Accurate Test of Chiral Dynamics in the \[^\text{\textoverline{y}}\]p[^\text{0}]p Reaction

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Accurate Test of Chiral Dynamics in the $\tilde{\gamma}p \rightarrow \pi^0 p$ Reaction


(A2 and CB-TAPS Collaborations)

1Mount Allison University, Sackville, New Brunswick, E4L 1E6, Canada
2Institut für Kernphysik, University of Mainz, D-55128 Mainz, Germany
3Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom
4Helmholtz-Institut für Strahlen- und Kernphysik, University of Bonn, D-53115 Bonn, Germany
5Petersburg Nuclear Physics Institute, Gatchina RU-188300, Russia
6II Physikalisches Institut, University of Giessen, D-35392 Giessen, Germany
7Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
8INFN Sezione di Pavia, I-27100 Pavia, Italy
9The George Washington University, Washington, D.C. 20052-0001, USA
10Lebedev Physical Institute, RU-119991 Moscow, Russia
11Institut für Physik, University of Basel, CH-4056 Basel, Switzerland
12Grupo de Física Nuclear, Universidad Complutense de Madrid, CEI Moncloa E-28040, Spain
13School of Physics, University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
14Institute for Nuclear Research, RU-125047 Moscow, Russia
15Rudjer Boskovic Institute, HR-10000 Zagreb, Croatia
16Kent State University, Kent, Ohio 44242-0001, USA
17University of California Los Angeles, Los Angeles, California 90095-1547, USA

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A precision measurement of the differential cross sections $d\sigma/d\Omega$ and the linearly polarized photon asymmetry $\Sigma = (d\sigma_+ - d\sigma_-)/(d\sigma_+ + d\sigma_-)$ for the $\tilde{\gamma}p \rightarrow \pi^0 p$ reaction in the near-threshold region has been performed with a tagged photon beam and almost 4 $\pi$ detector at the Mainz Microtron. The Glasgow-Mainz photon tagging facility along with the Crystal Ball/TAPS multiphoton detector system and a cryogenic liquid hydrogen target were used. These data allowed for a precise determination of the energy dependence of the real parts of the $S$- and all three $P$-wave amplitudes for the first time and provide the most stringent test to date of the predictions of chiral perturbation theory and its energy region of agreement with experiment.

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Low-energy pion photoproduction experiments are of special interest because the pion, the lightest hadron, is a Nambu-Goldstone boson that by its existence represents a clear signature of spontaneous chiral symmetry breaking in QCD [1]. The dynamic consequences are that the production and elastic scattering of neutral pions at low energies are weak in the $S$ wave and strong in the $P$ wave [1–4], as is seen in the $\gamma N \rightarrow \pi N$ reaction [5,6]. In neutral-pion photoproduction, the $S$-wave threshold amplitudes are small since they vanish in the chiral limit $(m_u, m_d \rightarrow 0)$; their small but nonvanishing values are consequences of explicit chiral symmetry breaking. In addition, they are isospin violating [2,3,7], since $m_u \neq m_d$ [8,9]. The magnitudes of low-energy scattering and production experiments are predicted by chiral perturbation theory (ChPT), an effective field theory of QCD based on spontaneous chiral symmetry breaking [1–5]. Our efforts have been focused on accurate measurements of low-energy $\gamma N \rightarrow \pi N$ reactions, including the sensitive spin observables that allow a unique separation of the
were sent through a 4-mm-diameter Pb collimator and two- and three-cluster events that satisfied the hypothesis of the reaction $\gamma p \rightarrow \pi^0 p \rightarrow \gamma\gamma p$ with a probability of more than 2% were accepted as candidates for this reaction. Background contamination of the event candidates was found to be from two sources: interactions of the bremsstrahlung photons in the target material different from liquid hydrogen and accidental coincidences between the tagger hits and the trigger based on the detector signals. The background was subtracted from the signal directly by using two different data samples, the first of which included only events with accidental coincidences, the second taken with an empty-target cell.

Acceptance of the detector system was determined by Monte Carlo simulation of $\gamma p \rightarrow \pi^0 p$ using an isotropic angular distribution. All events were propagated through a GEANT3.21 simulation of the experimental setup, folded with resolutions of the detectors and conditions of the trigger. Close to the reaction threshold, the production-angle acceptance was found to be almost uniform with a detection efficiency about 80%.

The systematic uncertainties in the absolute numbers of the differential cross sections for the reaction $\gamma p \rightarrow \pi^0 p$ obtained in the analysis of the data set were estimated to be not larger than 4%. Such a magnitude of the systematic uncertainty is mostly determined by the calculation of the photon-beam flux, the experimental detection efficiency, and the number of protons in the LH$_2$ target. The systematic uncertainties in the numbers for the photon asymmetry are on the level of 5%, where this value comes mostly from the uncertainty in the determination of the degree of the linear polarization of the incoming photons.

Results for the differential cross section and the photon asymmetry are presented in Fig. 1 as a function of the pion center-of-mass (c.m.) production angle $\theta$ at $E_\gamma = 163.4 \pm 1.2$ MeV and as a function of incident photon energy at $\theta \approx 90^\circ \pm 3^\circ$. Also shown are one empirical and two theoretical fits: (1) heavy baryon chiral perturbation theory (HBChPT) calculations to $O(q^4)$ [22] with the five empirical low-energy constants brought up to date by fitting these data [23], (2) relativistic ChPT calculations [also to $O(q^4)$] which as well have five low-energy constants fit to these data [24,25], and (3) an empirical fit with error bands calculated using the formalism from Refs. [26,27]. The error bands take into account the correlations among parameters; details on the method can be found in Appendix A of Ref. [23]. Fits have been performed employing a genetic algorithm combined with a gradient-based routine that is thoroughly discussed in Ref. [28].

Because of the high quality of the present data, it is, for the first time, possible to determine the energy range for which ChPT agrees with the data. The values of the low-energy constants were obtained from fits to the data in the range from 150 MeV to a variable $E_{\gamma\max}$. This was done [23] using the $O(q^4)$ formulas of heavy baryon calculations [22] and for the relativistic theory [24,25,29]. Figure 1(e) displays the $\chi^2$ per degree of freedom for the
baryon version, suggesting a better convergence for the relativistic ChPT method.

The next step in the interpretation of the data is to extract the multipole amplitudes and compare them to the theoretical calculations. To set the notation, the differential cross sections can be expressed in terms of the $S$- and $P$-wave multipoles ($E_{0^+}^s$, $P_1$, $P_2$, $P_3$) and can be written as

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{q}{k}(A + B\cos\theta + C\cos^2\theta),$$

where $q$ and $k$ denote the c.m. momenta of the pion and the photon, respectively. The coefficients are given by $A = |P_1|^2 + P_{23}^2$, with $P_{23}^2 = 1/2(|P_2|^2 + |P_3|^2)$, $B = 2\text{Re}(E_{0^+}^s P_1^*)$, and $C = |P_1|^2 - P_{23}^2$. The measurement of the cross sections of earlier experiments [11] permitted the extraction of $E_{0^+}^s$, $P_1$, and the combination $P_{23}$. In order to extract the values of Re$E_{0^+}^s$ and all three $P$ waves separately from the data, it is necessary to also measure the photon asymmetry

$$\Sigma = \frac{d\sigma_{\perp} - d\sigma_{||}}{d\sigma_{\perp} + d\sigma_{||}} = \frac{q}{2k}(|P_3|^2 - |P_2|^2)\sin^2\theta/\frac{d\sigma}{d\Omega}(\theta),$$

where $d\sigma_{\perp}$ and $d\sigma_{||}$ are the differential cross sections for photon polarization perpendicular and parallel to the reaction plane with the pion and the outgoing proton. To reiterate, the measurement of the differential cross section and $\Sigma$ allows for the separation of the four multipoles.

It is important to note that the determination reported here is more accurate than previous ones due to the far smaller uncertainties of the cross sections as well as the energy dependence of $\Sigma$. Furthermore, we note that the $D$ waves have been neglected in both (1) and (2), but they have recently been shown to be important in the near-threshold region [30]. Since there are insufficient data to determine the $D$-wave multipoles empirically, they have been taken into account by using their values in the Born approximation, which is sufficiently accurate for the present analysis.

The empirical fits to the data employ the following ansatz for the $S$- and $P$-wave multipoles

$$E_{0^+}^s(W) = E_{0^+}^{(0)} + E_{0^+}^{(1)}\left(\frac{E_{\gamma} - E_{\gamma}^{\text{thr}}}{m_{\pi^+}}\right) + i\beta \frac{q}{m_{\pi^+}},$$

$$P_i(W) = \frac{q}{m_{\pi^+}}\left[P_i^{(0)} + P_i^{(1)}\left(\frac{E_{\gamma} - E_{\gamma}^{\text{thr}}}{m_{\pi^+}}\right)\right],$$

where here $E_{\gamma}$ and $E_{\gamma}^{\text{thr}}$ are in the lab frame, and $E_{0^+}^{(0)}$, $E_{0^+}^{(1)}$, $P_i^{(0)}$, and $P_i^{(1)}$ (with $i = 1, 2, 3$) are constants that are fit to the data. The empirical values are in units of $10^{-3}/m_{\pi^+}$: $E_{0^+}^{(0)} = -0.369 \pm 0.027$, $E_{0^+}^{(1)} = -1.47 \pm 0.13$, $P_1^{(0)} = 9.806 \pm 0.068$, $P_1^{(1)} = 1.63 \pm 0.32$, $P_2^{(0)} = -10.673 \pm 0.070$, $P_2^{(1)} = -4.52 \pm 0.31$, $P_3^{(0)} = 9.671 \pm 0.060$, and $P_3^{(1)} = 15.87 \pm 0.29$. The pairs $(E_{0^+}^{(0)}, E_{0^+}^{(1)})$ and $(P_i^{(0)}, P_i^{(1)})$, $i = 1, 2, 3$, are highly correlated.

Based on unitarity, the cusp parameter in Eq. (3) has the value $\beta = m_{\pi^+} a_{\text{cusp}}(\pi^+ n \rightarrow \pi^0 p)\text{Re}E_{0^+}^s(\gamma p \rightarrow \pi^+ n)$ [31].

FIG. 1 (color online). Differential cross sections in (a) $\mu$b/str and (b) photon asymmetries for $\pi^0$ production as a function of pion c.m. production angle $\theta$ for an incident photon energy of $163.4 \pm 1.2$ MeV. Energy dependence of the (c) differential cross sections and (d) photon asymmetries at $\theta = 90^\circ \pm 3^\circ$. Errors shown are statistical only, without the systematic uncertainty of 4% for $d\sigma/d\Omega$ and 5% for $\Sigma$. The theory curves are dashed (black) for HBChPT [23], dash-dotted (blue) for relativistic ChPT [25,29], and solid (green) for an empirical fit with an error band. (e) $\chi^2$ per degree of freedom for fits to the data in the range from 150 MeV to $E_{\gamma}^{\text{max}}$ for HBChPT [23] (open black circles), relativistic ChPT [25,29] (open blue triangles), and an empirical fit (solid green dots), with lines drawn through the points to guide the eye. Note that in (c) and (d), the two points in incident photon energy below the $\pi^+$ threshold are included; these two points are excluded in the fits shown in (e) due to their large error bars.

Empirical fit and both ChPT calculations. For $E_{\gamma}^{\text{max}}$ up to $\approx 167$ MeV, the ChPT calculations are consistent with the empirical fit, but above this energy the relativistic calculation starts to deviate from the data, whereas the heavy baryon calculation begins to deviate at $\approx 170$ MeV. This is interesting, since the relative contributions of the terms containing the low-energy constants of the relativistic calculation are significantly smaller than those for the heavy baryon version, suggesting a better convergence for the relativistic ChPT method.
Using the experimental value of $a_{\text{cex}}(\pi^- p \rightarrow \pi^0 n) = -(0.122 \pm 0.002)/m_{\pi^-}$ obtained from the observed width in the $1s$ state of pionic hydrogen [32], assuming isospin is a good symmetry, i.e., $a_{\text{cex}}(\pi^+ n \rightarrow \pi^0 p) = -a_{\text{cex}}(\pi^- p \rightarrow \pi^0 n)$, and the latest measurement for $E_{\text{cex}}(p \rightarrow \pi^+ n) = (28.06 \pm 0.27 \pm 0.45) \times 10^{-3}/m_{\pi^-}$, [33], we obtain $\beta = (3.43 \pm 0.08) \times 10^{-3}/m_{\pi^-}$, which was employed in the empirical fit. If isospin breaking is taken into account [34,35], we obtain $\beta = (3.35 \pm 0.08) \times 10^{-3}/m_{\pi^-}$. In this experiment, we do not have access to the imaginary part of the $S$-wave amplitude and no difference is found if either the isospin-conserving, the isospin-breaking, or even other $\beta$ values, such as those for dispersive effective chiral theory $\beta = 3.10 \times 10^{-3}/m_{\pi^-}$ [36] or HBChPT $\beta = 2.72 \times 10^{-3}/m_{\pi^-}$, are employed. Hence, the uncertainty introduced by the errors in $\beta$ and isospin breaking is smaller than the statistical uncertainties of the multipole extraction depicted in Fig. 2.

The extracted multipoles are displayed in Fig. 2 along with the theoretical calculations. The points are single-energy fits to the real parts of the $S$- and $P$-wave multipoles, and the energy-dependent fits from Eqs. (3) and (4) are shown with the error band. The imaginary part of the $S$-wave multipole $E_{\text{cex}}$ was taken from unitarity (3) with the value of the cusp parameter explained above, the imaginary parts of the $P$ waves were assumed to be negligible, and the $D$-wave multipoles were calculated in the Born approximation. The impact of $D$ waves in the $P$-wave extraction is negligible [30], but in the $S$ wave it can be sizeable. In order to assess the uncertainties in the $S$-wave extraction associated to our $D$-wave prescription, we have estimated the uncertainty from the difference between the Born terms and the Dubna-Mainz-Tapei dynamical model in Ref. [37]. This error estimation is depicted in Fig. 2 as a gray area at the top of the first plot. Note that the $D$ waves have a negligible impact in the $P$-wave extraction (the uncertainty is smaller than the curve’s width) [30].

As was the case for the observables, there is very good agreement between the two ChPT calculations and the empirical values of the multipoles for energies up to $\approx 170$ MeV with the same pattern of deviations above that.

In conclusion, the combination of the photon asymmetry and improved accuracy in the differential cross section has allowed us to extract the real parts of the $S$-wave and all three $P$-wave multipoles as a function of photon energy for the first time. We have achieved an unprecedented accuracy in our empirical extraction of the multipoles from the data, providing a more sensitive test of the ChPT calculations than has previously been possible. What we have found is that none of the real parts of the multipoles $E_{\text{cex}}$, $P_1$, $P_2$, and $P_3$ is causing the gradual deviation from experiment (increasing $\chi^2$) with increasing energy. Rather, it is probably due to the gradually increasing importance of the higher-order terms neglected in the chiral series or to the fact that the $\Delta$ degree of freedom is not being taken into account in a dynamic way.

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[25] Section 36.3.2.4 in Ref. [8] with $\Delta \chi^2$ fixed to unity.


