developed to overcome the exponential increase of the required experimental configurations. Generally, these methods are tailored to estimate a polynomial number of effective parameters, such as gate fidelity [12] or when we can make a sparse quantum process or Hamiltonian assumption from a priori knowledge about the quantum system [17]. For example, the estimation of the dynamical parameters $T_1$ and $T_2$ (longitudinal and transverse relaxation times [1]) is a task involving two noncommuting observables (e.g., $\sigma_x$ and $\sigma_y$) that cannot be measured simultaneously. These parameters describe the influence of noise on atomic-, molecular-, and spin-based systems induced by the interaction with the environment. An alternative approach based on DCQD, henceforth called direct characterization of relaxation times (DCRT), enables the measurement of both $T_1$ and $T_2$ simultaneously with a single experimental configuration [18].

In this work, we apply the DCQD technique and extensions on a system of trapped $^{40}\text{Ca}^{+}$ ions. Single-qubit processes are reconstructed with four experimental configurations using DCQD, and alternatively with just a single configuration using a generalized measurement (GM). In addition, we quantify the relaxation times $T_1$ and $T_2$ in our system with a single configuration. This technique can also characterize more realistic environments affecting not only the probe but also the ancilla qubit collectively.

In the following, we consider quantum processes which can be described by a completely positive, convex-linear, and trace-preserving map $E$ mapping the input state $\rho$ onto the output state $\rho'$. For a single qubit this can be written as

$$E: \rho \rightarrow \rho' = \sum_{m,n=1}^{4} \chi_{m,n} \sigma_{m} \rho \sigma_{n}^\dagger,$$

with $\sigma_{m}$, $\sigma_{n}$ the Pauli operators $\{1, \sigma_x, \sigma_y, \sigma_z\}$ and $\chi$ a semipositive matrix containing complete information
The probabilities $p_{i,j}$ of measuring the Bell-state projector $P_i$ for each input state $\rho_j$ shown in Table I are determined, according to Refs. [7,19], by

$$p_{i,j} = \text{Tr}[P_i E(\rho_j)] = \sum_{m,n=1}^4 \chi_{m,n} \Lambda_{i,m}^{i,n} \Lambda_{i,m}^{i,n}.$$  

(2)

Therefore, the process matrix $\chi$ can be calculated directly by linear inversion of the matrix $\Lambda$. The set of input states $\rho_j$ and the Bell-state projectors $P_i$ have to be determined such that the 16 equations in Eq. (2) are linearly independent, which ensures that $\Lambda$ is invertible (Table I).

Our experiments were realized on a system consisting of $^{40}$Ca$^+$ ions confined to a string in a linear Paul trap [20]. Each ion represents a logical qubit which is encoded in the electronic levels $D_{5/2}(m = -1/2) = |0\rangle$ and $S_{1/2}(m = -1/2) = |1\rangle$. Each experimental cycle consists of an initialization of the ions in their internal electronic and motional ground states followed by a coherent manipulation of the qubits and finally a detection of the quantum state. State initialization is realized by optical pumping into the $S_{1/2}(m = -1/2)$ state after cooling the axial center-of-mass mode to the motional ground state. The manipulation of the qubits is implemented by coherently exciting the $S_{1/2} \leftrightarrow D_{5/2}$ quadrupole transition with laser pulses. Finally, the population of the qubit states is measured by exciting the $S_{1/2} \leftrightarrow P_{1/2}$ transition and detecting the fluorescence light, using electron shelving [21]. Our setup is capable of realizing collective qubit rotations $U(\theta, \phi) = \exp[-i\pi\sum_i \sin(\phi)\sigma_i^{(i)} + \cos(\phi)\sigma_i^{(i)}\hat{\sigma}_z^{(i)}]$ via a laser beam addressing the entire register as well as Mølmer-Sørensen entangling gates $MS(\theta, \phi) = \exp[-i\pi(\sum_i \sin(\phi)\sigma_i^{(i)} + \cos(\phi)\sigma_i^{(i)}\hat{\sigma}_z^{(i)})]$ [22, 23]. Additionally, we are able to perform single-qubit rotations on the $i$th ion of the form $U_{ij}^{(i)}(\theta) = \exp(-i\pi/2 \sigma_i^{(i)})$ by an off-resonant laser beam, which addresses individual ions. The input states for DCQD of Table I are prepared by applying collective entangling operations and qubit rotations as shown in Fig. 1(a). For example, the input state $\rho_2$ is created by the nonmaximally entangling operation $MS(\pi, 0)$. Our two-qubit entangling operation generates Bell states with a fidelity of $\approx 99\%$ in $120 \mu$s.

The BSM is experimentally realized by a maximally entangling operation $MS(\pi, \pi)$, which maps from the Bell-state basis to the computational basis ($|00\rangle, |01\rangle, |10\rangle, |11\rangle$), followed by individual-ion-resolving fluorescence detection with a CCD camera.

As an example of the reconstruction method, consider the first input state $\rho_1 = |\Phi^+\rangle\langle\Phi^+|$. If the process $E$ is the identity $1$, the expectation value of the BSM projector $P_i$ is 1, which is equivalent to detecting both ions in the state $|11\rangle$ after the BSM. If a bit flip occurs on the system ion, the output state is then mapped onto the state $|01\rangle$ by the BSM ($P_{1,2} = 1$). The considerations are similar for a phase-flip, or bit- and phase-flip processes. Therefore, the diagonal elements $\chi_{m,m}$ of the superoperator $\chi$ corresponding to $1$, $\sigma_x$, $\sigma_y$, and $\sigma_z$ are detected by a single input state in combination with one BSM.

We demonstrate the DCQD method by characterizing the full quantum process of implemented unitary rotations $\sigma_x$ and $\sigma_y$ as well as nonunitary processes such as amplitude and phase damping [24]. The $\chi$ matrices reconstructed from the measured probabilities are shown in Figs. 2(a) and 2(b) for $\sigma_x$ and $\sigma_y$ rotations. A single-qubit
Full QPT of a single-qubit process is also possible with a single experimental configuration by using additional ancillas and a GM. Here, we expand the dimension of the Hilbert space $H_A \otimes H_S$ with the system Hilbert space $H_S$ and the ancilla Hilbert space $H_A$ such that the dimension of the total Hilbert space is equal to the number of free parameters in the process matrix $\chi$ [6]. For a single-qubit process one has to determine all 16 superoperator elements $\chi_{m,n}$, which leads to an eight-dimensional ancilla Hilbert space. Therefore, we used three ancilla qubits $A_1$, $A_2$, and $A_3$ to quantify a full process $E$ acting on the system qubit $S$. This GM is realized by entangling the system and ancilla qubits using many-body interactions [22, 23] then applying the process $E$ on $S$ and finally performing BSMs on two pairs. Figure 1(b) shows the sequence implemented for this GM which proceeds as follows. First, we create an entangled input state using maximally and nonmaximally entangling Mølmer-Sørensen interactions in combination with global and addressed single-qubit rotations. After applying the process $E$ on $S$ we perform a pairwise BSM on the combined output state by implementing two nonmaximally entangling operations $\text{MS}(\frac{\pi}{2})$ and two addressed ac-Stark pulses $U_2^{(1)}(\pi)$ and $U_2^{(3)}(\pi)$, which separate the entangled system $H(S, A_1, A_2, A_3)$ into a product state of two subsystems $H(A_1, A_3) \otimes H(S, A_2)$. These operations are equivalent to two pairwise maximally entangling gates $\text{MS}(\pi)$ acting on the two subsystems $H(A_1, A_3)$ and $H(S, A_2)$. The 16 results of the measurement are directly linked to the 16 superoperator elements $\chi_{m,n}$ by a matrix $\Lambda$ similar to Eq. (2). Using this technique we reconstructed unitary processes $\{1, \sigma_x = U(\pi, 0), \sigma_y = U(\pi, \frac{\pi}{2}), \sigma_z = U(\pi, \frac{3\pi}{2})\}$ acting on a single qubit with a fidelity of $0.9970 \pm 0.02, 0.9730 \pm 0.02, 0.9980 \pm 0.01, 0.9440 \pm 0.02\%$. All processes were measured with a total of 5000 cycles.

In contrast to previous QPT measurements of engineered processes, the process of phase (amplitude) damping occurs naturally in our system due to magnetic field fluctuations (spontaneous decay) [29]. The dynamical parameters $T_1$ and $T_2$ can, however, be determined simultaneously with only the first input state $\rho_1$ being subject to the DCQD scheme even if the damping processes act collectively on both qubits (as in our experimental system [29]). This method, named DCRT above, consists of preparing an input Bell state $\rho_1 = |\Phi^+\rangle\langle\Phi^+|$, exposing both qubits to the damping processes for a time $t$, and a final BSM, which yields the diagonal elements $\chi_{ii}$ of the process matrix. As described in the Supplemental Material [24] and assuming Markovian noise, the dynamical parameters are then given by

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<th>Target process</th>
<th>DCQD, $F$ (%)</th>
<th>SQPT, $F$ (%)</th>
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<tr>
<td>$1$</td>
<td>$97.5 \pm 0.6$</td>
<td>$98.1 \pm 1.3$</td>
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<tr>
<td>$\sigma_x$</td>
<td>$96.5 \pm 1.0$</td>
<td>$98.1 \pm 1.3$</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>$96.6 \pm 1.4$</td>
<td>$97.5 \pm 1.4$</td>
</tr>
<tr>
<td>Amplitude damping</td>
<td>$95.3 \pm 1.9$</td>
<td>$95.2 \pm 2.7$</td>
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<tr>
<td>Phase damping</td>
<td>$97.4 \pm 0.8$</td>
<td>$95.7 \pm 0.8$</td>
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FIG. 2 (color online). Experimental results of DCQD for unitary and decoherence processes. (a), (b) Results of the measured superoperator $\chi$ for the rotation operations $U(\pi, 0)$ in (a) and $U(\pi, \pi/2)$ in (b). Ideally, the target processes have only nonzero elements at positions indicated by the gray (orange)-bordered bars. (c), (d) Bloch sphere representation of the ideal (c) and measured (d) amplitude damping process with 60% probability [22]. (e), (f) Bloch sphere representation of the ideal (e) and measured (f) phase damping process with 60% probability [22]. Bloch sphere axes in black evolve into the spheroid primed axes in blue. A slight imperfection due to residual light on the ancilla Bloch sphere axes in black evolve into the spheroid primed axes in blue.
measurement is realized by initializing the ion in the state \( \chi_{1,1} \) - \( \chi_{4,4} \)

\[
e^{-N\gamma t/T_2} = \chi_{1,1} - \chi_{4,4}
\]

\[
= \text{Tr}[[|\Phi^+\rangle\langle\Phi^+| - |\Phi^-\rangle\langle\Phi^-|]E(|\Phi^+\rangle\langle\Phi^+|)],
\]

(3)

\[
1 + 2e^{-2\gamma t/T_1} - 2e^{-\gamma t/T_1} = 1 - 2(\chi_{2,2} + \chi_{3,3}),
\]

(4)

with \( N \) the number of ions. From the entries of the \( \chi \) matrix corresponding to 1 and \( \sigma_z \) (\( \sigma_x \) and \( \sigma_y \)) operations, \( T_2(T_1) \) depends on the probability that no error or phase flips (bit flips) occur on the entire system. A fit of DCRT measurements \( \chi_{ij} \) to Eqs. (3) and (4) at different times \( t \) thus yields \( T_1 \) and \( T_2 \) using a single experimental configuration. We explored this DCRT technique in our experimental system. The measurement results of the decoherence estimation are shown in Fig. 3(a). The green dots show the difference between the diagonal elements \( \chi_{1,1} \) and \( \chi_{4,4} \) as a function of the waiting time \( t \). The spontaneous decay of the system is shown in Fig. 3(b) by plotting \( 1 - 2(\chi_{2,2} + \chi_{3,3}) \) as a function of time. For every waiting time \( t \) the experiment was repeated up to 250 times to gain significant statistics.

We can compare the DCRT technique with two traditional methods that use product input states: Ramsey-contrast measurements for phase-decoherence estimation and direct spontaneous-decay measurements [30]. A Ramsey-contrast measurement is realized by initializing the ion in the state \( |00\rangle + |11\rangle \) by a global rotation \( U(\pi, 0) \), followed by a waiting time \( t \) and finally applying a second rotation \( U(\pi, \phi) \) in which the phase \( \phi \) is varied. The observed contrast as a function of \( \phi \) corresponds to the preserved phase coherence. Spontaneous-decay measurements, instead, consist of measuring the probability of detecting both ions in the excited state \( |0\rangle \) as a function of time. The results of these Ramsey-contrast (spontaneous-decay) measurements are shown in Fig. 3(a) [Fig. 3(b)] as red diamonds (blue triangles). The measured relaxation times corresponding to the traditional methods are called \( T_1^{\text{rad}} \) and \( T_2^{\text{rad}} \). The exponential fit (solid green line) of Eq. (3) to the data was estimated with \( N = 2 \) (collective dephasing) and yields \( T_2^{\text{DCRT}} = 18.8(5) \) ms. The Ramsey-contrast measurements (red diamonds) were carried out on a single ion and yield a coherence time of \( T_2^{\text{rad}} = 19.4(8) \) ms. The dotted green line in Fig. 3(a) corresponds to the fitted function to Eq. (3) (solid green line) with \( N = 1 \) instead of \( N = 2 \) and shows good agreement with the single-ion Ramsey-contrast measurement. Therefore, the DCRT technique enables the characterization of the phase decoherence of the collective system (solid green line) and also gives a conclusion about the phase decoherence of a single ion (dotted green line). An exponential fit of the decay data of Fig. 3(b) to Eq. (4) gives the characteristic lifetime \( T_1^{\text{DCRT}} = 1130(47) \) ms for the DCRT technique (solid green line) and \( T_2^{\text{rad}} = 1160(30) \) ms for the traditional method (dotted blue line), which are in good agreement with previously measured values [31] of 1148(18) ms.

In summary, we have experimentally demonstrated two different approaches for the full characterization of single-qubit quantum processes, lowering the required experimental configurations from 12 to 4 using DCQD and a single configuration via the GM method. The reconstruction of coherent and incoherent processes was shown with fidelities of \( \approx 97\% \) using DCQD. In particular, we have observed a lower statistical uncertainty of the fidelity of some of the processes compared to the SQPT. Nevertheless, a matter of further investigation is a comparison of the scaling in the number of experimental cycles required for the SQPT and DCQD to achieve a target uncertainty in the fidelity (e.g., see identity process in Table II).

![Fig. 3](color online). Simultaneous measurement of phase decoherence (a) and the spontaneous decay (b) of a two-qubit system. The DCRT technique (green dots) is compared to a Ramsey-contrast measurement (red diamonds) and a spontaneous-decay measurement (blue triangles) (see text). The measurement using the DCRT method in (a) was carried out on the entangled two-qubit system \[ \exp(-4i/T_2^{\text{DCRT}}) \text{ scaling} \] whereas the red diamonds were measured on a single qubit with the Ramsey-contrast technique \[ \exp(-t/T_2^{\text{rad}}) \text{ scaling} \]. The shaded areas correspond to the envelope of the curves with the decay times \( T_2^{\text{DCRT}, \text{rad}} \pm \Delta T_2^{\text{DCRT}, \text{rad}} \), considering the statistical errors \( \Delta T_1^{\text{DCRT}, \text{rad}} \). The relaxation time measurements, using the DCRT method and, in comparison, the traditional Ramsey-contrast and spontaneous-decay measurement, yield \( T_2^{\text{DCRT}} = 18.8(5) \) ms, \( T_2^{\text{rad}} = 19.4(8) \) ms, \( T_1^{\text{DCRT}} = 1130(47) \) ms, and \( T_1^{\text{rad}} = 1160(30) \) ms.
Experimentally, a reduced number of experimental configurations implies a substantial reduction of measurement time for a full QPT using DCQD as compared with SQPT (e.g., from 35 days to 1 day; see Ref. [24]). In addition, the DCRT technique, based on the DCQD protocol, was used as a powerful tool to characterize the noise in our system by measuring the relaxation times $T_1$ and $T_2$ simultaneously with one experimental setting. This technique indicates good agreement with traditional methods as Ramsey-contrast and spontaneous-decay measurement. In principle, another application of DCRT could be for biological systems where dissipative dynamics play a crucial role [33,34]. The same measurement could be considered in several quantum architectures such as photons and ions.

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[3] Changing experimental configurations is an overhead to be considered in several quantum architectures such as photons and ions.