## Continuous-Variable Series-Elastic Actuator

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<td>Institution of Electrical and Electronics Engineers</td>
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Continuously-Variable Series-Elastic Actuator

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Abstract—Actuator efficiency is an important factor in the design of powered leg prostheses, orthoses, exoskeletons, and legged robots. A continuously-variable series-elastic actuator (CV-SEA) is presented as an efficient actuator for legged locomotion. The CV-SEA implements a continuously-variable transmission (CVT) between a motor and series elastic element. The CVT reduces the torque seen at the motor and allows the motor to operate in speed regimes of higher efficiency, while the series-elastic element efficiently stores and releases mechanical energy, reducing motor work requirements for actuator applications where an elastic response is sought. An energy efficient control strategy for the CV-SEA was developed using a Monte-Carlo minimization method that randomly generates transmission profiles and converges on those that minimize the electrical energy consumption of the motor. The CV-SEA is compared to a standard SEA and an infinitely variable series elastic actuator (IV-SEA). Simulations suggest that a CV-SEA will require less energy that an SEA or IV-SEA when used in a knee prosthesis during level-ground walking.

Keywords—continuously variable, series elastic actuator, efficiency, prostheses

I. INTRODUCTION

The performance and energetic efficiency of actuating legged locomotion are important factors to consider when designing powered leg prostheses, orthoses, exoskeletons, and legged robots. The energy required by a motorized legged robot can be reduced by choosing an actuator architecture that takes advantage of legged locomotion dynamics. Motors are most efficient when operating at intermediate speeds and low torque, but legged locomotion involves bidirectional joints experiencing both low and high torques \cite{1}. Legged locomotion also includes periods of both negative and positive mechanical power \cite{1}, so an efficient actuator should be able to store energy during periods of negative power, and release that energy during periods of positive power. Motors can act as generators, but the highest efficiency achieved for this process in legged robots is only 63 percent \cite{2}. An efficient motorized actuator for legged robots would both operate the motor at low torques and efficiently store energy.

Series-elastic elements improve the efficiency in legged locomotion by mechanically storing energy \cite{3}. Series-elastic actuators (SEAs) have been used in both ankle and knee prostheses to effectively reduce the energy consumption of the devices \cite{4}, \cite{5}. The addition of an SEA increases efficiency by mechanically storing energy during negative power portions of the gait cycle and then releasing the stored energy during subsequent portions of the cycle, reducing motor work requirements and/or adjusting the speed regime of the motor \cite{3}. Mechanically storing the energy in an elastic element circumvents the inefficiencies associated with energy domain transfers. In order to take advantage of spring-like torque-angle relationships observed in legged locomotion, clutches have also been used with elastic elements to store energy at a low electrical cost \cite{Herr, Paluska, Dilworth, Human Limbs}. Artificial Human Limbs and Joints Employing Actuators, Springs, and Variable Damper Elements. U.S. Patent Pending US 2006/0249315 A1. \cite{6}.

Although having series compliance within an actuator has many benefits, series compliance alone limits a designer’s capacity to fine tune motor operating speeds to optimize motor efficiency across a diverse set of actuator tasks. Forcing the motor to operate at extremes of motor speed results in a lower efficiency. Another approach to increasing the efficiency of an actuator is to provide a means to vary the transmission ratio, so that the motor can operate efficiently at both high speed/low torque regimes and at low speed/high torque regimes. Various forms of continuously variable transmissions (CVTs) have been explored \cite{6–8}. CVTs can be divided into two major categories, those that passively control the transmission ratio by reacting to velocity or torque \cite{6}, \cite{7}, and those that have active control over the transmission ratio \cite{8}.

The passive control of CVTs eliminates the need to develop a control strategy for the transmission ratio, but it does not guarantee an energy efficient transmission profile. Researchers have developed passively controlled CVTs \cite{6}, \cite{7}. Passively controlled CVTs can store some amount of energy, but the energy storage is a byproduct of the passive transmission ratio control. The energy storage is also coupled to both angle and torque, which limits the amount of energy that can be stored.

Actively controlling CVTs to reduce energy requirements have also been investigated in multiple areas of research. Previous research has been conducted on the appropriate control of CVTs when implemented in automobiles \cite{9}, but these methods focused on unidirectional motion and thus do not efficiently translate to cyclic bidirectional motion. A controllable CVT for quadruped motion was also developed. The device used a crank-type CVT with two equivalent
actuators and no elastic element [8]. The controllable CVT would allow a more energy efficient control strategy in principle, but the device did not provide a means for efficiently storing energy during negative power periods [8].

Infinitely variable transmissions (IVTs) have also been investigated. IVTs allow the input motor to run at a constant speed with the output producing a controllable velocity that can reverse directions. The active control of IVTs used in haptic devices has been extensively investigated [10–13]. These haptic control strategies mention the possible energy advantages, but focus on expanding the dynamic range of the device. Similar to a CVT, an IVT reduces the load on the motor, but it also cannot store elastic energy. Any negative mechanical power must be transformed into electrical energy by the motor.

Recently, Stramigiolo et al. presented a new actuator, the Very Versatile Energy Efficient (V2E2) actuator which utilizes a parallel motor and clutch in series with an elastic element and IVT [14]. Both [14] and [15] suggest that this novel architecture could significantly reduce the energy consumption of locomotive robotics, but the control strategy is neither fully developed nor compared to any other architecture. The proposed IVT used in the V2E2 also exhibits low efficiencies as the transmission reverses direction [16–18], a frequent occurrence in legged robotics. These regions of IVT inefficiency increase the energy of replicating legged locomotion, and will be further discussed in Section IV.

The capabilities of different variable stiffness actuators (VSAs) have been investigated by many groups [19]. A VSA can be constructed by placing a variable transmission between the load and an elastic element [14], [19]. A VSA can mechanically store energy in the elastic element during different dynamic regimes, but this architecture forces the variable transmission to be in between the load and elastic element. Therefore, the variable transmission must be able to accommodate the full torque seen at the elastic element. Placing a constant transmission between the variable transmission and the load can reduce the torque seen by the elastic element. However, this arrangement increases the deflection of the elastic element for a given amount of energy storage, increasing the size of the device. Therefore, the continuously-variable series-elastic actuator (CV-SEA) is introduced as an alternative architecture that places a variable transmission between the motor and elastic element. The CV-SEA does not vary the stiffness seen by the load, but controls the dynamics of the motor.

In this study, a novel actuator is presented: the continuously-variable series-elastic actuator (CV-SEA). The actuator has been developed with the intent of efficiently actuating cyclic, bidirectional motions, such as those observed during legged locomotion. Additionally, an energy efficient control strategy that utilizes a Monte-Carlo minimization method is presented to determine the minimum energy transmission profile of the CV-SEA for artificial knee actuation in a human walking gait cycle. Lastly, the efficiency of the CV-SEA in replicating legged locomotion is compared to a direct drive actuator, an SEA with a fixed transmission, and an infinitely variable series elastic actuator, IV-SEA.

![Fig. 1. The CV-SEA consists of three elements in series: an electromagnetic motor, a CVT and finally an elastic element. The series elastic element can reduce motor work, increase actuator efficiency and power, and limit motor/transmission shock, while the CVT can further optimize the speed profile of the motor for greater actuator efficiency.](image-url)
dynamics of the motor that corresponds with the clamped kinematic and kinetic trajectories. The voltage, \( v \), and current, \( i \), of a motor driving a given dynamic trajectory were estimated by (1) and (2):

\[
v = L_m \frac{dv}{dt} + R_m i + k_i \dot{\theta}_m
\]

(1)

\[
i = \frac{J_m \ddot{\theta}_m + \tau_m + v_m \dot{\theta}_m}{k_f}
\]

(2)

where \( L_m \) and \( R_m \) are the motor’s inductance and resistance, \( k_i \) is the back emf constant, \( k_f \) is the torque constant, \( J_m \) is the motor’s rotor inertia, \( v_m \) is the damping coefficient of the motor, \( \dot{\theta}_m \) and \( \ddot{\theta}_m \) are the motor’s velocity and acceleration, and \( \tau_m \) is the torque supplied by the motor.

Using (1) and (2) to estimate \( v \) and \( i \), the power of the motor, \( p_{motor} \) was defined as:

\[
p_{motor} = \begin{cases} 
iv \eta^{-1}_{act} & p > 0 \\
iv \eta_{act} \eta_{gen} & p < 0 
\end{cases}
\]

(3)

where \( \eta_{act} \) is the efficiency of the actuator and \( \eta_{gen} \) is the efficiency of converting mechanical power into electrical energy, which was conservatively set to 0.5 [2]. In order to account for inefficiencies in the actuator \( \eta_{act} \) was introduced, which is the ratio of the output power to the input power. Equation (3) shows that the required output power is multiplied by \( \eta_{CV-SEA} \) when the actuator is providing positive power, and the output power is multiplied by \( \eta_{CV-SEA} \) when power is flowing from the output back into the actuator. Equations (1), (2) and (3) estimate the power of the actuator given the dynamics of the motor, which can be related to the output dynamics and transmission ratio through (4-7). The output of the CVT, \( \theta_{CVT} \), can be defined as a function of both the load position, \( \theta_L \), and load torque, \( \tau_L \):

\[
\tau_L = k_s (\theta_L - \theta_{CVT})
\]

(4)

where \( k_s \) is the spring constant of the series-elastic element. The transmission ratio, \( N \), was the defined as:

\[
N = \frac{\dot{\theta}_M}{\dot{\theta}_{CVT}} = \frac{\tau_L}{\tau_M}
\]

(5)

where \( \dot{\theta}_{CVT} \) is the velocity of the CVT output. After differentiating (4) and eliminating \( \dot{\theta}_{CVT} \) with (5), \( \dot{\theta}_M \) and \( \dot{\theta}_M \) were defined as:

\[
\dot{\theta}_M = N k_s \dot{\theta}_L - \frac{\tau_L}{k_s}
\]

(6)

\[
\dot{\theta}_M = N k_s \dot{\theta}_L + \tau_M k_s \dot{\theta}_L - \frac{\tau_L}{k_s}
\]

(7)

Substituting the kinematics and kinetics of the motor shaft in (1) and (2) with (5-7) resulted in an estimation of the power that only depends on the load conditions and transmission ratio. The energy requirement of the CV-SEA was investigated using the kinematics and kinetics of biomechanical data as the load conditions.

B. Biomechanical Data Collection

In order to estimate the load conditions of the actuator, kinematic and kinetic data were collected at the Harvard University Skeletal Biology Lab in a study approved by the MIT Committee On the Use of Humans as Experimental Subjects (COUHES). After obtaining informed consent, the participants were asked to walk barefoot on an instrumented treadmill for two minutes at each of six speeds (0.75 m/s to 2.00 m/s, incrementing by 0.25 m/s). The kinetic data were collected using the force plates of the treadmill (Bertec Corporation, Columbus OH) and sampled at 1000 Hz while the kinematic data were collected synchronously via an infrared camera system (eight cameras, Qualisys Motion Capture Systems, Gothenburg, Sweden) and sampled at 500 Hz. The motion capture system utilized passive markers placed at 42 (bilateral) locations on the participant’s body according to the Helen Hayes marker model. The raw data were processed in SIMM (Software for Interactive Musculoskeletal Modeling, Musculographics Inc., Evanston, IL) to obtain joint trajectories and dynamics. The acquired data were used to provide insight into walking dynamics across a range of speeds and weights.

C. Monte-Carlo Energy Minimization

An energy efficient control strategy for the CVT was developed to minimize the energy requirements of the CV-SEA. Equation (3) is a nonlinear, time dependent, constrained, differential equation that is a function of both \( N \) and \( \dot{N} \). Finding the transmission ratio profile that minimizes the energy consumption for a given load profile is a nontrivial optimization problem. Therefore, approximate solutions were explored using a Monte-Carlo minimization method (MCMM).

The MCMM generates random transmission profiles that satisfy both transmission and operational constraints. In terms of transmission constraints, \( N \) must remain between \( N_{min} \) and \( N_{max} \) at all times. \( \dot{N} \) also must be constrained because there is an energy cost associated with changing the transmission ratio. Since the efficiency of the CV-SEA was explored for a generic CVT, the energy costs associated with controlling the CVT were estimated as viscous friction:

\[
P_{control} = v_{CVT} N^2
\]

(8)
where $\nu_{CVT}$ is the viscous damping coefficient of controlling the CVT. The magnitude of $\nu_{CVT}$ limits how quickly the transmission ratio can change and affects the shape of the transmission profile. The effects of $\nu_{CVT}$ were empirically studied, and the value was set to $10^{-6}$, so that the energy used to control the CVT would account for approximately 10 percent of the total actuator energy.

Along with transmission constraints, there were also operational constraints considered. Brushless motors have a rated voltage that they should remain under, so a penalty function was used to eliminate transmission profiles that result in a voltage greater than the motor’s rated voltage. Also, the transmission ratio must follow periodic boundary conditions at the same frequency of the load.

Taking the mentioned constraints into consideration, the random transmission profiles were generated by selecting 3-5 points on the plane defined by time, $t$, and $N$. Given that $t_f$ is the length of the gait cycle, $t \in (0,t_f)$ and $N \in (0,1)$. The points were then repeated backward in time and forward in time in order to create a periodic boundary condition. A cubic spline was used to create a smooth function defined by the randomly selected points. In order to ensure that $N$ is between $N_{min}$ and $N_{max}$, and evenly distributed, $N$ was randomly scaled and shifted to remain within the constraints of the transmission ratio.

The randomly generated transmission profiles were evaluated by the cost function:

$$E = \int \left( p_{motor} + p_{control} + g_N + g_V \right) dt$$

(9)

where $g_N$ and $g_V$ are penalty functions that constrain the transmission profile within the limits of the CVT, and the voltage of the motor within its specified limit.

**D. Motor Parameters**

Table I lists the motor parameters that were used in the simulations. The motor parameters were for a 24V Maxon EC-30 4-pole 200W brushless motor (Maxon Motor, Sachseln, Switzerland). The motor damping parameter is not provided by Maxon, but was estimated using no load conditions and (1).

**E. SEA Parameters**

The stiffness of the series elastic element, $k_s$, was set at 250 Nm/rad for all the simulations. This stiffness represents the characteristic torque-angle relationship observed during the early to midstance phase period of level ground walking. Using a series-elastic element with a stiffness similar to the characteristic linear torque-angle relationship observed during the human-knee stance period allows mechanical energy to be efficiently stored in the spring [23], minimizing motor work. The transmission ratio of both the SEA and direct drive actuator were optimized to reduce the average step energy of all three subjects over three different speeds. The SEA was modeled as having a transmission ratio of 188, and the direct drive had a slightly lower transmission ratio of 183.

**F. CVT Parameters**

The CVT used in both the CV-SEA and IV-SEA was constrained to have a transmission ratio between 75 and 300. This could be achieved with a typical belt CVT or toroidal CVT and a fixed gear transmission. The efficiency of the CVT was approximated as a constant 0.90 [20–22]. A more accurate model would model dynamic efficiency of the CVT [16], but since the CVT architecture is not defined, a constant efficiency was used as an approximation.

**G. IVT Parameters**

The IVT was modeled as a CVT connected in parallel to a fixed transmission and planetary gear system [18]. The IVT had a transmission ratio assumed to take any value within the ranges $(50,\infty)$ and $(-50,\infty)$. The efficiency of the IVT was

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**Table I. Maxon EC-30 4-pole 200W brushless motor**

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<th>Parameter</th>
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<th>Value</th>
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<td>No load speed</td>
<td>$\dot{\theta}_{NL}$</td>
<td>1750 rad/s</td>
</tr>
<tr>
<td>No load current</td>
<td>$I_{NL}$</td>
<td>0.728 A</td>
</tr>
<tr>
<td>Resistance</td>
<td>$R$</td>
<td>0.102 $\Omega$</td>
</tr>
<tr>
<td>Inductance</td>
<td>$L$</td>
<td>0.0163 mH</td>
</tr>
<tr>
<td>Damping</td>
<td>$\nu$</td>
<td>5.66 $\mu$Nm/(rad/s)</td>
</tr>
<tr>
<td>Rotor inertia</td>
<td>$J_M$</td>
<td>33.3 gcm$^2$</td>
</tr>
<tr>
<td>Torque constant</td>
<td>$k_T$</td>
<td>13.6 mNm/A</td>
</tr>
<tr>
<td>Back EMF constant</td>
<td>$k_B$</td>
<td>0.0136 V/(rad/s)</td>
</tr>
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![Fig. 2. The convergence of the MCMR on the optimal transmission ratio profile is depicted above. The five transmission profiles that resulted in the lowest energy consumption are shown. The subject was waking at 1.5 m/s and 0% gait cycle corresponds with heel strike.](image-url)
modeled according to the methods presented in [18], where the efficiencies of the CVT, fixed transmission, and planetary gear system were approximated as 0.90, 0.99, and 0.99 respectively. The efficiencies of the fixed gear and planetary gear are intentionally high to show that the inefficiency of the IVT does not arise from the complexity of the device, but from the inherent characteristics of the power flow. A constant voltage control strategy was optimized to minimize the energy required by the IV-SEA.

**IV. SIMULATION RESULTS**

The convergence of the MCMM was qualitatively observed by comparing the similarity of the transmission profiles that minimized the energy requirements of the CV-SEA. The convergence of the MCMM is depicted below in Fig 2. The MCMM shown used four randomly selected points to define the transmission ratio and executed $10^5$ iterations, the same number of iterations used in the comparison of the four different actuators.

The energy efficiency of the CV-SEA was compared to three other architectures: a direct drive with no elastic element, an SEA, and an IV-SEA. The energy requirement of each actuator to drive an artificial knee joint, assuming human-knee biomechanical torque and state output values during level ground walking, was simulated for three different subjects walking at three different speeds. The four actuators are compared in Fig. 3, where the step energy of the SEA, IV-SEA, and CV-SEA are normalized with respect to the step energy of the direct drive actuator.

**V. DISCUSSION**

The MCMM was developed as a tool to quickly compare the efficiencies of different actuator architectures. Although the MCMM requires many iterations to approximate the optimal transmission profile, the MCMM could be used to develop a control strategy for a consistent process, such as an autonomous legged robot. A faster control strategy would be required for legged robots that may not be as consistent, such as prostheses or orthoses. The MCMM would also not be able to predict transmission ratios that change at high frequencies. The number of defining points would be too high to converge quickly. High frequency transmission ratio control would only occur if the associated energy cost of changing the ratio was much lower than the energy being transmitted. However, the purpose of the MCMM was to serve as a tool to compare the energy efficiency of different actuator architectures for legged locomotion.

The control strategy for an IV-SEA to efficiently replicate legged locomotion has not yet been explored. Everarts et al. [16] suggested a constant voltage input control strategy for a similar architecture, where the series elastic element is between the motor and IVT. This control strategy takes advantage of an IVTs ability to reverse direction. Running the motor at a constant voltage eliminates any inefficiency associated with accelerating the rotor’s inertia. One would expect it to be most efficient to supply the motor with the highest voltage possible, so that it can supply the least amount of torque. Operating the motor at the highest voltage possible is not the most efficient when the viscous damping of the motor is considered. Therefore, the constant velocity of the motor was optimized to minimize the energy consumption. This control strategy utilized the fundamental advantage offered by using an IVT, but it is not guaranteed to be the most efficient.

The results presented in Fig. 3 suggest that the CV-SEA can consistently reduce the energy associated with actuating human knee motion during level ground walking. The CV-SEA optimizes the speed profile of the motor throughout the

![Fig. 3. The step energy of each transmission actuating a knee prosthesis was estimated for three different subjects walking at three different speeds. The step energy was normalized by the step energy required by a direct drive actuator.](image-url)
walking cycle so as to enhance motor efficiency and reduce resistive losses. The IVT-SEA is able to reduce any torque associated with accelerating the rotor’s inertia, but the inefficiency of the IVT at high transmission ratios greatly reduces its overall efficiency.

VI. FUTURE WORK

This study introduced the CV-SEA as a new actuator architecture for replicating legged locomotion. Simulations compared the CV-SEA to three other actuator architectures: a direct drive, a SEA, and an IV-SEA which uses an IVT instead of a CVT. The simulations suggest that the CV-SEA will most efficiently actuate a human knee prosthesis during level ground walking.

A miniature half-toroidal CVT is being developed to prototype a CV-SEA. Once the miniature CVT is developed, a more accurate model of the CVTs efficiency and control characteristics will be available.

While the MCM converges on transmission profiles that minimize the actuator energy, an analytical solution to find the transmission ratio that minimizes the energy consumption is also being explored. It is hoped that an analytical solution will provide further insight into actuator efficiency.

Along with developing a prototype and control strategy the application of the CV-SEA to other motions, such as ankle and hip, will also be explored.

ACKNOWLEDGMENT

The authors thank Dr. Elliott Rouse for his many insights and contributions to the project.

REFERENCES