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Relation between ordering and shear thinning in colloidal suspensions

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Colloidal suspensions exhibit shear thinning and shear thickening. The most common interpretation of these phenomena identifies layering of the fluid perpendicular to the shear gradient as the driver for the observed behavior. However, studies of the particle configurations associated with shear thinning and thickening cast doubt on that conclusion and leave unsettled whether these nonequilibrium phenomena are caused primarily by correlated particle motions or by changes in particle packing structure. We report the results of Stokesian dynamics simulations of suspensions of hard spheres that illuminate the relation among the suspension viscosity, shear rate, and particle configuration. Using a recently introduced sampling technique for nonequilibrium systems, we show that shear thinning can be decoupled from layering, thereby eliminating layering as the driver for shear thinning. In contrast, we find that there is a strong correlation between shear thinning and a two-particle measure of the shear stress. Our results are consistent with a recent experimental study.

Flowing colloidal suspensions exhibit many nonlinear response phenomena, prominent among which are shear thinning and thickening, i.e., the decrease and increase, respectively, of the suspension viscosity with increasing shear strength (1–5). In the last four decades, many experimental studies and computer simulations have attempted to identify changes in particle configurations responsible for these phenomena (6–16), but no conclusive signature has yet been identified. In early experimental studies of dense colloidal suspensions of hard spheres (8, 17) (solid or glass at equilibrium) and of dilute charge-stabilized suspensions that crystallize at equilibrium (18, 19), sliding layers were observed when the systems were subjected to shear flow. This layering structure, characterized by a nonuniform density distribution along the shear gradient direction, was independently predicted to occur by nonequilibrium molecular dynamics (NEMD) simulations without hydrodynamic interactions (20, 21). The organization of the suspension into layers was thought to explain the shear-thinning behavior observed, but later experimental studies of less dense systems (with liquid-like order at equilibrium) using light scattering (22, 23) or small-angle neutron scattering (24–27) yielded mixed signals relevant to the existence of this layering structure in the shear-thinning regime, thereby implying that layering might not be necessary for shear thinning. This implication is supported by Stokesian dynamics simulations (28, 29), which differ from earlier NEMD simulations by incorporating hydrodynamic interactions between the particles. The Stokesian dynamics simulations find no shear-induced layering of hard spheres in the shear-thinning regime in bulk systems that are liquid-like at equilibrium (9, 30).

It is certainly the case that diffraction from an assembly of particles is sensitive to ordering and many body correlations associated with complicated structures on many length scales. Determination of the structure of the assembly on those many length scales from diffraction data requires that the Fourier space complementary to real space is adequately sampled. The scattering studies of sheared colloidal systems that have been reported cover a small part of the Fourier space complementary to the real space particle arrangement, and are determined by particle correlations on the pair level only. In an equilibrium state of a fluid of particles with pair interactions that depend only on the pair separation, the pair correlation function determined from scattering measurements adequately describes the structure. In a nonequilibrium state of a fluid of particles, the statistical weight of each configuration depends explicitly on the way the system is driven out of equilibrium and cannot be obtained from the pair correlation function alone. Thus, although the light scattering and small-angle neutron scattering experiments hint that layering does not drive shear thinning, they do not provide unique evidence for that conclusion and the notion that shear thinning is associated with formation of particle layers persists (3, 14–16).

The experimental study of shear thinning has recently been advanced via simultaneous rheological and structural measurements on confined colloid suspensions, with single-particle resolution, using fast confocal microscopy (7). The results of this experiment, unlike the results of previous Stokesian dynamics simulations (9, 30), show simultaneous appearance of the layering structural change and the shear thinning. The difference in results with respect to the existence of layering in the fluid may be a consequence of the difference in boundary conditions between the experiment and the simulations. Specifically, confinement by planar boundaries, as used in the experiments, enhances layer formation in the dense colloidal suspension. However, the use of a periodic boundary condition in the velocity gradient direction (or in some cases Lees–Edwards boundary conditions (31)) used in the experiments does not drive shear thinning, they do not provide unique evidence for that conclusion and the notion that shear thinning is associated with formation of particle layers persists (3, 14–16).

Although the experimental data imply that the simultaneous appearance of layering and shear thinning is coincidental rather than causal, they are not adequate to resolve this issue because the shear strength and layering cannot be independently varied. Advances in simulation algorithms now make resolution of the relationship between shear thinning and layering possible. The Stokesian dynamics simulations (28, 29) we report in this paper exploit the same methodology as used in previous studies (9, 30) except that we use boundary conditions that are consistent with those used in the experiment by Cheng et al. (7). We furthermore use nonequilibrium umbrella sampling (32–34) to observe otherwise rare configurations. Our results show that shear thinning in a suspension of confined hard spheres is accompanied by other structural changes in addition to layering, and in particular a two-particle measure of the shear stress is demonstrated to be strongly correlated with shear thinning.
Results and Discussion

Observations of Shear Thinning. Our Stokesian dynamics simulations use the methodology developed by Brady and coworkers with boundary conditions as detailed in Methods. The shear applied to the $N$ hard spheres is in the $x$ direction, the shear gradient is in the $y$ direction, and the vorticity is in the $z$ direction (Fig. 1A). The sheared hard sphere fluid is characterized by the sphere diameter $2a$, the packing fraction $\phi = N(4\pi a^3)/3V$ and the Peclet number $Pe = 6\pi \eta_i a^2 \gamma/k_B T$, where $\eta_i$ is the viscosity of the supporting liquid and $\gamma$ is the shear rate. The Peclet number measures the magnitude of particle motion induced by shear relative to that associated with thermal motion. Simulations were carried out for packing fractions $\phi = 0.37$ and $\phi = 0.47$, similar to the packing fractions used in the experiments of Cheng et al. (7). These packing fractions lie below that at which the hard sphere fluid crystallizes. The dependence of the steady-state shear viscosity, $\eta_{NESS}$ (NESS stands for nonequilibrium steady state) on $Pe$ over the range $0.1 \leq Pe \leq 10$ is displayed in Fig. 1B and C. These data clearly show that up to $Pe = 10$ these suspensions are shear thinning, and the simulations are in general agreement with recent experimental data (7) for shear thinning and with the results of earlier simulations of bulk suspensions (29). The quantitative differences between our results and earlier results are attributable to the different boundary conditions used.

Study of the Relation Between Shear Thinning and Layer Ordering. Owing to the confinement, there is layering of the colloidal suspension in the equilibrium state, and that layering is enhanced with shear. Our simulations reproduce the enhanced layering structure in the sheared fluid observed in recent experiments (7). This layering structure can be characterized qualitatively by the peaks in the density profile of the system along the shear gradient direction $y$ (Fig. 2A and B). To quantitatively investigate how the shear thinning depends on layering structure, we quantify the strength of the layering by the scalar parameter $Q_L \equiv N^{-1} \sum_{i=1}^{N} \cos(2\pi y_i/L)$, where $y_i$ is the $y$-position of sphere $i$, and $L$ is the separation between two neighboring layers of particles (Fig. 2C). It is important to note that the increase in $Q_L$ is much stronger for $Pe > 1$ than for $Pe < 1$, yet the rate of decrease in suspension viscosity over the range $0.1 \leq Pe \leq 1$ is very close to that over the range $1 \leq Pe \leq 10$ (Fig. 1). This finding strongly implies that enhancement of layering with increasing $Pe$ is not essential to shear thinning.

To strengthen the conclusion implied by the results just described, we now demonstrate that the layer structure of a confined colloidal suspension and the decrease in viscosity with increasing Peclet number are uncorrelated. Using 100,000 steady-state colloid configurations for the suspension with $\phi = 0.47$ and $Pe = 10$, we construct a histogram of values of $Q_L$ (Fig. 3A). The distribution of values of $Q_L$ delineates the amplitude of fluctuations, and it is well represented by a Gaussian function with mean of 0.803 and SD of 0.040. If we can harvest a sufficient number of configurations in the small $Q_L$ tail of this distribution, we can calculate the viscosity associated with those configurations and compare the values found with the values for $\eta_{NESS}$ obtained at smaller $Pe$. Doing so is challenging because these configurations are rare. Consequently, we use nonequilibrium umbrella sampling (32–34) (Methods). This numerical technique preserves the dynamics of

Fig. 1. Characterization of shear thinning. (A) Illustration of the coordinate system. (B) Shear viscosity $\eta_{NESS}$ as a function of $Pe$ for suspensions with $\phi = 0.47$, showing strong shear-thinning behavior. (C) Shear viscosity as a function of $Pe$ for suspensions with $\phi = 0.37$, showing moderate shear-thinning behavior. Each data point shown in both $B$ and $C$ is the average of 100 independent simulations (Methods), and the bars show the SD.

Fig. 2. Characterization of layer ordering. Density profiles of the suspension along the $y$ direction with $\phi = 0.47$ (A) and $\phi = 0.37$ (B). Note the layer enhancement due to increased shear. (C) Scalar parameter $Q_L$ as a function of $Pe$, normalized by its value at $Pe = 0.1$ for suspensions with $\phi = 0.37$ (empty squares) and with $\phi = 0.47$ (black circles).
Mechanism of Shear Thinning: Study of a Two-Particle Structure. What then is the basis for the observed shear thinning? It is conventional to represent the shear stress \( S \) in the complex colloidal fluid as the sum of three contributions (28, 35): a thermodynamic component associated with the stochastic motions of particles, called the Brownian stress \( S^B \), an interaction component associated with the particle–particle forces, called the interaction stress \( S^i \), and a hydrodynamic component associated with the shear flow, called the hydrodynamic stress \( S^H \). The Brownian stress arises from the fluctuations in position of a colloidal particle that induce flows that interact with all other particles in the system, the spatial configuration of which also fluctuates on the same timescale as individual colloidal particle motion. Because of the many-particle dependence of the hydrodynamic resistance on the configuration, and the coupling of the fluctuation timescales, this contribution has a nonzero value when averaged over a time long compared with the Brownian relaxation time \( \tau_B = m / 6 \pi \eta_0 a \), where \( m \) is the mass of a colloidal particle. The interaction stress is zero for a hard-sphere system (9). In general, shear thinning is a consequence of the fact that the change in the thermodynamic component with Peclet number is unable to keep up with the increase of shear stress (9). Indeed, our Stokesian dynamics simulations reproduce the earlier results (9, 35) that both \( S^B \) and \( S^H \) increase with increasing \( Pe \), but the rate of increase of \( S^H \) cannot match the rate of increase of \( S^H \). As a result, the stochastic component of the viscosity, \( \eta^i \), decreases as \( Pe \) increases and eventually becomes negligible at large \( Pe \), while the hydrodynamic component of the viscosity, \( \eta^H \), stays almost constant up to \( Pe = 10 \) (Fig. 4A). Brady (36) has shown that at large packing fraction the Brownian stress is well approximated by the contact integral 
\[
-8kT \sum_{g_2} g_2(r) dS
\]
which is the pair correlation function.

We are concerned with the \( xy \)-component of the Brownian stress and its dependence on \( g_2(r) \), or more specifically 
\[
-8kT \sum_{g_2} \Delta g_2(\mathbf{r}) dS
\]
This integral is 0 for suspensions at equilibrium as a result of system symmetry. As the colloidal suspension is subject to simple shear motion, this symmetry is broken, and as a result \( g_2(r) \), when projected on the \( xy \)-plane, exhibits strong distortions as the colloidal suspension shear thins (7, 9). This observation

Fig. 3. Viscosity–layer relation. (A) Histogram of \( Q_L \) for 100,000 steady-state configurations of the suspension with \( \phi = 0.47 \) and \( Pe = 10 \) (bars), fitted to a Gaussian function (line) with average value \( Q_L = 0.803 \) and SD \( r = 0.040 \). (B) Dependence of \( \eta(Q_L) \) on \( Q_L \) for the suspension with \( \phi = 0.47 \) and \( Pe = 10 \) (empty squares) compared with \( \eta_{NESS} \), the average viscosity for the indicated \( Pe \) (Fig. 1B). Each data point shows the average of results from 100 independent simulations; the bars show the SD.

Fig. 4. Structural signature for shear thinning. (A) \( Pe \) dependence of the Brownian component (black squares) and the hydrodynamic component (empty circles) of the viscosity of the suspension with \( \phi = 0.47 \). Angular probability distributions of contact pairs in the \( xy \)-plane \( P(\psi; r = 2a) \) versus \( \psi \) for suspensions with \( Pe = 10 \) and \( \phi = 0.37 \) (B), and \( Pe = 10 \) and \( \phi = 0.47 \) (C). Inset in B defines the angle \( \psi \) for a contact pair when projected onto the \( xy \)-plane. Each data point shown in A is the average of 100 independent simulations, and the bars show the SD.
is also reproduced in our simulations for steady states under shear (Fig. 4B and C). To quantitatively examine this dependence, we introduce a scalar parameter defined as $Q_P \equiv -N^{-1} \sum_{i=1}^{N} \sum_{j=i+1}^{N} x_i y_j \exp \left[ (r_{ij}/a - 2)/L_{cut} \right]$, where $x_i = x_i - x_j$, $y_i = y_i - y_j$, $r_{ij} = |r_i - r_j|$, and the pair correlation function at contact is approximately obtained by counting pairs at surface distance $(r_{ij} = 2a)$ with a rapidly decaying weight $\exp \left[ (r_{ij}/a - 2)/L_{cut} \right]$, which assures that the dynamics of $Q_P$ is smooth. When projected onto the $xy$-plane, a pair at contact has a strong positive contribution to $Q_P$ when aligned along the $135^\circ$ axis and a strong negative contribution when aligned along the $45^\circ$ axis (Fig. 4B, Inset). We use $L_{cut} = 0.05$ for the calculation shown, and find that the results are insensitive to variations between 0.05 and 0.1.

As in the case of the analysis of $Q_L$, we constructed a histogram of values of $Q_P$ using 100,000 steady-state colloid configurations for the suspension with $\phi = 0.47$ and $Pe = 10$ (Fig. 5A), and using nonequilibrium umbrella sampling we sampled configurations that are associated with different values of $Q_P$. For each value of $Q_P$, the associated colloid configurations were analyzed to obtain the $xy$ component of the Brownian stress $\Sigma_{Q_P}^{\phi}(Q_P) = n^R(Q_P) \rho k_B T / 6 \pi \eta a^3$. A comparison of $\Sigma_{Q_P}^{\phi}(Q_P)$ with the values of $Q_P$ calculated for the steady states of the same suspension subject to different shear rates shows a strong correlation between $Q_P$ and the shear stress (Fig. 5B). We conclude that alternation of the pair structure, rather than the layering, in the suspension is the dominant mechanism underlying shear thinning. This interpretation is consistent with earlier suggestions that “entropic” (i.e., Brownian) contributions to the stress become small compared with hydrodynamic ones as the system is sheared more strongly (7, 9).
Given that the changes in the viscosity and layering are simultaneous, it is natural to ask whether they share the same origin: the alteration of the pair structure as characterized by $Q_\psi$. As we increase $Pe$, the steady-state average of $Q_\psi$ increases as contact particle pairs align along the 135° direction when projected onto the xy-plane. However, as indicated by our nonequilibrium umbrella sampling results, this structural change on the pair level has very little correlation with the layering (Fig. 5C). The mechanism that causes the enhancement of layering is of strong interest (37) but is not resolved by the present study.

Conclusions

The use of nonequilibrium umbrella sampling for studying rare events far from equilibrium allows us to decouple particle configurations from the shear strength. Our results have demonstrated that, for hard-sphere colloidal suspensions, the dominant mechanism underlying shear-thinning behavior is alteration of the pair distribution function, rather than layering, which is a single-particle measure. As also demonstrated from our results, the fact that the structural change on the pair level has no correlation with the layering is consistent with the observation that scattering measurements only yield information about the system on the pair level and make no prediction about the layering as the system is driven far from equilibrium by shear flow. In fact, we see that layering of the system can vary substantially with virtually no changes in the pair structure (Fig. 5B), and the pair structure of the system can vary substantially with almost no change in layering (Fig. 5C).

The particle structure within each layer is another aspect of the suspension that is of great interest. In our simulations, we observed a rich variety of structures within each layer as we vary the packing fraction and $Pe$. If we maintain $Pe = 10$, at $\phi = 0.37$ we observed a weak string configuration of particles (an anisotropic distribution in favor of the vorticity direction, as illustrated in Fig. 6A), in qualitative agreement with our earlier results (38). At higher density $\phi = 0.47$, which is very close to the bulk crystallization density for hard spheres, we observed distorted hexatic ordering (Fig. 6B). The existence of this hexatic ordering is in qualitative agreement with the results of earlier studies of suspensions that are solid at equilibrium. However, it is not clear to us whether the patterns observed in our system arise solely from shear. It is not clear whether the strong confinement of the suspension plays a key role. The distortion of the hexatic ordering observed can be written as a sum of a perfect hexatic part and a string phase part (Fig. 6B). Clearly, determining the detailed relationship between the particle structures within each layer and system parameters such as $Pe$ or packing fraction $\phi$ requires more quantitative studies.

It is important to note that our simulation applies a boundary condition, via an external potential, that confines only the colloidal spheres in the shear gradient direction but not the solvent flow. This boundary condition is not the same as that imposed by the planar walls used in the experiment (7). It has been shown that, for the systems we have studied, with moderate planar separations and high packing fractions, the behavior is dominated by the sphere–sphere interactions and the effect of hydrodynamic interactions between the spheres and the walls can be ignored (39). However, in other situations, for example with dilute suspension, the rheology of the system can be greatly influenced by the sphere–wall interactions.

Despite the fact that confinement induces or enhances layering within the suspension, we expect our conclusions to be generally valid in the absence of additional sources of stress (e.g., strong particle–particle magnetic repulsion) because we make comparisons between relative numbers for consistently defined systems. We also note that our results are complementary to those obtained from earlier simulations of bulk colloidal suspensions (9) that use periodic boundary conditions, which could repress layer structures. Looking forward, we plan to build on the present approach to address the broader problem of determining how to attribute suspension properties to particle interactions and structure changes in situations with additional sources of stress (40).

Methods

Stokesian Dynamics Simulations. We use Stokesian dynamics to simulate the dynamics of a monodisperse suspension of $N$ hard spheres of unit radius. The influence of hydrodynamic interactions on the particle motions is incorporated through the resistance tensor, which is evaluated based on the configuration at the beginning of each time step of length $\Delta t = 4 \times 10^{-4}$. The centers of the spheres are restricted to a rectangular box spanning $0 < x < 11.4, 0 < z < 11.4$, and $-5 < y < 5.0$. We apply periodic boundary conditions in the $x$ and $z$ directions and an external potential along the $y$ direction that is 0 for $|y| < 2y_0$ and $U = 500k_B T|y| - y_0^2$ otherwise. We have $N = 130$ and 167 and for $\phi = 0.37$ and 0.47, respectively, as packing fraction $\phi = N(4\pi a^3)/3V$ and volumetric $V = 11.4^211.4^2(2\times y_0 + 2)$. Although this potential is meant to mimic the confinement imposed by the shear cell used in ref. 7, it is important to note that the potential confines only the colloidal spheres and not the solvent flow. The unit time of the simulation is defined in terms of the characteristic timescales of the system: the diffusion constant of an isolated colloidal particle in the solvent $D_0$ when $Pe < 1$ and the inverse shear rate $1/\gamma$ when $Pe > 1$, where $D$ is the short time diffusion constant of an isolated colloidal particle in an infinite suspension. Spatial configurations of the suspension at successive time steps are recorded and used to compute structures and properties of the system. For example, the viscosity of the suspension is evaluated using eqs. 20 and 21 in ref. 28. To compute the average properties for the nonequilibrium steady state for each packing fraction and $Pe$, 100 independent simulation runs were initialized using the Monte Carlo method. In each run, the system was equilibrated for 30,000 time steps of $\Delta t = 4 \times 10^{-4}$ before applying shear. Another 50,000 time steps were taken after the onset of the shear motion to allow the system to settle into the nonequilibrium steady state, after which 100,000 time steps were taken for analysis of averaged properties.

Nonequilibrium Umbrella Sampling. We use nonequilibrium umbrella sampling for efficient sampling of rare configurations of our sheared system. In this approach, a space of order parameters (here $Q_\psi$ and $Q_\phi$) is divided into a number of regions with arbitrarily chosen sizes, and a copy of the system is simulated in each region. Simulations proceed independently according to the original dynamics except when a step would take a copy out of its region. In such an event, a series of bookkeeping variables for determining the copy is reinitialized from a previously saved entry into its region [see the study by Dickson et al. (34) for details]. In this way, the procedure enforces uniform sampling over the regions regardless of the probabilities of the ranges of order parameter values, without distorting the underlying dynamics.

We initialized the simulations for each order parameter from 100 independent configurations randomly chosen from the steady-state configurations of a Stokesian dynamics simulation with $\phi = 0.47$ and $Pe = 10$. From these configurations, we ran 100 independent nonequilibrium umbrella sampling simulations. In each simulation, the one-dimensional parameter space is divided into 30 regions, where the sizes of the middle regions range from 10 to 20% of the SD of the steady-state histogram of the corresponding order parameter with $\phi = 0.47$ and $Pe = 10$ (Figs. 3A and 5A). The regions without a copy initially were OFF and only turned ON once a copy naturally crossed into them [the progressive activation scheme in the study by Dickson et al. (34)]. The simulations run in terms of cycles. Within each simulation cycle, we simulated the dynamics within each active region for 40 time steps of $\Delta t = 4 \times 10^{-4}$, yielding on average about two crossings per active region. After all of the desired regions were activated, which took on average about 150 cycles, we ran an additional 100 cycles for viscosity and stress analysis.

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