Measurement of W-boson polarization in top-quark decay using the full CDF Run II data set

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Measurement of $W$-boson polarization in top-quark decay using the full CDF Run II data set


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031104-2
We measure the polarization of $W$ bosons from top-quark ($t$) decays into final states with a charged lepton and jets, $t \rightarrow W^+ bW^- b \rightarrow \ell b q \bar{q} b$, using the full Run II data set collected by the CDF II detector, corresponding to an integrated luminosity of $8.7 \text{ fb}^{-1}$. A model-independent method simultaneously determines the fraction of longitudinal ($f_0$) and right-handed ($f_+$) $W$ bosons to yield $f_0 = 0.726 \pm 0.066 \text{ (stat)} \pm 0.067 \text{ (syst)}$ and $f_+ = -0.045 \pm 0.044 \text{ (stat)} \pm 0.058 \text{ (syst)}$ with a correlation coefficient of $-0.69$. Additional results are presented under various standard model assumptions. No significant discrepancies with the standard model are observed.

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The top quark was first observed in the Tevatron Run I (1992–1996) data sets collected by the CDF and D0 experiments at Fermilab [1]. Because the mass of the top quark is large, beyond-the-standard-model physics contributions can affect the top-quark phenomenology in a wide variety of ways: the production mechanisms can be affected, the decay widths can be altered, its intrinsic properties can be changed, and the experimental signature can be mimicked by a new particle of similar mass. Thus a principal goal of the Tevatron Run II (2001–2011) program, which produced data samples O(100) times larger than Run I, was to thoroughly explore the properties of the top quark. This article reports a measurement of the W-boson polarization from top-quark decay using the full Run II data set collected by the CDF II experiment. The focus is on the lepton-plus-jets final state, $\tilde{t}\tilde{t}\rightarrow W^+bW^−\tilde{b}\rightarrow \ell vbgq\tilde{q}\tilde{b}$, which provides the most sensitive determination of the W polarization due to its high yield, low background, and constrained kinematics. At present the most precise experimental knowledge of the W-boson polarization comes from the ATLAS experiment [2] and from the combination of existing Tevatron results [3]. The results reported here supersede the previous CDF measurements in Ref. [4] and have a total uncertainty comparable to the Tevatron combination and a statistical precision a factor of 1.6 smaller than Ref. [4].

The top quark [5] almost always decays to a $W^+$ boson and a $b$ quark [6] via the charged-current weak interaction whose $V$-$A$ structure in the standard model (SM) specifies the $tWb$ coupling and the resulting W-boson polarization. Because of its large mass, the top-quark decays before hadronizing and thus offers a direct probe of the $tWb$ coupling. At first order in the SM perturbative expansion [7], the $W^+$ boson is expected to have longitudinal polarization $f_0 = 0.696$, left-handed polarization $f_- = 0.303$, and right-handed polarization $f_+ = 3.8 \times 10^{-4}$ for a top-quark mass $m_t = 172.5 \text{ GeV}/c^2$ [8], a $b$-quark mass $m_b = 4.79 \text{ GeV}/c^2$ [6], and a W-boson mass $M_W = 80.413 \text{ GeV}/c^2$ [9]. Higher-order quantum chromodynamic and electroweak radiative corrections, as well as the uncertainties on the values of $m_t$, $m_b$, and $M_W$, change these predictions at the 1%–2% relative level [6,10]. The presence of anomalous couplings at the $tWb$ vertex, due to contributions from beyond-the-standard-model physics, can modify the observed $W$ polarization with respect to the SM expectations [7].

In this article, three different measurements of the $W$-boson polarization are performed: a model-independent determination that simultaneously measures $f_0$ and $f_+$, a measurement of $f_0$ for fixed $f_+ = 0$, which enhances sensitivity to anomalous tensor couplings, and a measurement of $f_+$ for fixed $f_0 = 0.70$, which enhances sensitivity to anomalous right-handed couplings. The analysis assumes a top-quark mass of $m_t = 172.5 \text{ GeV}/c^2$, consistent with the world average value [8]. We use a data sample enriched in $\tilde{t}\tilde{t}\rightarrow W^+bW^−\tilde{b}\rightarrow \ell vbgq\tilde{q}\tilde{b}$ events, where one of the W bosons decays into quark pairs and the other into lepton pairs. The data was acquired by the Collider Detector at Fermilab (CDF II) [11], which recorded $p\bar{p}$ collisions from Fermilab’s Tevatron operating at $\sqrt{s} = 1.96 \text{ TeV}$. Most of the events used in the analysis were collected using inclusive-lepton online event selections (triggers) that required a high-transverse-momentum ($p_T$) electron or muon in the central (pseudorapidity $|\eta| < 1.1$) detector region [12]. The acceptance for $\tilde{t}\tilde{t}$ events is increased by also using muon events satisfying a trigger that requires large missing transverse energy $\slashed{E}_T$ [12] with either an energetic electromagnetic cluster or two separated jets [13] ($\slashed{E}_T + \text{jets}$ trigger). After all data quality requirements, the sample collected corresponds to an integrated luminosity of 8.7 fb$^{-1}$.

Candidate events are required to have a single isolated electron or muon candidate with $E_T > 20 \text{ GeV}$; missing transverse energy $\slashed{E}_T > 20 \text{ GeV}$ consistent with expectations from the undetectable high energy neutrino; and at least four energetic jets with $E_T > 20 \text{ GeV}$ and $|\eta| < 2$. Jets are reconstructed using a cone algorithm [14] with radius $\Delta R = 0.4$ in $\eta-\phi$ space, and their energies are corrected to the particle level by accounting for detector-response nonuniformities as a function of jet $\eta$, for effects from multiple $p\bar{p}$ interactions, and for the hadronic jet energy scale of the calorimeter [15]. At least one jet must be identified as having originated from a $b$ quark ($b$ tag) using an algorithm that exploits the long lifetime of $b$ hadrons and their large boost from the decay of the top quark. We require decay vertices displaced from the primary $p\bar{p}$-interaction vertex [11].

The backgrounds to the $\tilde{t}\tilde{t}$ signal are from multijet production (QCD), direct $W$-boson production in association with jets ($W + \text{jets}$), and electroweak backgrounds (EWK) composed of diboson ($WW$, $WZ$, $ZZ$) and single top-quark production. The $W + \text{jets}$ background includes events with $b$-tagged $b$-quark jets as well as erroneously $b$-tagged light-flavor or charm-quark jets. The method for estimating the background is described in detail in Ref. [16]. Table I shows the expected sample composition using a $\tilde{t}\tilde{t}$ cross section of 7.4 pb from Ref. [17]. Events that

<table>
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<th>Process</th>
<th>Central $e$</th>
<th>Central $\mu$</th>
<th>$\slashed{E}_T + \text{jets}$ $\mu$</th>
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<tr>
<td>$\tilde{t}\tilde{t}$</td>
<td>923 ± 93</td>
<td>696 ± 54</td>
<td>441 ± 44</td>
</tr>
<tr>
<td>$W + \text{jets}$</td>
<td>160 ± 41</td>
<td>125 ± 19</td>
<td>106 ± 21</td>
</tr>
<tr>
<td>EWK</td>
<td>36 ± 17</td>
<td>27 ± 11</td>
<td>16 ± 8</td>
</tr>
<tr>
<td>QCD</td>
<td>121 ± 48</td>
<td>6 ± 2</td>
<td>7 ± 3</td>
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<tr>
<td>Total expected</td>
<td>1239 ± 113</td>
<td>853 ± 59</td>
<td>569 ± 50</td>
</tr>
<tr>
<td>Observed</td>
<td>1226</td>
<td>804</td>
<td>544</td>
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satisfy one of the high-$p_T$ lepton triggers and the $E_T + \text{jets}$ trigger are assigned to the lepton-triggered sample and removed from the $E_T + \text{jets}$-triggered sample.

The $t\bar{t}$ signal events are modeled using the HERWIG [18] Monte Carlo (MC) generator. The QCD background is modeled using data control samples. The ALPGEN [19], MADEVENT [20] and MC@NLO [21] programs, with PYTHIA [22] or HERWIG supplying the parton-shower and fragmentation model, and the full PYTHIA [22] generator, are used to model the remaining backgrounds and for estimating systematic uncertainties. A GEANT-based simulation [23] is used to model the response of the CDF II detector for these simulated samples. The signal and background modeling has been extensively checked. The observed data and the predicted signal-plus-background distributions for various kinematic variables are compared in Fig. 1. We further validate the background model using a high-statistics background-dominated data control sample obtained vetoing events with a $b$-tagged jet.

To determine the polarization fractions $f_0$, $f_-$, and $f_+$, an unbinned likelihood technique is employed. The likelihood is calculated using the theoretical matrix elements for both the dominant signal process, $q\bar{q} \rightarrow t\bar{t}$, and the background process, inclusive production of $W + \text{jets}$. The method assumes that $p\bar{p} \rightarrow t\bar{t}$ production is accurately described by the SM and includes the physical constraint $\sum_i (p_{i}^\gamma) f_i = 1$. The technique was first developed for measuring the mass of the top quark and for determining $f_0$ when constraining $f_+$ to its SM value [24]. We have extended the technique to enable the simultaneous determination of $f_0$ and $f_+$ [4]. The $t\bar{t}$ matrix element is expressed in terms of the $W$-boson polarization fractions and the cosine of the angle $\theta^*$ between the momentum of the charged lepton or down-type quark from the $W$-boson decay in the $W$-boson rest frame and the direction of the top quark. For the signal $q\bar{q} \rightarrow t\bar{t}$ process [25], the leading-order matrix element is used,

$$|M|^2 = \frac{g_s^4}{9} F_\ell F_h (2 - \beta^2 \sin^2 \theta_{q\ell}),$$

where $g_s$ is the strong coupling constant, $\theta_{q\ell}$ describes the angle between the incoming parton and the top quark in the rest frame of the incoming partons, and $\beta = v/c$ where $v$ is the velocity of the top quark in the same rest frame. The factors $F_\ell$ and $F_h$ correspond to top quarks with a leptonic and a hadronic $W$-boson decay, respectively, such that

$$F_\ell = \frac{2 \pi g_s^4 m_\ell v}{3 m_t \Gamma} (2E^2_b + 3E^2_B m_\ell v + m_b^2) \left[ \left( 1 + \cos \theta^* \right)^2 f_+ + \frac{3}{4} \left( 1 - \cos^2 \theta^* \right) f_0 + \frac{3}{8} \left( 1 - \cos \theta^* \right)^2 (1 - f_0 - f_+) \right].$$

FIG. 1 (color online). The data (points) are compared to the MC prediction for different $W$ polarization fractions. The background predictions are shown as the shaded histograms while the signal-plus-background predictions are shown as the open histograms corresponding to $(f_0, f_+)$ values of (0.7, 0.0) and (0.7, 0.3) for the solid and dashed lines, respectively. The four kinematic variables displayed are the leading jet $p_T$, the lepton $p_T$, the invariant mass of the pair of light-quark jets from the hadronically decaying $W$ boson, and the $\cos \theta^*$ of the leptonically decaying $W$ boson. For the latter two distributions the jet-parton assignment most consistent with the signal hypothesis is shown.
where \( g_W \) is the weak coupling constant, \( m_{\tilde{t}} \) is the charged lepton-neutrino invariant mass, \( \Gamma_t \) is the decay width of the top quark, and \( E_b^* = \frac{m^2_{\tilde{t}} - m^2_{\tilde{b}} - m^2_{\tilde{t}}}{2m_{\tilde{t}}} \). The hadronic factor \( F_h \) is similar. Since we cannot distinguish between up-type and down-type quark jets from the hadronic \( W \)-boson decay, both jet-quark assignments are used and an average \( F_h \) is calculated from the two permutations. The background matrix element is approximated using the sum of \( W + \) jets matrix elements from the VECTBOS [26] program.

The polarization fractions are determined by maximizing the likelihood function \( L \) with respect to \( f_0, f_+ \), and the fraction of events consistent with the \( t\bar{t} \) signal hypothesis, \( C_s \),

\[
L(f_0, f_+, C_s; x) = \prod_{i=1}^{N} \left[ C_s \frac{P_s(x_i; f_0, f_+)}{\langle A_s(x_i; f_0, f_+) \rangle} + (1 - C_s) \frac{P_b(x_i)}{\langle A_b(x_i) \rangle} \right]
\]

where \( N \) is the number of observed events, \( x \) is the vector of observed momenta of the final state partons, and \( \langle A_s \rangle \) and \( \langle A_b \rangle \) are the average acceptances for \( t\bar{t} \) and \( W + \) jets background events, respectively. The dependence of \( \langle A_s \rangle \) on the polarization fractions is properly included. The signal probability density, \( P_s \), and background probability density, \( P_b \), are calculated as described in Ref. [27] and integrated over the relevant differential cross section, which depends on the matrix elements described above, convolved with the proton parton distribution functions (PDFs). Poorly known parton-level quantities are integrated out. The parton four-momenta are estimated from the measured momenta of the trigger lepton and the four highest-transverse-energy \( E_T \) jets in the event. Detector resolution effects are accounted for with transfer functions derived from simulated \( t\bar{t} \) samples. There is an ambiguity in the jet-parton assignments and all permutations are used for each event. When calculating \( P_s \) we fix the top-quark mass to \( m_t = 172.5 \text{ GeV}/c^2 \) and scan the \( (f_0, f_+) \) parameter space. The calculation of \( P_s \) is independent of \( m_t, f_0 \), and \( f_+ \).

The polarization fractions determined from the likelihood fit differ from the true polarization fractions because the signal and background probability densities contain approximations. For example, they do not accurately account for the effects of extra jets arising from initial and final state radiation (ISR/FSR) or for the full set of contributing background processes. Thus, the results of the likelihood fit are calibrated with samples of \( t\bar{t} \) and background events simulated using the sample composition of Table I and assuming a variety of input \( (f_0, f_+) \) values. The mean measured polarization fractions determined from the simulated samples are plotted against the true polarization fractions and a calibration function is determined from a linear fit to the resulting curve. For the one-dimensional polarization measurements a one-dimensional calibration function is employed, while for the simultaneous determination of \( (f_0, f_+) \) a two-dimensional calibration function is used. The resulting calibration functions are used to estimate the true polarization fractions from the measured polarization fractions determined from the three separate likelihood fits. The uncertainties on the coefficients of the calibration functions are included in the method-related systematic uncertainties. Even though the likelihood can be calculated only for the physical values of \( f_0 \) and \( f_+ \), after calibration the corrected measured values can be slightly outside their physical ranges.

The robustness of the fitting and calibration procedure is tested over all physical values of \( (f_0, f_+) \) using simulated experiments constructed to have the number of observed data events and the sample composition of Table I. No significant biases are observed. However, near the physical boundaries the statistical uncertainty is underestimated by as much as a factor of 2. A correction to the statistical uncertainty is applied in these regions. Assuming the SM, the expected statistical uncertainties for the simultaneous measurement, after all corrections, are ±0.075 and ±0.047 for \( f_0 \) and \( f_+ \), respectively.

The sources of systematic uncertainty affecting the measurements are summarized in Table II. All systematic uncertainties are determined by performing simulated experiments in which the systematic parameter in question is varied, the default method and calibrations are applied, and the shifts in the mean measured polarization fractions are used to quantify the uncertainty. All shifts are evaluated using the SM polarization fractions. The dominant source of systematic uncertainty for the simultaneous measurement of \( f_0 \) and \( f_+ \) is due to uncertainties on the background shape and normalization. For the model-dependent measurements several sources of systematic uncertainty contribute at a comparable level.

The uncertainty on the background model is determined by simulating experiments with the mean number of total background events increased and decreased by 1 standard deviation while keeping the relative contributions of the various background sources fixed as given in Table I. Half the mean difference between these two sets of simulated experiments is assigned as the systematic uncertainty. The background model uncertainty also adds in quadrature the

| TABLE II. Summary of systematic uncertainties. |
| --- | --- | --- | --- |
| Source | \( \Delta f_0 \) | \( \Delta f_+ \) | \( \Delta f_0 \) Simultaneous |
| Background model | 0.007 | 0.011 | 0.049 | 0.036 |
| ISR/FSR | 0.011 | 0.017 | 0.022 | 0.023 |
| MC generator | 0.012 | 0.009 | 0.023 | 0.011 |
| Color reconnection | 0.013 | 0.010 | 0.020 | 0.016 |
| Method-related | 0.014 | 0.020 | 0.018 | 0.016 |
| Jet energies | 0.016 | 0.017 | 0.010 | 0.022 |
| PDF | 0.024 | 0.013 | 0.009 | 0.016 |
| Multiple interactions | 0.009 | 0.013 | 0.008 | 0.014 |
| Total | 0.040 | 0.040 | 0.067 | 0.058 |
largest observed change when varying the normalization for each background source in turn by 1 standard deviation, while keeping the total background fixed thereby affecting the shape of the background distributions. The ISRF/FSR uncertainty is evaluated using MC samples generated with ISRF/FSR settings that are amplified or damped relative to the default settings. The MC generator uncertainty is evaluated by comparing between $t\bar{t}$ MC generated by PYTHIA and MC@NLO with parton showering done by HERWIG; it includes uncertainties from not using the next-to-leading-order matrix element in the generator, choice of parton-shower model and modeling of the $t\bar{t}$ spin correlation. In Ref. [4] we only listed uncertainty from the choice of parton-shower model. The color reconnection systematic uncertainty [28] is evaluated using MC samples generated with and without color reconnection effects adopting different configurations [29] of PYTHIA. The method-related uncertainty includes propagating the uncertainty on the fit parameters of the calibration functions, including their correlations. It also includes the uncertainties related to the statistics of the MC samples used to perform the calibration. The uncertainties in the jet energy scale corrections are propagated through the analysis by varying the corrections within 1 standard deviation and recording the resulting shifts in the polarization fractions. Variations associated with the choice of PDF and their uncertainties affect the $t\bar{t}$ acceptance and are included as a systematic uncertainty. The luminosity profile of the MC samples does not exactly match that of the data. The associated systematic uncertainty is evaluated by varying the MC distribution of events containing multiple $p\bar{p}$ interactions so that it matches the data distribution.

Using the 2574 data events that meet all selection criteria we perform three measurements of the $W$-boson polarization fractions. For the model-independent measurement we simultaneously determine $f_0$ and $f_+$ to be

$$f_0 = 0.726 \pm 0.066\text{(stat)} \pm 0.067\text{(syst)},$$

$$f_+ = -0.045 \pm 0.044\text{(stat)} \pm 0.058\text{(syst)},$$

after all corrections. The correlation between $f_0$ and $f_+$ is $-0.69$.

The two-dimensional likelihood contour obtained from the data only includes the statistical uncertainty. The final contour, including statistical and systematic uncertainties, is obtained by analytic convolution of the data likelihood with a two-dimensional Gaussian representing the systematic uncertainties, resulting in a new likelihood $L_{\text{syst}}$. Figure 2 shows the point estimate with error bars, corresponding to one-dimensional 68.27% confidence level uncertainties, and the two-dimensional 68.27% confidence level region, obtained using $-\ln(L_{\text{syst}}/L_{\text{MAX}}) = 0.5$ and 1.15, respectively, where $L_{\text{MAX}}$ is the maximum value for the likelihood $L_{\text{syst}}$. We estimate a shift of $+ (0.010 \pm 0.004)$ in $f_0$ and $+ (0.012 \pm 0.002)$ in $f_+$ per $\pm 1$ GeV/c$^2$ shift in the top-quark mass from the central value of 172.5 GeV/c$^2$.

For the measurement fixing $f_+ = 0$, we obtain after all corrections $f_0 = 0.683 \pm 0.042\text{(stat)} \pm 0.040\text{(syst)}$. For the measurement fixing $f_0 = 0.70$, we measure after all corrections $f_+ = -0.025 \pm 0.024\text{(stat)} \pm 0.040\text{(syst)}$. We estimate a shift of $+ (0.007 \pm 0.002)$ in $f_0$ and $+ (0.008 \pm 0.001)$ in $f_+$ per $\pm 1$ GeV/c$^2$ shift in the top-quark mass from the central value of 172.5 GeV/c$^2$.

In summary, we present measurements of the polarization of $W$ bosons in top-quark decays using the lepton-plus-jets final state and the full CDF Run II data sample corresponding to an integrated luminosity of 8.7 fb$^{-1}$. A matrix-element technique is used to significantly improve the statistical precision relative to previously used techniques and is extended to allow for a simultaneous determination of $f_0$ and $f_+$ in a model-independent manner. This result improves the statistical precision on both the model-independent and model-dependent determinations of $f_0$ and $f_+$ by a factor of 1.6 compared to the previous CDF measurement [4] in the lepton plus jets channel. Our result is the first $W$-polarization measurement using the full data set from Tevatron Run II, and is the most precise single-channel measurement to date from the Tevatron. The results from the model-independent and model-dependent measurements of $W$ polarization are limited by the size of the systematic uncertainties, and have a precision comparable to the combination reported in Ref. [3]. All the results are consistent with the SM.
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[5] Charge-conjugate modes are included implicitly throughout this paper.
[12] We use a cylindrical coordinate system where the $z$ axis is along the proton beam direction, $\phi$ is the azimuthal angle, and $\theta$ is the polar angle. Pseudorapidity is $\eta = -\ln \tan (\theta/2)$, while transverse momentum is $p_T = |p| \sin \theta$, and transverse energy is $E_T = E \sin \theta$. Missing transverse energy, $\not{E}_T$, is defined as the magnitude of $-\sum E_i \hat{n}_i$, where $\hat{n}_i$ is the unit vector in the azimuthal plane that points from the primary $p \bar{p}$ collision vertex to the $i$th energy deposition in the CDF II calorimeter.