Experimental Signatures of Critically Balanced Turbulence in MAST

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1103/PhysRevLett.110.145002">http://dx.doi.org/10.1103/PhysRevLett.110.145002</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>American Physical Society</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Sun Apr 24 05:47:13 EDT 2016</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/81999">http://hdl.handle.net/1721.1/81999</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.</td>
</tr>
<tr>
<td>Detailed Terms</td>
<td></td>
</tr>
</tbody>
</table>
Experimental Signatures of Critically Balanced Turbulence in MAST

Y.-c. Ghim,1,2,3,* A. A. Schekochihin,1,4 A. R. Field,2 I. G. Abel,1,4 M. Barnes,5,6 G. Colyer,1,2 S. C. Cowley,2,7 F. I. Parra,5 D. Dunai,8 S. Zoletnik,8 and the MAST Team2

1Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, United Kingdom
2EURATOM/CCFE Fusion Association, Culham Science Centre, Abingdon OX14 3DB, United Kingdom
3Department of Nuclear and Quantum Engineering, KAIST, Daejeon 305-701, Republic of Korea
4Merton College, Oxford OX1 4JD, United Kingdom
5Plasma Science and Fusion Center, MIT, Cambridge, Massachusetts 02139, USA
6Oak Ridge Institute for Science and Education, Oak Ridge, Tennessee 37831, USA
7Blackett Laboratory, Imperial College, London SW7 2AZ, United Kingdom
8Wigner Research Centre for Physics, Association EURATOM/HAS, P.O. Box 49, H-1525 Budapest, Hungary

(Received 29 August 2012; published 1 April 2013)

Beam emission spectroscopy (BES) measurements of ion-scale density fluctuations in the MAST tokamak are used to show that the turbulence correlation time, the drift time associated with ion temperature or density gradients, the particle (ion) streaming time along the magnetic field, and the magnetic drift time are consistently comparable, suggesting a “critically balanced” turbulence determined by the local equilibrium. The resulting scalings of the poloidal and radial correlation lengths are derived and tested. The nonlinear time inferred from the density fluctuations is longer than the other times; its ratio to the correlation time scales as \( \nu_i^{-0.8\pm0.1} \), where \( \nu_i \) is the ion collision rate/streaming rate. This is consistent with turbulent decorrelation being controlled by a zonal component, invisible to the BES, with an amplitude exceeding those of the drift waves by \( \sim \nu_i^{-0.8} \).

Introduction.—Microscale turbulence hindering energy confinement in magnetically confined plasmas is driven by gradients of equilibrium quantities such as temperature and density. These gradients give rise to instabilities that inject energy into fluctuations (“drift waves”) at scales just above the ion gyroscale. The most effective of these is believed to be the ion-temperature-gradient (ITG) instability [1–3]. A turbulent state ensues, giving rise to the “anomalous transport” of energy [4]. It is of both practical and fundamental interest what the structure of this turbulence is and how it and the resulting transport depend on the local equilibrium parameters.

Fluctuations in a magnetized toroidal plasma are subject to a number of physical effects, which can be classified in terms of various time scales: the drift times associated with the temperature and density gradients, the particle streaming time along the magnetic field as it takes them around the torus toroidally and poloidally, the magnetic drift times of particles moving across the field, the nonlinear time of the fluctuations being advected across the field by the fluctuating \( E \times B \) velocity, the collision time, and the shear time of the plasma rotation. There has been a growing understanding [5], driven largely by theory [6–9], observations [10–12], and simulations of magnetohydrodynamic [13–15] and kinetic [7,16] plasma turbulence in space, that, if a medium can support parallel (to the magnetic field) propagation of waves (and/or particles) and nonlinear interactions in the perpendicular direction, the turbulence in such a medium would normally be “critically balanced,” meaning that the characteristic time scales of propagation and nonlinear interaction would be comparable to each other and (therefore) to the correlation time of the fluctuations. This means that the turbulence is not weak and not two dimensional, unless artificially constrained [9].

In this Letter, we use beam emission spectroscopy (BES) measurements [17–20] in the MAST tokamak [21], along with the local equilibrium parameters calculated by other diagnostics, to estimate and compare the characteristic time scales of the turbulent fluctuations in the energy-containing range. We obtain, for the first time, direct evidence that the correlation, drift, and parallel streaming time scales are indeed comparable across a range of equilibrium parameters (cf. Refs. [22,23]) and that the magnetic drift time is part of this “grand critical balance” as well. We also find indirect evidence that the decorrelation rate of turbulence is controlled by a zonal component whose relative importance to the drift-wave-like fluctuations scales with the ion collisionality.

Before presenting this evidence and its implications (e.g., dependence of the correlation lengths on equilibrium parameters), let us describe how it was obtained.

Experimental data and its analysis.—The data presented here were collected from 39 neutral-beam heated “double-null-diverted” discharges (including L- and H-modes and internal transport barriers), with no pellet injection and no resonant magnetic perturbations. The BES system on MAST [21] collects photons from a 2D array of

0031-9007/13/110(14)/145002(6) 145002-1 Published by the American Physical Society

PACS numbers: 52.35.Ra, 52.35.Kt, 52.55.Fa
8 radial × 4 vertical locations in the outboard midplane of the tokamak, with a 2 cm separation between adjacent channels in either direction. The detected intensity (mean + fluctuating, \(I + \delta I\)) is used to infer, at each location, the density fluctuation level \(\delta n/n = (1/\beta)(\delta I/I)\) [17], where \(\beta\) depends on the mean density \(n\) and is estimated based on the Hutchinson model [24]. (Dependence on the mean temperature is weak.) As the BES array was moved radially for different discharges, our database contains cases with radial viewing positions 10 cm < \(r\) < 50 cm from the magnetic axis (the minor radius of the plasma is \(r = 60\) cm).

The local equilibrium parameters are measured by standard diagnostics: mean electron densities \(n_e\) and temperatures \(T_e\) by the Thomson scattering system [25], impurity ion (\(\text{C}^{\text{ii+}}\)) mean temperatures (assumed to equal the bulk ion temperature \(T_i\)) and toroidal flow velocity \(U_{\phi}\) by the charge exchange recombination spectroscopy system [26], and local magnetic pitch angle \(\alpha\) by the motional Stark effect (MSE) system [27]; further equilibrium magnetic field information is obtained from pressure- and MSE-constrained EFIT equilibria [28].

We filter the BES data to the frequency interval [20, 100] kHz [29] and calculate the spatiotemporal correlation function

\[
\mathcal{C}(\Delta x, \Delta Z, \Delta t) = \frac{\langle \delta I(x, Z, t) \delta I(x + \Delta x, Z + \Delta Z, t + \Delta t) \rangle}{\sqrt{\langle \delta I^2(x, Z, t) \rangle \langle \delta I^2(x + \Delta x, Z + \Delta Z, t + \Delta t) \rangle}}, \tag{1}
\]

where \(x\), \(Z\), and \(t\) are the radial, vertical, and time coordinates, respectively, and \(\Delta x\), \(\Delta Z\), and \(\Delta t\) are the corresponding channel separations and the time lag; the angled brackets represent the time average over 5 ms periods. At \(\Delta x = \Delta Z = 0\), the autocovariances \(\langle \delta I(x, Z, t) \delta I(x, Z, t + \Delta t) \rangle\) contain not only the physical signal but also photon and electronic noise. We remove this effect by applying LED light to the BES channels, obtaining 150 different dc levels of BES signal from 0 to 1.5 V, calculating the noise autocovariance \(C_N(\Delta t)\) at each dc level with the same band frequency filter of [20, 100] kHz, and then finding \(C_N(\Delta t)\), whose dc level of the signal matches the dc level of the BES data from the MAST discharges, and subtracting it from the calculated autocovariances. From the correlation function (1) (illustrated in Fig. 1), we calculate the local characteristics of the density fluctuations.

The fluctuation level at each radial location is obtained from the (noise-subtracted) autocovariance function \(\delta n/n = (1/\beta)\sqrt{\langle \delta I^2(x, Z, t) \rangle}/I\) at all 32 locations and then averaged over the four poloidally separated channels at the same radial location.

The correlation length \(\ell_x\) perpendicular to the magnetic field within a flux surface is obtained from the vertical (poloidal) correlation length \(\ell_z\) via \(\ell_x = \ell_z \tan \alpha\), assuming that the parallel correlation length is sufficiently long: \(\ell_x \gg \ell_z \tan \alpha\). The correlation length \(\ell_z\) is estimated using four poloidal channels at each radial location (the top channel is the reference channel) by fitting \(\mathcal{C}(\Delta x = 0, \Delta Z, \Delta t)\) to the function \(f_z(\Delta Z) = p_z + (1 - p_z) \cos[2\pi \Delta Z/\ell_z] \exp[-|\Delta Z|/\ell_z]\), where \(p_z\) is a fitting constant that serves to account for global structures such as coherent magnetohydrodynamics modes [for which \(\mathcal{C}(\Delta x = 0, \Delta Z = \infty, \Delta t = 0) = p_z \neq 0\)]. In choosing \(f_z(\Delta Z)\), we assumed wavelike fluctuations in the poloidal direction [18] (drift-wave turbulence), with the wavelength and correlation length comparable to each other. It is not possible to distinguish meaningfully between the two with only four poloidal channels. Assuming wavelike structure is essential, as, in most cases, we find that \(\mathcal{C}(\Delta x = 0, \Delta Z, \Delta t = 0)\) goes negative and/or is nonmonotonic over the vertical extent of the BES array.

The radial correlation length \(\ell_r\) is estimated using eight radial channels at each poloidal location (the reference channel is the fourth from the inside). The correlation function \(\mathcal{C}(\Delta x, \Delta Z = 0, \Delta t = 0)\) is fitted to the function \(f_r(\Delta z) = p_z + (1 - p_z) \exp[-|\Delta z|/\ell_r]\), where \(p_z\) plays the same role as \(p_z\) did for \(f_z\). The values of \(\ell_r\) from four poloidal locations are averaged, assuming that the radial correlations do not change significantly within the poloidal extent of the BES array. Because we have to use the entire array to estimate \(\ell_r\), the number of data points for \(\ell_r\) is 8 times smaller than for \(\ell_z\).

To estimate the correlation time \(\tau_x\), we use the fact that the fluctuating density patterns are advected poloidally past the BES array with an apparent velocity \(v_{\text{BES}} = U_{\phi} \tan \alpha\) due to the toroidal rotation velocity \(U_{\phi}\) [29]. We fit \(\mathcal{C}(\Delta x = 0, \Delta Z, \Delta t) = \Delta t_{\text{peak}}(\Delta Z)\) taken at the time delay \(\Delta t_{\text{peak}}(\Delta Z)\) when the correlation function is maximum at a given \(\Delta Z\) [30], to the function \(f_r(\Delta Z) = \exp[-\Delta t_{\text{peak}}(\Delta Z)/\tau_x]\). This method relies on the temporal decorrelation dominating over the parallel spatial decorrelation, viz., we require \(\tau_x \ll \ell_z \cos \alpha / U_{\phi}\). Anticipating the critical balance assumption \(\tau_x \sim \ell_z / v_{\text{thi}}\).
where \( v_{thi} = \sqrt{2T_i/m_i} \) is the ion thermal speed, and denoting the Mach number \( \text{Ma} = U_B/v_{thi} \) we estimate that the fractional error in \( \tau_c \) is \( \approx \text{Ma}/\cos \alpha \), which was never more than 20% in the discharges we used.

The quantities \( \delta n/n, \ell_x, \ell_y, \ell_z \), and \( \tau_c \) are calculated every 5 ms for all 39 discharges. All fitting is done using the MPFIT procedure [31]. We consider a data point unreliable and discard it if (i) \( I < 0.3 \ V \) (the signal-to-noise ratio is too low, i.e., SNR < 150), (ii) the estimated correlation lengths are smaller than the size of the point spread function [32] \( \ell_z < 2 \) or \( \ell_y < 5 \ cm \), (iii) the assumption that plasma rotation is mostly toroidal is suspect, viz., \| (v_{BES} - U_B \tan \alpha)/v_{BES} \| \geq 0.2 \) (see Ref. [29]), where \( v_{BES} \) is calculated at each radial location using the cross-correlation time delay method [30], (iv) the estimated error in the calculation of \( v_{BES} \) is \( > 20\% \), and (v) \( p_Z \) or \( \eta_p > 0.5 \). The last two exclusion criteria pick out the cases when magnetohydrodynamics modes are too strong; they are known to degrade the reliability of the BES data [29].

The remaining database contains 448 points.

**Correlation time vs drift time.—** The turbulence can be driven by radial gradients in the mean ion and electron temperatures \( T_i;e \) and density \( n \). Denoting \( L_{-1} = |\nabla \ln T_i;e| \) and \( L_0 = |\nabla \ln n| \), the associated time scales are the inverse drift frequencies

\[
\tau_{-1} = \frac{\rho_i;e}{\ell_y L_{-1}}, \quad \tau_{0} = \frac{\rho_i;e}{\ell_y L_0},
\]

\( \ell_y > 0.0 \) (see Ref. [29]).

In Fig. 2(a), we compare the drift times with the correlation time \( \tau_c \). We find that \( \tau_c = (0.7 \pm 0.3) \tau_e \), where \( \tau_e = \min(\tau_{ii}, \tau_{en}) \) and the spread is calculated as the root mean square deviation from the mean value. Thus, the turbulence appears to be driven by the larger of the ion temperature or density gradient [33]. We find no clear correlation of \( \tau_e \) with \( \tau_c \) or with any of the other time scales discussed below.

**Critical balance.—** The standard argument behind the critical balance conjecture is causality [9]: Two distant points on a field line cannot stay correlated if information cannot be exchanged between them over a turbulence correlation time. Assuming information travels at \( v_{thi} \) [1–5], one gets \( \ell_{thi} \sim v_{thi} \tau_c \). This cannot be checked directly because there are no diagnostics capable of measuring \( \ell_{thi} \) on MAST. Considering that the inboard side of the torus is a region of “good” (stabilizing) curvature, not much turbulence is expected there, so we assume that, at the energy injection scale, \( \ell_{thi} \sim \Lambda \) [5], where the distance along the field line that takes a particle from the outer to the inner side of the torus is \( \Lambda = \pi r B_p/B_r \) (\( r \) is the minor radius at the BES position on the outer side and \( B_p \) the poloidal component of the magnetic field) [34]. Then, critical balance means that \( \tau_c \) should be comparable to

\[
\tau_{st}^{-1} = \frac{\rho_{thi}}{\Lambda} = \frac{v_{thi}}{\pi r} B_p \sim \frac{v_{thi}}{\ell_{thi}}, \quad \tau_c.
\]

\( \ell_y > 0.0 \) (see Ref. [29]).

The balance \( \tau_{st} \sim \tau_c \) implies that the poloidal correlation scale is \( \ell_y/\rho_i \sim \Lambda/L_s \), where \( L_s = \min(L_T, L_n) \) [5]. This is tested in Fig. 3(a), showing that, while the two quantities are certainly of the same order, we do not have enough of a range of equilibrium parameters to state conclusively that this theoretically predicted scaling works.

**Magnetic drift time and radial correlation scale.—** The time scale of the magnetic (\( \nabla B \) and curvature) drifts is

\[
\tau_{s}^{-1} = \frac{\rho_{thi}}{\Lambda} = \frac{v_{thi}}{\pi r} B_p \sim \frac{v_{thi}}{\ell_{thi}}, \quad \tau_c.
\]
where we have assumed that the scale length of the background magnetic field is $R$ (major radius at the viewing location) and $\ell_x < \ell_y$ (this will shortly prove correct). It is clear that this scale cannot be shorter than $\tau_e$, because damping due to the drift resonance would eliminate such fluctuations. While magnetic drift physics may matter (in a torus, curvature contributes to the ITG drive [4]), it does not have to affect scalings, as, e.g., it did not in the numerical simulations of [5]. In contrast, Fig. 2(c) shows that, in the MAST discharges we have analyzed, $\tau_M$ is not negligible and scales with $\tau_e$, similarly to $\tau_r$ and $\tau_a$. We find $\tau_M = (1.6 \pm 0.7) \tau_e$. Thus, a “grand critical balance” appears to hold in MAST, viz., $\tau_c \sim \tau_r \sim \tau_a \sim \tau_M$.

This suggests that the balance of relevant time scales determines correlation lengths of the turbulence in all three spatial directions. Indeed, balancing $\tau_M \sim \tau_{nl}$, we find the radial correlation scale $\ell_x / \rho_i \sim \Lambda / R$, the scaling tested in Fig. 3(b), with a degree of success. Thus, density fluctuations in MAST are not isotropic in the perpendicular plane but elongated poloidally: $\ell_y / \ell_x \sim R / L_x \sim 5$ in our data. Interestingly, this clashes with the approximate isotropy ($\ell_x \sim \ell_y$) reported in Cyclone Base Case simulations [5] and in measured DIII-D tokamak turbulence (where $\ell_x / \ell_y \sim 1.4$ [35] and $\ell_x$ appears independent of $B_p$ [36]). Whether this is a difference between spherical and conventional tokamaks is not as yet clear (cf. Ref. [37]).

**Nonlinear time.—**Since we know the fluctuation amplitude, we can directly estimate the time scale associated with the advection of the fluctuations ($\Delta u \sim \mathbf{V} \delta n$) by the fluctuating $E \times B$ velocity $\Delta u / c B = \mathbf{V} \phi / B^2$. The electrostatic potential $\phi$ is not directly measured but can be estimated by assuming the Boltzmann response of the electrons: $\delta n / n = e \phi / T_e$. This estimate ignores trapped particles and, more importantly, as we are about to argue, also does not apply to ion-scale zonal flows (poloidally and toroidally symmetric perturbations of $\phi$ with $\delta n = 0$ [38,39]). Thus, the nonzonal nonlinear time is

$$\tau_{nl}^{-1} = \frac{\nu_{hi} \rho_i}{\ell_x} \frac{T_e}{T_i} \frac{\delta n}{n}.$$  

Figure 4(a) shows that $\tau_{nl}^{-1}$ is always larger than $\tau_e$ (or the other time scales discussed above) and, furthermore, observed to have an inverse rather than a direct correlation with it. Since turbulence clearly cannot be saturated by linear physics alone, this means that our estimate does not capture the correct nonlinear time. We conjecture that it is in fact the coupling to the zonal flows, invisible (directly) to BES (because their $\delta n = 0$), that dominates over the nonlinear interaction between the drift-wave-like fluctuations represented by $\tau_{nl}^{-1}$ [38,40–46]. It has long been suspected that the relative amplitude of the zonal flows compared to that of the drift waves depends on the ion collisionality [38,47–50]. We can test this expectation by assuming that $\tau_e$ is the characteristic time associated with the coupling of the drift waves to the zonal flows and so depends on their amplitude. Figure 4(b) indeed shows a strong collisionality dependence: using the MPFIT procedure [31], $\tau_e / \tau_{nl}^{-1} \sim \nu_{ei}^{0.8 \pm 0.1}$, where $\nu_{ei} = \nu_{ji} \tau_{ei}$. (The ion collision time itself $\nu_{ei}^{-1}$ is at least an order of magnitude longer than the time scales that participate in the “grand critical balance.”) If $\tau_e^{-1} \sim (\nu_{hi} \rho_i / \ell_x \nu_i \tau_{ei}) e \phi / T_i$, where $\phi^{ZF}$ (or the amplitude of the zonal potential, this result implies that the ratio of the zonal to the nonzonal components of the turbulence is $\phi^{ZF} / \phi^{NL} \sim \nu_{ei}^{-0.8 \pm 0.1}$ [51].

We note that this situation is qualitatively distinct from what is seen in numerical simulations of ITG turbulence far from the threshold [5], where the drift-wave nonlinearity appears to dominate ($\tau_{nl}^{-1} \sim \tau_c$). However, the turbulence in a real tokamak is likely to be close to marginal and so possibly in the state of reduced transport controlled by weakly collisionally damped zonal flows [39] and usually associated with the so-called “Dimits upshift” of the stiff-transport threshold [43,44,50,52].

**Discussion.—**Our results support the notion that the statistics of turbulence are determined by the local equilibrium properties of the plasma [53]. We find little correlation between the quantities reported above and the radial location [54]. (Note that we have limited our consideration to temporal and spatial scales and did not touch on the fluctuation amplitudes or transport properties, which do of course depend on radius.) Our results also appeared insensitive to (i.e., not measurably correlated with) three other parameters that might in principle have proven important: $T_i / T_e$ (varied between 0.5 and 2), the magnetic shear $\tilde{\chi} = d \ln q / d \ln r$ (varied between $-1$ and 5), and the perpendicular component of the toroidal velocity shear $\tau_{sh}^{-1} = (B_p / B) d U_b / d r$. In much of our data, $\tau_{sh} \geq \tau_e$, $\tau_{sh}$ [see Fig. 2(d)], so it stands to reason that the statistics of the turbulence would not be dramatically affected; in the instances of $\tau_{sh} \sim \tau_{nl}$, the effect of $\tau_{sh}$ could not be isolated [55]. It would be interesting to investigate higher-rotation plasmas, as $\tau_{sh}^{-1}$, when sufficiently large, is expected to
have a dramatic effect on transport [41,56–62]; even in our database, there is in fact some evidence that velocity shear might raise the critical temperature gradients [63], but we see no signature of this effect in the correlation properties of the turbulence.

Conclusion.—We have presented experimental results that are statistically consistent with a turbulent state in MAST set by the local equilibrium and in which the time scales of the linear drive, turbulence decorrelation, ion streaming, and magnetic drifts are all similar and scale together as equilibrium parameters are varied. This “grand critical balance” implies a three-dimensionally anisotropic turbulence, with parallel, poloidal, and radial correlation lengths having different parameter dependences and $\ell_{\parallel} \gg \ell_y \gg \ell_x$. Our results also suggest the presence of a zonal component with an amplitude $n_{\|}^{0.8 \pm 0.1}$ greater than the drift-wave density fluctuations. Note that these results are entirely consistent with a drift-wave turbulence obeying the gyrokinetic ordering [5,53,64] but provide a more detailed view of the dependence of this turbulence on the local equilibrium parameters.


*Present address: Department of Nuclear and Quantum Engineering, Korea Advanced Institute of Science and Technology, Deajeon, 305-701, Republic of Korea. ycghim@kaist.ac.kr

[33] However, for $\tau_e \leq 10 \mu$ sec, $\tau_{\|} \sim \tau_{\perp}$, and, for $\tau_e \geq 10 \mu$ sec, $\tau_{\parallel} \sim \tau_{\perp}$, so we cannot rule out the ion-scale electron drive.
In a conventional tokamak, $\Lambda \approx \pi qR$, where $q$ is the safety factor and $R$ the major radius, but, in a spherical tokamak, our local estimate is more appropriate.

There is a slight bias in Fig. 4 for larger $\nu_{st}$ to be found farther from the magnetic axis.

In general, we expect that a strong velocity shear would change $\ell_{||}$ via a modified critical balance: If $\tau_{th} < \tau_{ri}$, then $\tau_c \sim \tau_{ab} \sim \ell_{||}/v_{thi}$, so $\ell_{||} \sim v_{thi} \tau_{ab} < \Lambda$.

A scaling popular in theoretical models of zonal-flow ITG turbulence is $\nu_{ri}^{-1/2}$ [38].

A scaling popular in theoretical models of zonal-flow ITG turbulence is $\nu_{ri}^{-1/2}$ [38].

[54] There is a slight bias in Fig. 4 for larger $\nu_{st}$ to be found farther from the magnetic axis.

In general, we expect that a strong velocity shear would change $\ell_{||}$ via a modified critical balance: If $\tau_{th} < \tau_{ri}$, then $\tau_c \sim \tau_{ab} \sim \ell_{||}/v_{thi}$, so $\ell_{||} \sim v_{thi} \tau_{ab} < \Lambda$.

[55] In general, we expect that a strong velocity shear would change $\ell_{||}$ via a modified critical balance: If $\tau_{th} < \tau_{ri}$, then $\tau_c \sim \tau_{ab} \sim \ell_{||}/v_{thi}$, so $\ell_{||} \sim v_{thi} \tau_{ab} < \Lambda$.

In general, we expect that a strong velocity shear would change $\ell_{||}$ via a modified critical balance: If $\tau_{th} < \tau_{ri}$, then $\tau_c \sim \tau_{ab} \sim \ell_{||}/v_{thi}$, so $\ell_{||} \sim v_{thi} \tau_{ab} < \Lambda$.