Sensitivity analysis of fracture scattering

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1190/geo2011-0521.1">http://dx.doi.org/10.1190/geo2011-0521.1</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>Society of Exploration Geophysicists</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Mon Apr 01 10:02:01 EDT 2019</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/79723">http://hdl.handle.net/1721.1/79723</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>Article is made available in accordance with the publisher’s policy and may be subject to US copyright law. Please refer to the publisher’s site for terms of use.</td>
</tr>
<tr>
<td>Detailed Terms</td>
<td></td>
</tr>
</tbody>
</table>
Sensitivity analysis of fracture scattering

Xinding Fang\(^1\), Michael Fehler\(^1\), Tianrun Chen\(^2\), Daniel Burns\(^1\), and Zhenya Zhu\(^1\)

**ABSTRACT**

We use 2D and 3D finite-difference modeling to numerically calculate the seismic response of a single finite fracture with a linear-slip boundary in a homogeneous elastic medium. We use a point explosive source and ignore the free surface effect, so the fracture scattered wavefield contains two parts: P-to-P scattering and P-to-S scattering. The elastic response of the fracture is described by the fracture compliance. We vary the incident angle and fracture compliance within a range considered appropriate for field observations and investigate the P-to-P and P-to-S scattering patterns of a single fracture. P-to-P and P-to-S fracture scattering patterns are sensitive to the ratio of normal to tangential fracture compliance and incident angle, whereas the scattering amplitude is proportional to the compliance, which agrees with the Born scattering analysis. We find that, for a vertical fracture, if the source is located at the surface, most of the energy scattered by the fracture propagates downwards. We also study the effect of fracture height on the scattering pattern and scattering amplitude.

**INTRODUCTION**

The understanding of seismic scattering from fractures is very important in reservoir fracture characterization. Analytical solutions for scattering from realistic fractures are not available. Scattering from a system of fractures involves not only scattering from individual fractures, but also interaction of the scattered wavefields with other fractures in the system. Examination of the wavefield scattered by fractures has been carried out by Willis et al. (2006) and Grandi-Karam (2008) who showed how to process the complex scattered wavefield to determine some characteristics of a fractured layer. They were only interested in the spatial-temporal pattern of the wavefield and did not study how scattering is influenced by the properties of individual fractures. Scattering of seismic waves from an object is influenced by the ratio of the characteristic scale of the object to the incident wavelength, the geometry of the object, and the mechanical properties of the object. We seek to better understand the scattering from a single fracture, which will provide important insights for interpreting the observed scattered waves. The first step is to understand scattering from individual fractures. Thus, in this paper, we present a numerical study of the seismic response of a finite fracture.

A fracture usually refers to a localized zone containing lots of microcracks or inclusions, each with size that is orders of magnitude smaller than the seismic wavelength. The boundary element method (Benites et al., 1997; Yomogida et al., 1997; Liu and Zhang, 2001; Yomogida and Benites, 2002), which solves for the accurate solution by satisfying the boundary conditions, and the Born approximation method (Hudson and Heritage, 1981; Wu, 1982, 1989), which considers only the single scattered field and ignores the multiply scattered field, have been used for the study of the seismic response of cracks. The seismic response of a fracture can be approximated as the superposition of the responses of individual cracks on the fracture plane. However, it is not practical to represent a fracture as a system of microcracks in a reservoir scale simulation due to the tremendous computational cost. In numerical simulation, a fracture is usually represented as an imperfectly bounded interface (Schoenberg, 1980) between two elastic media.

The way fractures affect seismic waves depends on fracture mechanical parameters, such as compliance and saturating fluid, and on their geometric properties, such as dimensions and spacing. When fractures are small relative to the seismic wavelength, waves are weakly affected by individual fractures. Effective medium theory can be used to model the fractured layer as a zone comprised of many small fractures, which is equivalent to a homogeneous anisotropic zone without fractures (Hudson, 1991; Coates and Schoenberg, 1995; Schoenberg and Sayers, 1995; Grechka and Kachanov, 2006; Grechka, 2007; Sayers, 2009). When fractures are much larger than the seismic wavelength, then we can take fracture...
interfaces as infinite planes and apply plane wave theory to calculate their reflection and transmission coefficients and interface wave characteristics (Schoenberg, 1980; Fehler, 1982; Pyrak-Nolle and Cook, 1987; Gu et al., 1996). In field reservoirs, fractures having characteristic lengths on the order of the seismic wavelength can be the scattering sources that generate seismic codas. Sánchez-Sesma and Iturrarán-Viveros (2001) derived an approximate analytical solution for scattering and diffraction of SH waves by a finite fracture, and Chen et al. (2010) derived an analytical solution for scattering from a 2D elliptical crack in an isotropic acoustic medium. However, there are no analytical elastic solutions for scattering from a finite fracture with a linear-slip boundary and characteristic length on the order of the seismic wavelength. Although fractures are usually present as fracture networks in reservoirs, the interaction between fracture networks and seismic waves is very complicated, usually present as fracture networks in reservoirs, the interaction between fracture networks and seismic waves is very complicated, scattering from a single fracture can be considered as the first-order effect on the scattered wavefield. Therefore, study of the general elastic response of a single finite fracture is important for reservoir fracture characterization, and we will do it numerically in this study.

Here, we adopt Schoenberg (1980)’s linear-slip fracture model and use the effective medium method (Coates and Schoenberg, 1995) for finite-difference modeling of fractures. In this model, a fracture is considered as an interface across which the traction is discontinuous, and they are equidistant (500 m) from the fracture center, stars indicate sources at different angles of incidence to the fracture. The distance between receivers and fracture center is 2.5 times of the fracture height which is 200 m. Incident angles are measured from the normal of the fracture (e.g., a source directly above the fracture is considered to have a 90° incident angle).

**METHODOLOGY**

We assume the complete wavefield recorded at receivers in the fracture model shown in Figure 1a is \( \bar{u}(r, t, \theta_{\text{inc}}) \), and the corresponding wavefield recorded in the reference model, Figure 1b, without a fracture, is \( \bar{u}_0(r, t, \theta_{\text{inc}}) \), then

\[
\bar{u}(r, t, \theta_{\text{inc}}) = \bar{u}_0(r, t, \theta_{\text{inc}}) + \bar{s}(r, t, \theta_{\text{inc}})
\]

(1)

where \( \bar{s}(r, t, \theta_{\text{inc}}) \) is the scattered wavefield, \( \theta_{\text{inc}} \) is the incident angle.

In the frequency domain, equation 1 can be written as

\[
\bar{U}(r, \omega, \theta_{\text{inc}}) = \bar{U}_0(r, \omega, \theta_{\text{inc}}) + \bar{S}(r, \omega, \theta_{\text{inc}})
\]

(2)

where \( \bar{U}, \bar{U}_0, \) and \( \bar{S} \) are the Fourier transformations of \( \bar{u}, \bar{u}_0, \) and \( \bar{s} \), respectively, and \( \omega \) is angular frequency.

Thus, the scattered wavefield can be expressed as

\[
\bar{S}(r, \omega, \theta_{\text{inc}}) = \bar{U}(r, \omega, \theta_{\text{inc}}) - \bar{U}_0(r, \omega, \theta_{\text{inc}}).
\]

(3)

We assume the source is a pressure point source and we ignore the earth’s free surface, so the scattered wavefield \( \bar{S}(r, \omega, \theta_{\text{inc}}) \) includes two parts: P-to-P scattered wavefield \( \bar{S}_{pp}(r, \omega, \theta_{\text{inc}}) \) and P-to-S scattered wavefield \( \bar{S}_{ps}(r, \omega, \theta_{\text{inc}}) \).

In a homogeneous isotropic medium, the total displacement field in the frequency domain can be expressed as

\[
\bar{U}(r, \omega) = \left( \frac{V_p}{\omega} \right)^2 \nabla [\nabla \bar{U} \times (\bar{r}, \omega)] + \left( \frac{V_S}{\omega} \right)^2 \nabla \times [\nabla \bar{U}(\bar{r}, \omega)]
\]

(4)

where \( V_p \) and \( V_S \) are P- and S-wave velocities.

In a homogeneous isotropic medium, we can separate the P- and S-wave energy by calculating the divergence and curl of the whole displacement field, thus, equation 4 can be written as

\[
\bar{U}(r, \omega, \theta_{\text{inc}}) = \bar{U}_P(r, \omega, \theta_{\text{inc}}) + \bar{U}_S(r, \omega, \theta_{\text{inc}})
\]

(5)

with

\[
\bar{U}_P(r, \omega, \theta_{\text{inc}}) = -\left( \frac{V_p}{\omega} \right)^2 \nabla [\nabla \bar{U}(\bar{r}, \omega)]
\]

(6a)

as the P-wave displacement, and

\[
\bar{U}_S(r, \omega, \theta_{\text{inc}}) = \left( \frac{V_S}{\omega} \right)^2 \nabla \times [\nabla \bar{U}(\bar{r}, \omega)]
\]

(6b)

as the S-wave displacement.

Therefore,

\[
\bar{S}(r, \omega, \theta_{\text{inc}}) = \bar{S}_{pp}(r, \omega, \theta_{\text{inc}}) + \bar{S}_{ps}(r, \omega, \theta_{\text{inc}})
\]

(7)

with

\[
\bar{S}_{pp}(r, \omega, \theta_{\text{inc}}) = \bar{U}_P(r, \omega, \theta_{\text{inc}}) - \bar{U}_0(r, \omega, \theta_{\text{inc}})
\]

(8)

\[
\bar{S}_{ps}(r, \omega, \theta_{\text{inc}}) = \bar{U}_S(r, \omega, \theta_{\text{inc}}).
\]

(9)

Note that \( \bar{U}_0(r, \omega, \theta_{\text{inc}}) \) is the reference wavefield, and it has no S-wave component.
Equations 8 and 9 are frequency dependent, and we wish to obtain the far field fracture response function which is independent of the source pulse used in simulation. Thus, we write

$$ |\tilde{S}_{PP}(\theta, \omega, \theta_{inc})| = a F_{PP}(\theta, \omega, \theta_{inc})| \tilde{I}(\omega, \theta_{inc})| $$

(10)

and

$$ |\tilde{S}_{PS}(\theta, \omega, \theta_{inc})| = a F_{PS}(\theta, \omega, \theta_{inc})| \tilde{I}(\omega, \theta_{inc})| $$

(11)

with

$$ a = \begin{cases} 1/\sqrt{r}, & \text{for 2D} \\ 1/r, & \text{for 3D}. \end{cases} $$

(12)

where $F_{PP}(\theta, \omega, \theta_{inc})$ and $F_{PS}(\theta, \omega, \theta_{inc})$ are P-to-P and P-to-S fracture response functions, respectively, $a$ is the geometrical spreading factor which is a function of the distance ($r$) from the receiver to the fracture center, and $I(\omega, \theta_{inc})$ is the incident wavefield recorded at the center of the fracture, $\theta$ and $\theta_{inc}$ are the radiation angle and incident angle, respectively.

We will fix $\theta_{inc}$ when we calculate $F_{PP}$ and $F_{PS}$ by evaluating the wavefield at a fixed $r$ from the fracture for all angle $\theta$. For convenience, hereafter $F_{PP}$ and $F_{PS}$ will only be expressed as functions of $\theta$ and $\omega$, but they depend on the incident angle.

The fracture response function $F_{PP}(\theta, \omega)$ and $F_{PS}(\theta, \omega)$ can be expressed as

$$ F_{PP}(\theta, \omega) = \frac{[\tilde{U}_P(\theta, \omega, \theta_{inc}) - \tilde{U}_I(\theta, \omega, \theta_{inc})]}{a| \tilde{I}(\omega, \theta_{inc})|} $$

(13)

and

$$ F_{PS}(\theta, \omega) = \frac{[\tilde{U}_S(\theta, \omega, \theta_{inc})]}{a| \tilde{I}(\omega, \theta_{inc})|}. $$

(14)

Here, we want to emphasize that fracture response functions 13 and 14 are frequency-dependent, but are source-wavelet independent. We can get the same solution for a given frequency even though we use different source wavelets to numerically calculate 13 and 14. $F_{PP}(\theta, \omega)$ and $F_{PS}(\theta, \omega)$ could be functions of frequency, radiation angle, incident angle, fracture compliance, and wavelength to fracture-length ratio. We visualize the scattering pattern by plotting them in polar coordinates.

### NUMERICAL RESULTS

We first conduct 2D simulations using $V_p = 4$ km/s, $V_s = 2.4$ km/s and $\rho = 2.3$ g/cm$^3$, and we investigate scattering for frequencies ranging from 0 to 50 Hz. The source wavelet is a Ricker wavelet with 40 Hz central frequency in our simulation. In all the 2D simulations, sources and receivers are 550 and 500 m away from the fracture center, respectively, so the incident wavefield magnitude and geometrical spreading factor $a$ are identical for all simulations. Results for 3D simulations are discussed later.

### Fracture scattering pattern

We first study the scattering from a single 200 m long fracture in a homogeneous 2D medium. In this paper, we study fractures with $Z_T$ varying from $10^{-12}$ to $10^{-9}$ m/Pa (Worthington, 2007) and $Z_N/Z_T$ varying from 0.1 to 1 (Lubbe et al., 2008; Gurevich et al., 2009). We first investigate the influence of $Z_N/Z_T$ on the scattering pattern by fixing the tangential compliance $Z_T$ at $10^{-10}$ m/Pa and varying the normal compliance $Z_N$ from $10^{-11}$ to $10^{-10}$ m/Pa. Figure 2 shows the P-to-P fracture response functions for five different $Z_N/Z_T$ at four different incident angles $\theta_{inc}$. For 30° and 60° incidences, P-to-P back scattering changes significantly for different $Z_N/Z_T$. For 0° and 90° incidences, the P-to-P fracture scattering

<table>
<thead>
<tr>
<th>$Z_N/Z_T$</th>
<th>$\theta_{inc} = 0^\circ$</th>
<th>$\theta_{inc} = 30^\circ$</th>
<th>$\theta_{inc} = 60^\circ$</th>
<th>$\theta_{inc} = 90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$F_{PPmax} = 0.283$</td>
<td>$F_{PPmax} = 0.452$</td>
<td>$F_{PPmax} = 0.378$</td>
<td>$F_{PPmax} = 0.0327$</td>
</tr>
<tr>
<td>0.3</td>
<td>$F_{PPmax} = 0.819$</td>
<td>$F_{PPmax} = 0.829$</td>
<td>$F_{PPmax} = 0.523$</td>
<td>$F_{PPmax} = 0.0982$</td>
</tr>
<tr>
<td>0.5</td>
<td>$F_{PPmax} = 1.35$</td>
<td>$F_{PPmax} = 1.21$</td>
<td>$F_{PPmax} = 0.667$</td>
<td>$F_{PPmax} = 0.163$</td>
</tr>
<tr>
<td>0.7</td>
<td>$F_{PPmax} = 1.88$</td>
<td>$F_{PPmax} = 1.59$</td>
<td>$F_{PPmax} = 0.812$</td>
<td>$F_{PPmax} = 0.228$</td>
</tr>
<tr>
<td>1</td>
<td>$F_{PPmax} = 2.67$</td>
<td>$F_{PPmax} = 2.15$</td>
<td>$F_{PPmax} = 1.03$</td>
<td>$F_{PPmax} = 0.325$</td>
</tr>
</tbody>
</table>

Figure 2. The $F_{PP}(\theta, \omega)$ for different $Z_N/Z_T$ at different incident angles. Incident angles, which are shown on top of the figure, are 0°, 30°, 60°, and 90° for each column. The value of $Z_N/Z_T$ for each row is shown at the left side of each row. The $F_{PP}(\theta, \omega)$ is plotted in polar coordinates, the radial and angular coordinates are frequency $(\omega/2\pi)$ and $\theta$, respectively. The range of frequency in each panel is from 0 to 50 Hz. The magnitude of each fracture response function is normalized to one in plotting. $F_{PPmax}$ denotes the P-to-P maximum scattering strength of each panel. Here, $Z_T$ is fixed at $10^{-10}$ m/Pa, $Z_N$ varies.
patterns are similar for different $Z_N/Z_T$. For all angles of incidence, the P-to-P scattering magnitude increases with the increasing of $Z_N/Z_T$. When $Z_N/Z_T$ changes from 0.1 to 1, the P-to-P scattering increases by about an order of magnitude for 0° and 90° incidences. For 30° and 60° incidences, the P-to-P scattering magnitude, respectively, increases by about five times and three times when $Z_N/Z_T$ varies from 0.1 to 1. Figure 3 shows the corresponding P-to-S forward scattering patterns. In contrast to P-to-P scattering, the P-to-S forward scattering patterns change with $Z_N/Z_T$ whereas P-to-S back scattering patterns are similar for different $Z_N/Z_T$ when the incident angle is larger than 0°. For 30° and 60° incidences, P-to-S back scattering is much stronger than P-to-S forward scattering and the P-to-S scattering magnitude increases by about 2.5 and 3 times, respectively, when $Z_N/Z_T$ varies from 0.1 to 1. For P-to-P and P-to-S scattering, the scattering strength increases with increasing compliance magnitude.

Figure 4 shows the comparison of $F_{PP}(\theta, \omega)$ and $F_{PS}(\theta, \omega)$ for different compliance values with $Z_N/Z_T = 1$. By using $Z_T$ to normalize $F_{PP}(\theta, \omega)$ and $F_{PS}(\theta, \omega)$, the scattering patterns for fractures with different compliance values are almost identical, but the normalized amplitude of the case of $10^{-9}$ m/Pa compliance is smaller than that of the others. In our numerical study, we compare $F_{PP}(\theta, \omega)$ and $F_{PS}(\theta, \omega)$ for fractures with $Z_T$ varying from $10^{-12}$ to $10^{-9}$ m/Pa and $Z_N/Z_T$ varying from 0.1 to 1. We find that, for a given incident angle, if we only consider the influence of fracture compliance on the fracture response functions (keeping other conditions, such as background medium, fracture height, etc., unchanged), then the fracture scattering pattern is controlled by the ratio of normal-to-tangential compliance ($Z_N/Z_T$). In other words, $F_{PP}(\theta, \omega)$ and $F_{PS}(\theta, \omega)$ of different fractures have similar scattering patterns if their $Z_N/Z_T$ has the same value, but the scattering strength depends on the magnitude of $Z_N$ and $Z_T$. In Figure 4, the magnitude of the fracture response function is linearly proportional to the fracture compliance when the compliance is less than $10^{-9}$ m/Pa. A fracture of $10^{-9}$ m/Pa compliance represents a strong scatterer which makes its scattering strength depart from the linear trend variation. We will discuss this in a later section.

The belt-shaped pattern shown in Figure 3 at 0° incidence is caused by the interference of the scattered waves from the two fracture tips. Figure 5 is a cartoon showing the ray paths of the scattered waves from the fracture tips to a receiver for the case of 0° incidence. Constructive interference appears at the receiver if the

<table>
<thead>
<tr>
<th>$Z_N/Z_T$</th>
<th>$\theta_{in}=0^\circ$</th>
<th>$\theta_{in}=30^\circ$</th>
<th>$\theta_{in}=60^\circ$</th>
<th>$\theta_{in}=90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$F_{PP(\theta, \omega)}$ = 0.174</td>
<td>$F_{PP(\theta, \omega)}$ = 0.912</td>
<td>$F_{PP(\theta, \omega)}$ = 0.649</td>
<td>$F_{PP(\theta, \omega)}$ = 0.0986</td>
</tr>
<tr>
<td>0.3</td>
<td>$F_{PP(\theta, \omega)}$ = 0.253</td>
<td>$F_{PP(\theta, \omega)}$ = 1.22</td>
<td>$F_{PP(\theta, \omega)}$ = 0.926</td>
<td>$F_{PP(\theta, \omega)}$ = 0.296</td>
</tr>
<tr>
<td>0.5</td>
<td>$F_{PP(\theta, \omega)}$ = 0.372</td>
<td>$F_{PP(\theta, \omega)}$ = 1.53</td>
<td>$F_{PP(\theta, \omega)}$ = 1.21</td>
<td>$F_{PP(\theta, \omega)}$ = 0.493</td>
</tr>
<tr>
<td>0.7</td>
<td>$F_{PP(\theta, \omega)}$ = 0.491</td>
<td>$F_{PP(\theta, \omega)}$ = 1.84</td>
<td>$F_{PP(\theta, \omega)}$ = 1.5</td>
<td>$F_{PP(\theta, \omega)}$ = 0.689</td>
</tr>
<tr>
<td>1</td>
<td>$F_{PP(\theta, \omega)}$ = 0.668</td>
<td>$F_{PP(\theta, \omega)}$ = 2.31</td>
<td>$F_{PP(\theta, \omega)}$ = 1.93</td>
<td>$F_{PP(\theta, \omega)}$ = 0.982</td>
</tr>
</tbody>
</table>

Figure 3. Same as Figure 2, except that $F_{PS}(\theta, \omega)$ is plotted. $F_{PS(\theta, \omega)}$ max is the P-to-S maximum scattering strength of each panel. Figure 4. Comparison of $F_{PP(\theta, \omega)}$ (first row) and $F_{PS(\theta, \omega)}$ (second row) for fractures with different compliance values at 30° incidence. The $Z_N$ is equal to $Z_T$ for these models. The $F_{PP(\theta, \omega)}$ and $F_{PS(\theta, \omega)}$ are normalized by the corresponding $Z_T$ value in plotting. The $F_{PP(\theta, \omega)}$ max and $F_{PS(\theta, \omega)}$ max are, respectively, the P-to-P and P-to-S maximum scattering strength of each panel.

Figure 5. Cartoon showing the scattering from two fracture tips. The $l_1$ and $l_2$ represent the paths from the two fracture tips to the receiver, $l_1$ indicates the distance from receiver to fracture center, $d$ is fracture height, $\theta$ is the radiation angle with respect to the fracture normal.
distance difference $\delta l = |l_1 - l_2|$ is integer multiple of wavelength $\lambda$. The $\delta l$ can be expressed as

$$\delta l = |l_1 - l_2| = \sqrt{l^2 + l \cdot d \cdot \sin \theta + d^2/4} - \sqrt{l^2 - l \cdot d \cdot \sin \theta + d^2/4}. \quad (15)$$

The scattered waves from the fracture tips with wavelength $\lambda = \frac{d}{n}$ ($n = 1, 2, \ldots$) have constructive interference at the receiver. We calculate the constructive interference frequencies in all directions and plot them in polar coordinates in Figure 6. In Figure 6, radial and angular coordinates, respectively, are frequency and azimuth. The blue lines, which represent the constructive interference frequencies, agree with the P-to-S scattering pattern at 0° incidence. The fracture tip P-to-P scattering should have a similar interference pattern, but the fracture tip P-to-P scattering at 0° incidence is relatively weak compared to the entire scattered wavefield. The fracture tip P-to-P scattering becomes weak compared to the entire scattered wavefield. Gibson (1991) modeled Born scattering from fractured media using an approach similar to that described here.

From Figures 3 and 4, we can find that, when the incident angle is between 0° and 90°, P-to-P forward scattering is much stronger than back scattering; however, for P-to-S scattering, back scattering is much stronger than forward scattering. In this paper, forward scattering and back scattering refer to the scattering energy propagating to the right side and left side (source on the left side) of the fracture, respectively. In the field, most fractures are close to vertical (Bredehoeft et al., 1976; Rives et al., 1992) and the seismic source is at the surface. In this case, as illustrated in Figure 7, the observed scattered seismic waves at the surface can be separated into three parts based on the complexity of their wave path: (1) the upward-propagating scattered waves (mainly P-to-P scattering) from fracture tips; (2) the downward-propagating P-to-P scattered waves; (3) the downward-propagating P-to-S converted scattered waves. The downward-propagating P-to-P and P-to-S scattered waves are reflected back to surface by reflectors below the fracture zone.

![Figure 6](image-url)  
*Figure 6. Fracture tip P-to-S scattering interference at 0° incidence. Radial and angular coordinates are frequency and azimuth. Red star indicates the source at 0° incidence. Blue lines show the constructive interference frequencies, of which the corresponding wavelengths satisfy $\lambda = \frac{d}{n}$ ($n = 1, 2, \ldots$), of the P-to-S scattered waves from the two fracture tips at all directions.*

The P-to-P and P-to-S scattering energy, respectively, propagates in the forward and backward directions with respect to the source, as shown in Figure 7. Fracture tip scattering and downward-propagating scattered waves have been observed in the laboratory experiments of Zhu et al. (2011).

Figure 8 shows a numerical simulation of wave propagation in a uniform medium containing 21 nonparallel fractures. Figure 8a shows the geometry of the model, Figure 8b and 8c shows snapshots of the divergence of the scattered displacement wavefield and curl of the scattered field at 0.52 s (the scattered field is obtained by subtracting the whole field from the reference field of the same model without fractures). We can see that most of the P-to-P scattering energy propagates down and forward and most of the P-to-S scattering energy propagates down and backward. In the field, we may have reflectors below the fracture zone to reflect the down-going scattered waves back to the surface. Therefore, in the field, most scattered signals observed at the surface come from fracture tips and scattering energy reflected off events below the fracture zone. If we want to use P-to-P and P-to-S scattered waves to study fractures, we should search for P-to-P and P-to-S scattered waves at different offsets based on their scattering direction.

**Scattering strength**

We define scattering strength for a given frequency as the average of the fracture response function over all radiation angles. Figure 9 shows the scattering strength of P-to-P scattering for different $Z_I$ and $Z_N/Z_T$. We find that P-to-P scattering is usually stronger at small angles of incidence, except for the case of a small $Z_N/Z_T$ (~0.1). Figure 10 shows the corresponding P-to-S scattering strength, where regardless of the variation of $Z_N/Z_T$, P-to-S scattering is always strongest when the incident angle is about 40°. The strength of P-to-P scattered waves at small incident angles is mainly determined by $Z_N$, but the influence of $Z_N$ on P-to-P scattered waves decreases with increasing incident angle, and at intermediate incident angles, $Z_T$ has stronger impact on the scattered wavefield because P-to-S conversion at the fracture surface is more efficient at intermediate incident angles (Gu et al., 1996), so stronger P-to-S scattered waves are generated at intermediate incident angles. When $Z_N \ll Z_T$ (e.g. $Z_N/Z_T \leq 0.1$), the scattered wavefield is mostly
influenced by $Z_T$, $Z_N$ has relatively small impact on the incident wavefield; therefore, the P-to-P scattering is strong only at intermediate incident angles when $Z_N/Z_T = 0.1$.

By comparing P-to-P and P-to-S scattering strength in Figures 9 and 10, we find that P-to-S scattering is stronger than P-to-P scattering when $Z_N/Z_T$ is smaller than 0.5. For P-to-P and P-to-S scattering, the scattering strength increases about one order of magnitude when the compliance increases one order, the scattering strength and compliance have a very good linear relationship when the compliance is smaller than $10^{-9}$ m/Pa, whereas the scattering strength deviates from this linear relationship a little when the compliance is $10^{-9}$ m/Pa. We will discuss this discrepancy in the Born scattering analysis section.

The ratio $Z_N/Z_T$ is strongly influenced by the way the fracture surfaces interact, so this ratio may be of use for fluid identification. Numerical simulations (Gurevich et al., 2009; Sayers et al., 2009) and laboratory measurements (Lubbe et al., 2008; Gurevich et al., 2009) suggest that the compliance ratio $Z_N/Z_T$ of reservoir fractures should be less than one. Based on laboratory experimental data, Lubbe et al. (2008) suggested that a $Z_N/Z_T$ ratio of 0.5 is appropriate for simulation of gas-filled fractures, and $Z_N/Z_T$ can be less than 0.1 for fluid saturated fractures.

Figure 11 shows the comparison of P-to-P and P-to-S scattering at 20 Hz, of which the corresponding P-wave wavelength is the same as the fracture height, for different $Z_N/Z_T$ at 20° and 60° incidences. At 20° incidence, P-to-S scattering is stronger than P-to-P scattering when $Z_N/Z_T$ is small, whereas this reverses with the

![Figure 8](image8.png)

Figure 8. (a) A homogeneous isotropic model with 21 nonparallel fractures; red lines indicate fractures and asterisk is the source. Parameters for the background medium are shown in (a) and $Z_N$ and $Z_T$ are $5 \times 10^{-10}$ and $10^{-9}$ m/Pa, fracture height is 200 m, the source wavelet is a Ricker wavelet with a 40 Hz central frequency; (b) and (c) show snapshots of the divergence and curl of the scattered displacement field at 0.52 s.

![Figure 9](image9.png)

Figure 9. P-to-P scattering strength for different $Z_T$ and different $Z_N/Z_T$. Horizontal and vertical axes are incident angle and frequency. The scattering strength for each panel is normalized to one for plotting. The number above each panel is the scaling factor (maximum scattering strength).

![Figure 10](image10.png)

Figure 10. Same as Figure 9, but for P-to-S scattering strength.
increasing of $Z_N/Z_T$. At 60° incidence, P-to-S scattering is always stronger than P-to-P scattering regardless of the change of $Z_N/Z_T$. From our simulations, we find that, when $Z_N/Z_T \leq 0.5$, generally, P-to-S scattering is stronger than P-to-P scattering when the incident angle is larger than 20°. This implies that we could detect strong P-to-S scattered waves at the surface although S-waves generally attenuate more rapidly than P-waves.

The effect of fracture height on fracture scattering

Figures 12 and 13 show comparisons of the fracture response functions of models with two different fracture heights at different incident angles. Fracture heights are 100 m and 200 m, which correspond to the wavelength of waves at frequencies of 40 and 20 Hz, respectively. In these two models, except for the fracture height, all other model parameters are the same. From these two figures, we can see that the dominant scattering directions of two fractures with different fracture heights are very similar, but the P-to-P and P-to-S scattering patterns for the 100 m long fracture have a broader distribution of high amplitude regions. So the scattering of a fracture will approach that of the scattering from a point scatterer when the length of the fracture decreases, so the scattering pattern of a shorter fracture at a given frequency is comparable to that of a longer fracture at lower frequency. We also find that the scattering strength increases with the increasing fracture height. Such a pattern is evident in Figures 9 and 10, which show that higher frequencies (shorter wavelengths) are more strongly scattered by a fracture. Although we only show the comparison of two models, the same phenomena can be observed from all other models.

Single fracture scattering in three dimensions

We began our study on single fracture scattering using the 2D case because it is easier to calculate and visualize the fracture response function in 2D. We now discuss scattering in 3D. In our 3D simulation, a fracture is represented as a rectangular plane with finite area, all model parameters are the same as those used for the 2D simulation, sources are 550 m away from the fracture center, and receivers cover a spherical surface centered at the fracture center with a 500 m radius. Because it is very time-consuming to compute the 3D fracture response function, we compute a suite of models with different incident angles, then the fracture response function at any incident angle is obtained through interpolation. Because the incident and radiation directions are unit 3D vectors, it is difficult to visualize the fracture response functions. From our numerical study, we find that the P-to-P and P-to-S fracture-scattered waves have characteristics that are generally similar to those shown in Figure 7. A point source and a finite fracture in a 2D model are equivalent to a line source and an infinite long fracture in 3D. To study the influence of the finite fracture length on the

![Figure 12](image-url)

Figure 12. Comparison of $F_{PP}(\theta, \omega)$ of two fracture models with different fracture heights at four different incident angles. The first and second rows, respectively, show the $F_{PP}(\theta, \omega)$ of fractures with 100 m and 200 m lengths at incident angles of 0°, 30°, 60°, and 90°. In each panel, $F_{PP}(\theta, \omega)$ is normalized by its maximum scattering strength, which is shown by the number below each panel. The $Z_N$ and $Z_T$ of these two models are $10^{-10}$ m/Pa, other model parameters are the same as the model shown in Figure 1.

![Figure 13](image-url)

Figure 13. Same as Figure 12, but for $F_{PS}(\theta, \omega)$.
scattered wavefield, in the 3D model, we vary the fracture length in the fracture strike direction (the direction perpendicular to the 2D plane shown in Figure 1) from 50 to 200 m and keep the fracture height (200 m) equal to that of the 2D model. Figure 14 shows the comparison of 2D and 3D fracture response functions for a fracture with $Z_T = 10^{-9}$ m/Pa and $Z_N = 5 \times 10^{-10}$ m/Pa. We only show a profile of the 3D fracture response function, which contains the source and is perpendicular to the fracture plane and has the same geometry as the 2D model. From Figure 14, we can see that, compared to the 2D case, the 3D fracture response functions have a broader distribution of high amplitude regions, the 3D effect is similar to shortening the fracture height in 2D. When the fracture length is changed from 200 to 50 m, the scattering pattern does not change whereas the scattering strength decreases.

To quantitatively study the scattering strength, we average the fracture response function of the 200 $\times$ 200 m fracture over different incident angles and radiation angles and take the mean value as the average scattering strength. Figure 15 shows the average scattering strength as a function of frequency and fracture compliance. For each panel, $Z_N/Z_T$ is fixed; black, blue, and red curves show the scattering strength for different $Z_T$. For a given $Z_T$, the P-to-P and P-to-S scattering strength increase with increasing $Z_N$, and the curves of P-to-P and P-to-S scattering strength merge when the frequency increases. The P-to-S scattering is stronger than the P-to-P scattering for all $Z_N/Z_T$. For P-to-P and P-to-S scattering, the scattering strength is almost linearly dependent on the compliance.

**Born scattering analysis**

We have numerically investigated the characteristics of single-fracture scattering. In this section, we use the Born scattering approximation to derive the wave equation for fracture-scattered waves, which can help us to gain insights into our observations from the numerical simulations.

The average strain of a rock containing fractures can be expressed as (Schoenberg and Sayers, 1995)

$$\epsilon = (S_b + S_f) \cdot \sigma,$$  \hspace{1cm} \text{(16)}

where $\sigma$ and $\epsilon$ are stress and strain, $S_b$ and $S_f$ are the background medium compliance and the fracture induced extra compliance, respectively. Function $S_f$ is a linear function of $Z_N$ and $Z_T$; its expression can be found in the paper of Schoenberg and Sayers (1995).

After simple algebraic manipulations, we can rewrite equation 16 as

$$\sigma \approx (C_b + C_f) \cdot \epsilon,$$ \hspace{1cm} \text{(18)}

where $C_f = -C_b \cdot S_f \cdot C_b$ is the extra stiffness caused by the presence of fractures. Negative $C_f$ means the medium becomes more compliant due to fractures.

Thus, the wave equation for the displacement wavefield $\ddot{u}(x, t)$ is

$$\rho \ddot{u}(x, t) - \partial_j [(c_{ijkl}^b + c_{ijkl}^f) \cdot \varepsilon_{kl}] = 0,$$ \hspace{1cm} \text{(19)}

where $c_{ijkl}^b$ and $c_{ijkl}^f$ are the fourth rank tensor notations of $C_b$ and $C_f$, and $\partial_j$ represents $\partial/\partial_j$.

We can solve equation 19 by using the Born approximation (Sato and Fehler, 2009). The total wavefield $\ddot{u}$ is written as a sum of an incident wave $\ddot{u}^0$ and a scattered wave $\ddot{u}^s$.

---

![Figure 14](image-url)  
**Figure 14.** Comparison of 2D and 3D fracture response functions at 30° incidence. Here, $Z_T$ and $Z_N$ are $10^{-9}$ and $5 \times 10^{-10}$ m/Pa, respectively. Lengths $L = 200, 100$, and 50 m indicate the fracture length in the fracture strike direction for three different models. Note that a point source in 2D is equivalent to a line source in 3D, so the scattering strength of 2D results is larger.

![Figure 15](image-url)  
**Figure 15.** P-to-P and P-to-S scattering strength as a function of frequency. The scattering strength is normalized by the maximum strength of all cases. Solid and dashed curves correspond to P-to-P and P-to-S scattering strength, respectively. The value of $Z_N/Z_T$ is shown on top of each panel.
\[ \ddot{u} = \ddot{u}^0 + \ddot{u}^1. \]  

(20)

The incident wave satisfies the homogeneous wave equation

\[ \rho u_{ij}^0(\vec{x}, t) - \delta_j (\epsilon_{ijkl} \cdot \dot{e}_{kl}^0) = 0, \]  

(21)

where \( \dot{e}_{kl}^0 \) is the strain associated with \( \ddot{u}^0 \).

Substituting equation 20 in equation 19 and using equation 21 and assuming \( |\ddot{u}^0| \ll |\ddot{u}| \), we get the wave equation for the scattered wave

\[ \rho \ddot{u}_{ik}^1(\vec{x}, t) - \delta_j (\epsilon_{ijkl} \cdot \dot{e}_{kl}^1) = \delta f_i(\vec{x}, t), \]  

(22)

where the cross term \( \epsilon_{ijkl} \cdot \dot{e}_{kl}^1 \) is neglected, \( \delta f_i(\vec{x}, t) = \delta_j (\epsilon_{ijkl} \cdot \dot{e}_{kl}^0) \) is the equivalent body force for the scattered wavefield and \( \dot{e}_{kl}^1 \) is the strain associated with \( \ddot{u}^1 \).

Because \( S_f \) is a linear function of \( Z_N \) and \( Z_T \), and if \( Z_N/Z_T \) is fixed, then \( \delta f_j(\vec{x}, t) \) has the same form for different compliance values, except for being scaled by a constant factor, which explains why the scattering pattern is independent of the compliance value when \( Z_N/Z_T \) is fixed.

From equation 22, we can infer that

\[ \ddot{u}^1 \propto \delta f \propto C_f = -C_b \cdot S_f \cdot C_b \propto S_f, \]  

(23)

Therefore, the amplitude of scattered waves is proportional to the fracture compliance.

However, the Born scattering approximation is based on a weak scattering assumption. If the extra fracture stiffness is comparable to or larger than the background stiffness, a fracture is a strong scatterer. Born scattering approximation does not hold for this case, and extra nonlinear terms have to be considered in the scattering wave equation, which breaks the linear relationship between scattered waves and fracture compliance in equation 22. This explains the discrepancies, which are observed in the numerical study, between the relationship of the fracture response functions and compliance values of very compliant fractures (>10^{-10} m/Pa) and less compliant fractures (<10^{-10} m/Pa). For a fracture system with large compliance, waves are trapped among fractures, so in a sense, multiple scattering becomes important.

CONCLUSIONS

We studied scattering from a single fracture using numerical modeling and found that the fracture scattering pattern is controlled by \( Z_N/Z_T \) and the fracture scattering strength varies linearly with fracture compliance for fractures with compliance \( \leq 10^{-10} \) m/Pa. This is well explained by the Born approximation. For compliance greater than about \( 10^{-10} \) m/Pa, scattering strength does not scale with compliance. For small value of \( Z_N/Z_T \), P-to-S scattering dominates P-to-P scattering when the incident angle is larger than 20\(^\circ\); for large value of \( Z_N/Z_T \), P-to-P scattering dominates. Due to the gravity of overburden and the regional stress field, fractures tend to be vertical in a fractured reservoir. Because seismic sources are normally located at the surface, waves scattered from vertical fractures propagate downward, specifically, the P-to-P scattering energy propagates down and forward whereas the P-to-S scattering energy propagates down and backward. Therefore, for a vertical fracture system, most of the fracture-scattered waves observed at the surface are first scattered by fractures and then reflected back to the surface by reflectors located below the fracture zone, so the fracture scattered waves have complex wave paths and are influenced by the reflectivity of the reflectors.

ACKNOWLEDGMENTS

This work was funded by the Eni Multiscale Reservoir Science Project within the Eni-MIT Energy Initiative Founding Member Program. Tianrun Chen was supported by an ERL Founding Member postdoctoral fellowship. We thank Ru-Shan Wu, Mark Willis, Richard Gibson, and an anonymous reviewer for comments that helped us improve the manuscript.

REFERENCES


