Measurement of CP asymmetries and branching fractions in charmless two-body B-meson decays to pions and kaons

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Measurement of $CP$ asymmetries and branching fractions in charmless two-body $B$-meson decays to pions and kaons

We present improved measurements of CP-violation parameters in the decays $B^0 \rightarrow \pi^+ \pi^-$, $B^0 \rightarrow K^+ \pi^-$, and $B^0 \rightarrow \pi^0 \pi^0$, and of the branching fractions for $B^0 \rightarrow \pi^0 \pi^0$ and $B^0 \rightarrow K^0 \pi^0$. The results are obtained with the full data set collected at the Y(4S) resonance by the BABAR experiment at the PEP-II asymmetric-energy $B$ factory at the SLAC National Accelerator Laboratory, corresponding to $(467 \pm 5) \times 10^6$ $B\bar{B}$ pairs. We find the CP-violation parameter values and branching fractions: $S_{\pi^+ \pi^-} = -0.68 \pm 0.10 \pm 0.03$, $C_{\pi^+ \pi^-} = -0.25 \pm 0.08 \pm 0.02$, $\lambda_{K^+ \pi^-} = -0.107 \pm 0.016 \pm 0.006$, $C_{\pi^0 \pi^0} = -0.43 \pm 0.26 \pm 0.05$, $B(B^0 \rightarrow \pi^0 \pi^0) = (1.83 \pm 0.21 \pm 0.13) \times 10^{-6}$, $B(B^0 \rightarrow K^0 \pi^0) = (10.1 \pm 0.6 \pm 0.4) \times 10^{-6}$, where in each case, the first uncertainties are statistical and the second are systematic. We observe CP violation with a significance of 6.7 standard deviations for $B^0 \rightarrow \pi^+ \pi^-$ and 6.1 standard deviations for $B^0 \rightarrow K^+ \pi^-$, including systematic uncertainties. Constraints on the unitarity triangle angle $\alpha$ are determined from the isospin relations among the $B \rightarrow \pi \pi$ rates and asymmetries. Considering only the solution preferred by the Standard Model, we find $\alpha$ to be in the range $[71^\circ, 109^\circ]$ at the 68% confidence level.

I. INTRODUCTION

Large $CP$-violating effects [1] in the $B$-meson system are among the most remarkable predictions of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing model [2]. These predictions have been confirmed by the BABAR and Belle Collaborations, most precisely in $b \to c\bar{c}s$ decays of $B^0$ mesons to $CP$ eigenstates [3,4].

Effective constraints on physics beyond the Standard Model (SM) are provided by high-precision measurements of quantities whose SM predictions are subject to only small theoretical uncertainties. Many experimental and theoretical uncertainties partially cancel in the calculation of $CP$-violating asymmetries. This makes $CP$-violation measurements a sensitive probe for effects of yet-undiscovered additional interactions and heavy particles that are introduced by extensions to the SM. All measurements of $CP$ violation to date, including those involving the decay modes studied here [5–9], are in agreement with the indirect predictions from global SM fits [10,11], which are based on measurements of the magnitudes of the elements $V_{ij}$ of the CKM quark-mixing matrix. This strongly constrains [12] the flavor structure of SM extensions.

The CKM-matrix unitarity triangle angle $\alpha \equiv \arg[-V_{u3}V_{d3}^*/V_{u3}V_{d3}]$ is measured through interference between two decay amplitudes, where one amplitude involves $B^0$-$\bar{B}^0$ mixing. Multiple measurements of $\alpha$, with different decays, further test the consistency of the CKM model. The time-dependent asymmetry in $B^0 \to \pi^+\pi^-$ decays is proportional to $\sin 2\alpha$ in the limit that only the $b \to u$ ("tree") quark-level amplitude contributes to this decay. In the presence of $b \to d$ ("penguin") amplitudes, the time-dependent asymmetry in $B^0 \to \pi^+\pi^-$ is modified to

$$a(\Delta t) = \frac{|\bar{A}(\Delta t)|^2 - |A(\Delta t)|^2}{|\bar{A}(\Delta t)|^2 + |A(\Delta t)|^2} = S_{\pi^+\pi^-} \sin(\Delta m_d \Delta t) - C_{\pi^+\pi^-} \cos(\Delta m_d \Delta t),$$

where $\Delta t$ is the difference between the proper decay times of the $B$ meson that undergoes the $B \to \pi^+\pi^-$ decay (the signal $B$) and the other $B$ meson in the event (the tag $B$), $\Delta m_d$ is the $B^0$-$\bar{B}^0$ mixing frequency, $A$ is the $B^0 \to \pi^+\pi^-$ decay amplitude, $\bar{A}$ is the $CP$-conjugate amplitude, and

$$C_{\pi^+\pi^-} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2},$$

$$S_{\pi^+\pi^-} = \sqrt{1 - C_{\pi^+\pi^-}^2} \sin(2\alpha - 2\Delta\alpha_{\pi\pi}).$$

Both the direct $CP$ asymmetry $S_{\pi^+\pi^-}$ and the phase $\Delta\alpha_{\pi\pi}$ may differ from zero due to the penguin contribution to the decay amplitudes.

The magnitude and relative phase of the penguin contribution to the asymmetry $S_{\pi^+\pi^-}$ may be determined with an analysis of isospin relations between the $B \to \pi\pi$ decay amplitudes [13]. The amplitudes $A_{ij}^\pm$ of the $B \to \pi^+\pi^\mp$ decays and $\bar{A}_{ij}$ of the $\bar{B} \to \pi^+\pi^\mp$ decays satisfy the relations

$$A^+ = \frac{1}{\sqrt{2}} A^{+-} + A^{00}, \quad A^- = \frac{1}{\sqrt{2}} A^{+-} + A^{00}.$$
an electromagnetic calorimeter (EMC) consisting of 6580 CsI(Tl) crystals. The photon energy resolution is \( \sigma_E/E = (2.3/E(\text{GeV})^{1/4} \times 1.4)\% \), and the photon angular resolution relative to the interaction point is \( \sigma_\theta = 4.16/\sqrt{E(\text{GeV})} \) mrad [24].

The data used in this analysis were collected during the period 1999–2007 with the BABAR detector at the PEP-II asymmetric-energy B-meson factory at the SLAC National Accelerator Laboratory. A total of \((467 \pm 5) \times 10^6 \) \( B\bar{B} \) pairs were used. Relative to previous BABAR measurements [5–7], roughly 22% more \( B\bar{B} \) pairs have been added to the analyzed data set, and improvements have been introduced to the analysis technique, boosting the signal significance. These improvements include better reconstruction of charged-particle tracks, improved hadronidentification and flavor-tagging algorithms, and optimal selection of tracks and calorimeter clusters for calculation of event-shape variables.

Samples of Monte Carlo (MC) simulated events are analyzed with the same reconstruction and analysis procedures as used for the data, following a Geant4-based [25] detailed detector simulation [23]. The MC samples include \( e^+e^- \rightarrow q\bar{q} \) continuum background events generated with JETSET [26] and \( Y(4S) \rightarrow B\bar{B} \) decays generated with EvtGen [27] and JETSET, including both signal and background B-meson decays.

### III. EVENT SELECTION AND ANALYSIS METHOD

Many elements of the measurements discussed in this paper are common to the decay modes [28] \( B^0 \rightarrow h^+h^- \) (where \( h, h' = \pi \) or \( K \)), \( B^0 \rightarrow \pi^0\pi^0 \), and \( B^0 \rightarrow K^0_S\pi^0 \). The signal B-meson candidates \( (B_{\text{rec}}) \) are formed by combining two particles, each of which is a charged-particle track, a \( \pi^0 \) candidate, or a \( K^0_S \) candidate. The event selection differs for each mode and is described below.

The number of \( B \) decays and the corresponding CP asymmetries are determined with extended unbinned maximum likelihood (ML) fits to variables described below. The likelihood is given by the expression

\[
L = \exp \left( -\sum_{i}^{M} n_{i} \prod_{j}^{N} \sum_{\tilde{\alpha}} P_{\tilde{\alpha}}(\tilde{x}_{j}; \tilde{\alpha}_{i}) \right),
\]

where \( N \) is the number of events, the sums are over the event categories \( M \), \( n_{i} \) is the event yield for each category as described below, and the probability-density function (PDF) \( P_{\tilde{\alpha}} \) describes the distribution of the variables \( \tilde{x}_{j} \) in terms of parameters \( \tilde{\alpha}_{i} \). The PDF functional forms are discussed in Secs. III C and III D.

### A. Track and \( K^0_S \) selection

In the \( B^0 \rightarrow h^+h^- \) mode, we require charged-particle tracks to have at least 12 DCH hits and to lie in the polar-angle region \( 0.35 < \theta < 2.40 \) with respect to the beam direction. The track impact parameter relative to the \( e^+e^- \) collision axis must be smaller than 1.5 cm in the plane perpendicular to the beam axis and 2.5 cm in the direction along the axis.

In order for DIRC information to be used for particle identification, we require that each track have its associated Cherenkov angle \( (\theta_C) \) measured with at least six Cherenkov photons, where the value of \( \theta_C \) is required to be within 4.0 standard deviations \( (\sigma) \) of either the pion or kaon hypothesis. This removes candidates containing a high-momentum proton. Tracks from electrons are removed based primarily on a comparison of the track momentum and the associated energy deposition in the EMC, with additional information provided by DCH \( dE/dx \) and DIRC \( \theta_C \) measurements.

The ionization energy loss in the DCH is used either in combination with DIRC information or alone. This leads to a 35% increase in the \( B^0 \rightarrow h^+h^- \) reconstruction efficiency relative to the use of only tracks with good DIRC information. A detailed DCH \( dE/dx \) calibration developed for the \( B^0 \rightarrow h^+h^- \) analysis takes into account variations in the mean and resolution of \( dE/dx \) measurement values with respect to changes in the DCH running conditions over time, as well as the track’s charge, polar and azimuthal angles, and number of ionization samples. The calibration is performed with large high-purity samples (with more than \( 10^6 \) events) of protons from \( \Lambda \rightarrow p\pi^- \), pions and kaons from \( D^+ \rightarrow D^0\pi^+ (D^0 \rightarrow K^-\pi^+) \), and \( K^0_S \rightarrow \pi^0\pi^- \) decays that occur in the vicinity of the interaction region.

Candidates for the decay \( K^0_S \rightarrow \pi^+\pi^- \) are reconstructed from pairs of oppositely charged tracks. The two-track combinations are required to form a vertex with a \( \chi^2 \) probability greater than 0.001 and a \( \pi^+\pi^- \) invariant mass within 11.2 MeV/c\(^2\), corresponding to 3.7\( \sigma \), of the nominal \( K^0_S \) mass [29].

### B. \( \pi^0 \) selection

We form \( \pi^0 \rightarrow \gamma\gamma \) candidates from pairs of clusters in the EMC that are isolated from any charged track. Clusters are required to have a lateral profile of energy deposition consistent with that of a photon and to have an energy \( E_{\gamma} > 30 \) MeV for \( B^0 \rightarrow \pi^0\pi^0 \) and \( E_{\gamma} > 50 \) MeV for \( B^0 \rightarrow K^0_S\pi^0 \). We require \( \pi^0 \) candidates to lie in the invariant-mass range \( 110 < m_{\gamma\gamma} < 160 \) MeV/c\(^2\).

For the \( B^0 \rightarrow \pi^0\pi^0 \) mode, we also use \( \pi^0 \) candidates from a single EMC cluster containing two adjacent photons (a merged \( \pi^0 \)), or one EMC cluster and two tracks from a photon conversion to an \( e^+e^- \) pair inside the detector. To reduce the background from random photon combinations, the angle \( \theta_{\gamma} \) between the photon momentum vector in the \( \pi^0 \) rest frame and the \( \pi^0 \) momentum vector in the laboratory frame is required to satisfy \( |\cos\theta_{\gamma}| < 0.95 \). The \( \pi^0 \) candidates are fitted
kinematically with their mass constrained to the nominal \( \pi^0 \) mass [29].

Photon conversions are selected from pairs of oppositely charged electron-electron tracks with an invariant mass below 30 MeV/c\(^2\) whose combined momentum vector points away from the beam spot. The conversion point is required to lie within detector material layers. Converted photons are combined with photons from single EMC clusters to form \( \pi^0 \) candidates.

Single EMC clusters containing two photons are selected with the transverse second moment, \( S = \sum E_i \times (\Delta \alpha_i)^2/E_i \), where \( E_i \) is the energy in each CsI(Tl) crystal and \( \Delta \alpha_i \) is the angle between the cluster centroid and the crystal. The second moment is used to distinguish merged \( \pi^0 \) candidates from both single photons and neutral hadrons.

**C. \( B^0 \to \pi^+ \pi^- \), \( B^0 \to K^+ \pi^- \), and \( B^0 \to \pi^0 \pi^0 \)**

Two kinematic variables are used in the \( B^0 \to h^+ h^- \) and \( B^0 \to \pi^0 \pi^0 \) analyses to separate \( B \)-meson decays from the large \( e^+ e^- \to q\bar{q}(q = u, d, s, c) \) combinatoric background [23]. One variable is the beam-energy-substituted mass \( m_{ES} = \sqrt{s/2 + p_B^2/E_B^2 - p_B^2} \), where \( \sqrt{s} \) is the total \( e^+ e^- \) c.m. energy, \( (E_i, p_i) \) is the four-momentum of the initial \( e^+ e^- \) system in the laboratory frame, and \( p_B \) is the laboratory momentum of the \( B \) candidate. The second variable is \( \Delta E = E_B^0 - \sqrt{s}/2 \), where \( E_B^0 \) is the energy of the \( B \) candidate in the c.m. frame.

To further separate \( B \) decays from the \( q\bar{q} \) background, we use two additional topological variables that take advantage of the two-jet nature of \( q\bar{q} \) events and the isotropic particle distribution of \( e^+ e^- \to BB \) events. The first variable is the absolute value of the cosine of the angle \( \theta_3 \) between the sphericity axis [30] of the decay products of the \( B \) candidate and the sphericity axis of the remaining tracks and neutral clusters in the event, computed in the c.m. frame. The distribution of this variable peaks at 1 for the jetlike \( q\bar{q} \) events and is uniform for \( B \) decays. We require \( |\cos \theta_3| < 0.91 \) for \( B^0 \to h^+ h^- \) and \( |\cos \theta_3| < 0.7 \) for \( B^0 \to \pi^0 \pi^0 \), a tighter requirement is needed due to the higher background. For the \( B^0 \to h^+ h^- \) mode, we remove a small remaining background from \( e^+ e^- \to \pi^+ \pi^- \) events by further requiring that the normalized second Fox-Wolfram moment [31] satisfy \( R_2 < 0.7 \).

To improve the discrimination against \( q\bar{q} \) events, a Fisher discriminant \( F \) is formed as a linear combination of the sums \( L_0^+ = \sum |p_i^+| \) and \( L_0^- = \sum |p_i^-| \cos^2 \theta_i^* \), where \( p_i^+ \) are the momenta and \( \theta_i^* \) are the angles with respect to the thrust axis [32] of the \( B \) candidate, both in the c.m. frame, of all tracks and clusters not used to reconstruct the signal \( B \)-meson candidate. The \( F \) variable takes advantage of the fact that much of the momentum flow in \( q\bar{q} \) events is along the thrust axis. In the case of \( B^0 \to \pi^0 \pi^0 \), we improve the sensitivity to signal events by combining \( F \) with three other event-shape variables in a neural network. The first variables is \(|\cos \theta_3|\), described above. The second is \(|\cos \theta_3^*|\), where \( \theta_3^* \) is the angle between the momentum vector of the signal \( B \) and the beam axis. The \( |\cos \theta_3^*|\) distribution of \( q\bar{q} \) events is uniform, while that of signal events is proportional to \( \sin^2 \theta_3^* \). The third variable is \(|\cos \theta_{K}^*|\), where \( \theta_{K}^* \) is the angle between the thrust axis of the signal \( B \)-meson’s daughters and the beam axis. Both \( \theta_3^* \) and \( \theta_{K}^* \) are calculated in the c.m. frame. The characteristics of the \( |\cos \theta_{K}^*|\) distributions are similar to those of \( |\cos \theta_3|\).

1. **\( B^0 \to \pi^+ \pi^- \) and \( B^0 \to K^+ \pi^- \)**

We reconstruct the candidate decays \( B_{\text{rec}} \to h^+ h^- \) from pairs of oppositely charged tracks that are consistent with originating from a common decay point with a \( \chi^2 \) probability of at least 0.001. The remaining particles are examined to infer whether the other \( B \) meson in the event (\( B_{\text{tag}} \)) decayed as a \( B^0 \) or \( \bar{B}^0 \) (flavor tag). We perform an unbinned extended ML fit to separate \( B^0 \to \pi^+ \pi^- \) and \( B^0 \to K^+ \pi^- \) decays and determine simultaneously their \( CP \)-violating asymmetries \( S_{\pi^+ \pi^-}, C_{\pi^+ \pi^-}, \) and

\[
A_{K^- \pi^-} = \frac{B(B \to K^- \pi^-) - B(B \to K^+ \pi^-)}{B(B \to K^- \pi^-) + B(B \to K^+ \pi^-)},
\]

as well as the signal and background yields and PDF parameters. The fit uses \( \theta_C, dE/dx, \Delta E, m_{ES}, F, B_{\text{tag}} \) flavor, and \( \Delta \) information.

The value of \( \Delta E \) is calculated assuming that both tracks are charged pions. The \( B^0 \to \pi^+ \pi^- \) signal is described by a Gaussian distribution for \( \Delta E \), with a resolution of 29 MeV. For each kaon in the final state, the \( \Delta E \) peak position is shifted from zero by an amount that depends on the kaon momentum, with an average shift of \(-45\) MeV. We require \( | \Delta E | < 0.150 \) GeV. The wide range in \( \Delta E \) allows us to separate \( B^0 \) decays to the four final states \( \pi^+ \pi^- \), \( K^+ \pi^- \), \( \pi^+ K^- \), and \( K^+ K^- \) in a single fit. The analysis is not optimized for measuring the \( K^+ K^- \) final state, which is treated as background. The \( m_{ES} \) resolution is 2.6 MeV/c\(^2\). We require \( m_{ES} > 5.20 \) GeV/c\(^2\), with events in the large range below the signal peak allowing the fit to effectively determine the background shape parameters.

We construct \( \theta_C \) PDFs for the pion and kaon hypotheses, and \( dE/dx \) PDFs for the pion, kaon, and proton hypotheses, separately for each charge. The \( K^- \pi^- \) separations provided by \( \theta_C \) and \( dE/dx \) are complementary: for \( \theta_C \), the separation varies from \( 2.5\sigma \) at 4.5 GeV/c to \( 13\sigma \) at 1.5 GeV/c, while for \( dE/dx \) it varies from less than 1.0\sigma at 1.5 GeV/c to 1.9\sigma at 4.5 GeV/c (Fig. 1). For more details, see Ref. [5].

We use a multivariate technique [33] to determine the flavor of the \( B_{\text{tag}} \). Separate neural networks are trained to identify leptons from \( B \) decays, kaons from \( D \) decays, and soft pions from \( D^* \) decays. Events are assigned to one of seven mutually exclusive tagging categories (one category being untagged events) based on the estimated average
FIG. 1 (color online). The average expected $K$-$\pi$ separation, in units of uncertainty, provided by the DIRC angle $\theta_C$ and DCH $dE/dx$ for kaons and pions from $B^0 \rightarrow K^+\pi^-$ decays in the laboratory-frame polar angle range $0.35 < \theta < 2.40$, as a function of laboratory-frame momentum.

mistag probability and the source of the tagging information. The quality of tagging is expressed in terms of the effective efficiency $Q = \sum \epsilon_k (1 - 2w_k)^2$, where $\epsilon_k$ and $w_k$ are the efficiencies and mistag probabilities, respectively, for events tagged in category $k$. The difference between the mistag probabilities for $B^0$ and $\bar{B}^0$ mesons is given by $\Delta w = w_\text{pK} - w_\text{p}\bar{K}$. Table 1 summarizes the tagging performance measured in a large data sample of fully reconstructed neutral $B_{\text{flav}}$ decays to $D^{(*)}(\pi^+\pi^-)$, $\rho^+\pi^-$, and $K^+\cdots$ background events, where the (anti)proton has no $\text{DIRC}$ information. The $K^\pm \pi^\mp$ event yields $n_{K^\pm\pi^\mp}$ are parametrized in terms of the asymmetry $A_{\text{raw}}^{K^\pm\pi^\mp}$ and average yield $n_{K^\mp\pi^\pm}$ as $n_{K^\mp\pi^\pm} = n_{K^\pm\pi^\mp}(1 + A_{\text{raw}}^{K^\pm\pi^\mp})/2$. All other event yields are products of the fraction of events in each tagging category taken from $B_{\text{flav}}$ events, and the total event yield. The background PDFs are a threshold function [36] for $m_{ES}$ and a second-order polynomial for $\Delta E$. The $J^*$ PDF is a sum of two asymmetric Gaussians for both signal and background. We use large samples of simulated $B$ decays to investigate the effects of backgrounds from other $B$ decays on the determination of the $CP$-violating asymmetries in $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow K^+\pi^-$, and find them to be negligible.

2. $B^0 \rightarrow \pi^0\pi^0$

$B^0 \rightarrow \pi^+\pi^-$ events are identified with a ML fit to the variables $m_{ES}$, $\Delta E$, and the output $NN$ of the event-shape neural network. We require $m_{ES} > 5.20 \text{ GeV}/c^2$ and $|\Delta E| < 0.2 \text{ GeV}$. Since tails in the EMC response produce a correlation between $m_{ES}$ and $\Delta E$, a two-dimensional binned PDF derived from the signal MC sample is used to describe signal PDF. The $NN$ distribution is divided into ten bins (with each bin approximately equally populated by signal events) and described by a nine-bin step-function PDF with values taken from the MC and fixed in the fit. $B_{\text{flav}}$ data are used to verify that the MC accurately reproduces the $NN$ distribution. The $q\bar{q}$ background PDFs are a threshold function [36] for $m_{ES}$, a second-order polynomial for $\Delta E$, and a parametric step function for $NN$. For $q\bar{q}$ events, $NN$ is not distributed uniformly across the bins but rises sharply toward the highest bins. We see a small correlation of 2.5% between the shape parameter of the $m_{ES}$ threshold function and the $NN$ bin number, and this relation is taken into account in the fit. All $q\bar{q}$ background PDF-parameter values are determined by the ML fit.

where $f^k_\pm (f^k_\mp)$ indicates a $B^0 (\bar{B}^0)$ flavor tag and the index $k$ indicates the tagging category. The resolution function $R(\Delta t_{\text{meas}} - \Delta t)$ for signal candidates is a sum of three Gaussian functions, identical to the one described in Ref. [35], with parameters determined from a fit to the $B_{\text{flav}}$ sample, which includes events in all seven tagging categories. The background $\Delta t$ distribution is modeled as the sum of three Gaussians, with parameters, common for all tagging categories, determined simultaneously with the $CP$-violation parameters in the ML fit to the $B_{\text{rec}} \rightarrow h^+h'^-$ sample.

The ML fit PDF includes 28 components. Of these, 24 components correspond to $B^0$ signal decays and background events with the final states $\pi^+\pi^-$, $K^+\pi^-$, $K^-\pi^+$, and $K^+K^-$, where either the positively charged track, the negatively charged track, or both have good $\text{DIRC}$ information (2 x 4 x 3 = 24 components). Four additional components correspond to $p\pi^-$, $pK^-$, $\pi^+\bar{p}$, and $K^-\bar{p}$ background events, where the (anti)proton has no $\text{DIRC}$ information. The $K^\pm \pi^\mp$ event yields $n_{K^\pm\pi^\mp}$ are parametrized in terms of the asymmetry $A_{\text{raw}}^{K^\pm\pi^\mp}$ and average yield $n_{K^\mp\pi^\pm}$ as $n_{K^\mp\pi^\pm} = n_{K^\pm\pi^\mp}(1 + A_{\text{raw}}^{K^\pm\pi^\mp})/2$. All other event yields are products of the fraction of events in each tagging category taken from $B_{\text{flav}}$ events, and the total event yield. The background PDFs are a threshold function [36] for $m_{ES}$ and a second-order polynomial for $\Delta E$. The $J^*$ PDF is a sum of two asymmetric Gaussians for both signal and background. We use large samples of simulated $B$ decays to investigate the effects of backgrounds from other $B$ decays on the determination of the $CP$-violating asymmetries in $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow K^+\pi^-$, and find them to be negligible.

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<th>Category</th>
<th>$\epsilon(%)$</th>
<th>$w(%)$</th>
<th>$\Delta w(%)$</th>
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<td>LEPTON</td>
<td>8.96 ± 0.07</td>
<td>2.9 ± 0.3</td>
<td>0.2 ± 0.5</td>
<td>7.95 ± 0.11</td>
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<td>0.0 ± 0.6</td>
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<td>KAO PION</td>
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<td>23.3 ± 0.4</td>
<td>-0.6 ± 0.7</td>
<td>3.91 ± 0.12</td>
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<tr>
<td>PION</td>
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<td>5.1 ± 0.7</td>
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<tr>
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<td>3.8 ± 0.8</td>
<td>0.28 ± 0.04</td>
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<tr>
<td>Total</td>
<td>31.1 ± 0.3</td>
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</table>
The decays $B^+ \to \rho^+ \pi^0$ and $B^0 \to K_S^0 \pi^0 (K^0_S \to \pi^0 \pi^0)$ add $71 \pm 10$ background events to $B^0 \to \pi^0 \pi^0$ and are included as an additional component in the ML fit. We model these $B$-decay background events with a two-dimensional binned PDF in $m_{KS}$ and $\Delta E$, and with a step function for $N/N$. The shapes of these PDFs are taken from MC simulation, and their event yields and asymmetries are fixed in the fit and are later varied to evaluate systematic uncertainties.

The time-integrated $CP$ asymmetry is measured by the $B$-flavor tagging algorithm described above. The fraction of events in each tagging category is constrained to the corresponding fraction determined from MC simulation. The PDF event yields for the $B^0 \to \pi^0 \pi^0$ signal are given by the expression

$$n_{\pi^0 \pi^0,k} = \frac{1}{2} f_k N_{\pi^0 \pi^0}[1 - s_j(1 - 2\chi)(1 - 2w_k)C_{\pi^0 \pi^0}],$$

where $f_k$ is the fraction of events in tagging category $k$, $N_{\pi^0 \pi^0}$ is the number of $B^0 \to \pi^0 \pi^0$ candidate decays, $\chi$ is the time-integrated $B^0$ mixing probability [29], $s_j = +1(-1)$ when the $B_{tag}$ is a $B^0$ ($\bar{B}^0$), and

$$C_{\pi^0 \pi^0} = \frac{|A^{0}\pi^0|^2 - |\bar{A}^{0}\pi^0|^2}{|A^{0}\pi^0|^2 + |\bar{A}^{0}\pi^0|^2}$$

is the direct $CP$ asymmetry in $B^0 \to \pi^0 \pi^0$.

**D. $B^0 \to K^0_S \pi^0$**

$CP$-violation parameters for $B^0 \to K^0_S \pi^0$ have been reported in Ref. [4]. Here we describe the measurement of the branching fraction for this mode.

For each $B^0 \to K^0_S \pi^0$ candidate, two independent kinematic variables are computed. The first variable is the invariant mass $m_B$ of the $B_{rec}$. The second variable is the invariant (missing) mass $m_{miss}$ of the $B_{tag}$, computed from the difference in the $m_B$ of the candidate decays and the decay no charged particles originate from the decay vertex, as in the reconstructed $K_S^0$ decay vertex and momentum vector, and the average $e^+e^-$ interaction point [38]. We have verified that all correlations between the fit variables are negligible and so construct the likelihood function as a product of one-dimensional PDFs. Residual correlations are taken into account in the systematic uncertainty, as explained below.

The PDFs for signal events are parametrized based on a large sample of fully reconstructed $B$ decays in data and from simulated events. For background PDFs, we take the functional form from the background-dominated sideband regions in the data. The likelihood function is

$$L(S_{K^0_S \pi^0}, C_{K^0_S \pi^0}, N_S, N_B, f_s f_B, \bar{a})$$

$$= e^{-(N_S + N_B)} N! \prod_{i \in g} (N_S f_s^g e_s^g P_S(\bar{x}_i; \bar{y}_i; S_{K^0_S \pi^0}, C_{K^0_S \pi^0}))$$

$$+ N_B f_B^g e_B^g P_B(\bar{x}_i; \bar{y}_i; \bar{a}) \prod_{i \in b} (N_S f_s^b e_s^b P_s(\bar{x}_i; C_{K^0_S \pi^0}))$$

$$+ N_B f_B^b e_B^b P_B(\bar{x}_i; \bar{a})],$$

where the $N$ selected events are partitioned into two subsets: the index $i \in g$ indicates events that have $\Delta t$ information, while $i \in b$ events do not have $\Delta t$ information. Here, $f_s^g (f_B^g)$ is the fraction of signal (background) events that are in the subset $g$, and $f_s^b = 1 - f_s^g (f_B^b = 1 - f_B^g)$ are the corresponding signal (background) fractions in the subset $b$. The parameter $N_S$ ($N_B$) is the number of signal (background) events. The probabilities $P_S$ and $P_B$...
TABLE II. Results for the $B^0 \to h^+ h^-$ decay modes. Uncertainties on the signal yields $N_{sig}$ are statistical. For the
$CP$-violation parameters, the first uncertainties are statistical, and the second are systematic.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$N_{sig}$</th>
<th>$CP$-violation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to \pi^+ \pi^-$</td>
<td>1394 ± 54</td>
<td>$S_{\pi^+ \pi^-} = -0.68 \pm 0.10 \pm 0.03$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_{\pi^+ \pi^-} = -0.25 \pm 0.08 \pm 0.02$</td>
</tr>
<tr>
<td>$B^0 \to K^+ \pi^-$</td>
<td>5410 ± 90</td>
<td>$A_{K^+ \pi^-} = -0.107 \pm 0.016 \pm 0.006$</td>
</tr>
<tr>
<td>$B^0 \to K^+ K^-$</td>
<td>7 ± 17</td>
<td></td>
</tr>
</tbody>
</table>

are products of PDFs for the signal and background hypotheses evaluated for the measurements $\hat{x}_i = \{m_B, m_{miss}, \frac{L_2}{L_0}, \cos \theta^*_B\}$, flavor tag, tagging category) and $\hat{y}_i = \{\Delta t, \sigma_\Delta\}$. The corresponding PDFs for events without $\Delta t$ information are $P^0_S$ and $P^0_B$. Detailed descriptions of $P_S$, $P_B$, $P^0_S$, and $P^0_B$ are given in Ref. [4]. The vector $\hat{\alpha}$ represents the set of parameters that define the shapes of the PDFs. Along with the $CP$ asymmetries
$S_{\pi^+ \pi^-}$ and $C_{\pi^+ \pi^-}$, the fit extracts the yields $N_S$ and $N_B$, the fraction of events $f_S$ and $f_B$, and the parameters of the background PDFs.

IV. RESULTS AND SYSTEMATIC UNCERTAINTIES

A. $B^0 \to \pi^+ \pi^-$ and $B^0 \to K^+ \pi^-$ results

The event yields and $CP$-violation parameters are listed in Table II. The correlation coefficient between $S_{\pi^+ \pi^-}$ and $C_{\pi^+ \pi^-}$ is found to be $-0.056$, and the correlation between $C_{\pi^+ \pi^-}$ and $A_{K^+ \pi^-}$ is 0.019. We show the $m_{ES}$, $\Delta E$, and $F$ distribution for the $B \to \pi \pi$, $B \to K \pi$, and $q\bar{q}$ background in Fig. 2, where the Plots [39] weighting and background-subtraction technique is used to display a distribution for a particular type of event. The direct $CP$ asymmetry in $B^0 \to K^+ \pi^-$ is apparent in the $\Delta E$ distributions, which are plotted separately for $B^0$ and $\bar{B}^0$ decays in Fig. 3. We show the distributions of $\Delta t$ for $B^0 \to K^+ \pi^-$ signal and background decays in Fig. 4. In Fig. 5, we show the distribution of $\Delta t$ separately for $B^0 \to \pi^+ \pi^-$ events

![Fig. 2](color online). Plots of (left column) $m_{ES}$, (center column) $\Delta E$, and (right column) Fisher discriminant $F$ distributions for (top row) $B^0 \to \pi^+ \pi^-$, (middle row) $B^0 \to K^+ \pi^-$, and (bottom row) $q\bar{q}$ background candidates. The points with error bars show the data, and the lines represent the PDFs used in the fit and reflect the fit result. The structure to the left of the signal $\Delta E$ peak for $B^0 \to \pi^+ \pi^-$ is consistent with the expected background from other charmless modes, which is negligible for $\Delta E > -0.10$ GeV. In the calculation of $\Delta E$ for $B^0 \to K^+ \pi^-$, the kaon candidate is assigned the pion mass.
FIG. 3 (color online). Plots of the $\Delta E$ distribution for signal $K^+\pi^-$ events comparing (blue solid lines, filled circles) $B^0$ and (red dashed lines, empty circles) $\bar{B}^0$ decays. The points with error bars show the data, and the lines represent the PDFs used in the fits and reflect the results of the fits.

tagged as $B^0$ or $\bar{B}^0$, as well as the asymmetry $a(\Delta t)$ of Eq. (1). The results for $S_{\pi^+\pi^-}$ and $C_{\pi^+\pi^-}$ are shown in Fig. 6, along with confidence-level contours corresponding to statistical significances ranging from 1$\sigma$ to 7$\sigma$. Our measurement excludes the absence of $CP$ violation in $B^0 \rightarrow \pi^+\pi^-$ ($S_{\pi^+\pi^-}=0, C_{\pi^+\pi^-}=0$) at a confidence level corresponding to 6.7$\sigma$, including systematic uncertainties.

Systematic uncertainties for the direct $CP$ asymmetry $A_{K^-\pi^+}$ are listed in Table III. Here, $A_{K^-\pi^+}$ is the fitted value of the $K^+\pi^-$ event-yield asymmetry $A_{K^-\pi^+}^{raw}$, shifted by $+0.005^{+0.005}_{-0.003}$ to account for a bias that arises from the difference between the cross sections of $K^+$ and $K^-$ hadronic interactions within the BABAR detector. We determine this bias from the MC. The bias is independently verified with a calculation based on the known material composition of the BABAR detector [23] and the cross sections and material properties tabulated in Ref. [29]. The corrected $K^+\pi^-$ event-yield asymmetry in the background where no observable $CP$ violation is expected is $-0.005 \pm 0.004^{(stat)}\pm 0.003^{(syst)}$ consistent with zero. Uncertainties on the $\theta_C$ and $dE/dx$ distributions are obtained from the $D^0 \rightarrow K^-\pi^+$ control sample, and contribute 0.002 to the systematic uncertainty on $A_{K^-\pi^+}$.

An additional uncertainty of the same magnitude is obtained by adding a bifurcated-Gaussian component to the two-Gaussian $\theta_C$ PDF. We use a combination of

FIG. 4 (color online). Plots of the $\Delta t$ distribution for (top) signal $K^+\pi^-$ and (bottom) background events. The points with error bars show the data, and the lines represent the PDFs used in the fit and reflect the fit result.

FIG. 5 (color online). Plots of the $\Delta t$ distributions for signal $\pi^+\pi^-$ events tagged as (top) $B^0$ or (middle) $\bar{B}^0$, and (bottom) their asymmetry $a(\Delta t)$, from Eq. (1). The points with error bars show the data, and the lines represent the PDFs used in the fit and reflect the fit result.
MC events and parametrized experiments to test for a potential bias in the fit, for which we estimate an uncertainty of 0.001.

Systematic uncertainties for the CP asymmetries \( S_{\pi^+\pi^-} \) and \( C_{\pi^+\pi^-} \) are listed in Table IV. The largest uncertainties on \( S_{\pi^+\pi^-} \) are due to the \( \Delta t \) and \( B \)-flavor-tagging parameters, and are determined by varying the \( \Delta t \) resolution function parameters and the flavor-tagging parameters by their uncertainties. The largest \( C_{\pi^+\pi^-} \) uncertainty is due to the effect of CP violation in the \( B_{\text{tag}} \) decays [40]. The effect of SVT misalignment is determined by reconstructing events with shifted alignment parameters, and the uncertainties due to the machine boost and detector size are obtained by scaling \( \Delta t \) by 1.0046. We evaluate uncertainties due to the measurement of the beam spot by shifting its position in the vertical direction by 20 \( \mu \)m, and those due to the knowledge of

The \( B^0 - \bar{B}^0 \) mixing frequency and the \( B^0 \) lifetime are determined by varying these parameters within their uncertainties [29]. The uncertainties due to particle identification and potential fit bias are evaluated as described above for \( \mathcal{A}_{K^+\pi^+} \).

### B. \( B^0 \to \pi^0\pi^0 \) results

Results from the ML fit for the \( B^0 \to \pi^0\pi^0 \) decay mode are summarized in Table V. Plots of \( m_{\text{ES}} \), \( \Delta E \), and \( NN \) for \( B^0 \to \pi^0\pi^0 \) are shown in Fig. 7, and for the \( q\bar{q} \) background in Fig. 8.

The various systematic uncertainties for the \( B^0 \to \pi^0\pi^0 \) decay mode are listed in Tables VI and VII. The uncertainty in the efficiency is dominated by a 3\% systematic uncertainty per \( \pi^0 \), which is estimated from a study of \( \tau \to \pi^0\pi^0\nu_\tau \) decays. An uncertainty of 1.0\% is due to the resolution of the signal shape, and an additional uncertainty of 0.5\% is due to the limited knowledge of the \( m_{\text{ES}} \) and \( \Delta E \) peak positions in data. These are estimated by shifting the \( m_{\text{ES}} \) and \( \Delta E \) means and resolutions by amounts determined from MC-data comparison in a control sample of \( B^+ \to \pi^+\pi^0 \) events. An uncertainty of 1.5\%, which is determined from the \( B_{\text{flav}} \) sample, is due to the \( |\cos \theta_3| \) requirement. A 1.1\% uncertainty is assigned to the number of \( BB \) events in the data sample. Systematic uncertainties involving the ML fit are evaluated by varying the PDF parameters and refitting the data. These contribute an uncertainty of 8.3\% to the branching-fraction measurement and an uncertainty of 0.055 to \( C_{\pi^0\pi^0} \).

### C. \( B^0 \to K^0_S\pi^0 \) results

The efficiency and branching fraction measured for the \( B^0 \to K^0_S\pi^0 \) decay mode are summarized in Table V (CP-violation parameters have been reported in Ref. [4]).

We show plots of \( m_{\text{miss}} \), \( m_B \), \( L_2/L_0 \), and \( \cos \theta_B^* \) for signal events in Fig. 9 and for background events in Fig. 10.

![Graph](image-url)
The systematic uncertainties on the branching fraction $B(B^0 \to K_S^0 \pi^0)$ are summarized in Table VIII. The uncertainty on the efficiency of the $K_S^0$ reconstruction is obtained from detailed comparison of inclusive $K_S^0$ candidates in data and MC. The $\pi^0$ efficiency uncertainty is evaluated from the ratio of branching fractions $B(D^0 \to K^- \pi^+ \pi^0)/B(D^0 \to K^- \pi^+)$. To compute the systematic uncertainty associated with the statistical precision on the parameters of the likelihood function, we shift each parameter by its associated uncertainty and repeat the fit. For $\Delta t$ and the tagging parameters, the uncertainty is obtained from the fit to the $B_{\text{res}}$ sample, while for the other parameters it is obtained from MC. This uncertainty accounts for the size of the sample used for determining the shape of the likelihood function in Eq. (9). A systematic uncertainty associated with the data-MC agreement in the shape of the signal PDFs is evaluated by taking the largest deviation observed when the parameters of the individual signal PDFs for $m_{\text{miss}}$, $m_B$, $L_2/L_0$, and $\cos \theta_B^*$ are allowed to vary in the fit. The output values of the PDF parameters

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$B^0 \to \pi^0 \pi^0$ & $247 \pm 29$ & 28.8 $\pm$ 1.8 & 1.83 $\pm$ 0.21 $\pm$ 0.13 & $-0.43 \pm 0.26 \pm 0.05$ \\
$B^0 \to K_S^0 \pi^0$ & $556 \pm 32$ & 34.2 $\pm$ 1.2 & 5.1 $\pm$ 0.3 $\pm$ 0.2 \\
\hline
\end{tabular}
\caption{Results for the $B^0 \to \pi^0 \pi^0$ and $B^0 \to K_S^0 \pi^0$ decay modes. For each mode, we show the signal yield $N_{\text{sig}}$, the efficiency, the branching fraction, and the $CP$-violation parameter $C$. When two uncertainties are given, the first is statistical and the second is systematic. Uncertainties for the signal yields are statistical, and those for the efficiencies are systematic.}
\end{table}
are also used to assign a systematic uncertainty to the efficiency of the event selection requirements on the likelihood variables, by comparing the efficiency in the data to that in the MC. We evaluate the systematic uncertainty due to the neglected correlations among fit variables using a set of MC experiments, in which we embed signal events from a full detector simulation with events generated from the background PDFs. Since the shifts are small and only marginally significant, we use the average relative shift in the yield as the associated systematic uncertainty.

In the fit we neglect background from \( B \) decays, which is estimated from simulation to contribute of order 0.1\% of the total background. To account for a bias due to this, we study in detail the effect of a number of specific \( B \) decay channels that dominate this type of background, notably \( B^+ \to \rho^+ K^0_S, \ B^+ \to K^{*+}\pi^0, \) and \( B^+ \to K^0_S\pi^0\pi^+ \). We embed these simulated \( B \)-background events in the data set and find the average shift in the fit signal yield to be +5.2 events. We adjust the signal yield accordingly and use half of the bias as a systematic uncertainty.

For the branching fraction, additional systematic uncertainties originate from the uncertainty on the selection efficiency, the number of \( BB \) pairs in the data sample (1.1\%), and the branching fractions \( B(K^0_S \to \pi^+\pi^-) \) and \( B(\pi^0 \to \gamma\gamma) \) [29].

### Table VI. Systematic uncertainties on the \( B^0 \to \pi^0\pi^0 \) signal yield \( N_{\pi^0\pi^0} \) and direct \( CP \) asymmetry \( C_{\pi^0\pi^0} \). The total uncertainty is the sum in quadrature of the individual uncertainties.

<table>
<thead>
<tr>
<th>Source</th>
<th>( N_{\pi^0\pi^0} )</th>
<th>( C_{\pi^0\pi^0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peaking background</td>
<td>4.9</td>
<td>0.030</td>
</tr>
<tr>
<td>Tagging</td>
<td>0.35</td>
<td>0.034</td>
</tr>
<tr>
<td>Background shape</td>
<td>5.5</td>
<td>0.023</td>
</tr>
<tr>
<td>Signal shape</td>
<td>3.8</td>
<td>0.020</td>
</tr>
<tr>
<td>Total fit systematic uncertainty</td>
<td>8.3</td>
<td>0.055</td>
</tr>
</tbody>
</table>

### Table VII. Relative systematic uncertainties on the \( B^0 \to \pi^0\pi^0 \) branching fraction. The total uncertainty is the sum in quadrature of the relative uncertainties on the signal yield (from Table VI), the signal efficiency, and the number of \( BB \) pairs.

<table>
<thead>
<tr>
<th>Source</th>
<th>( B(B^0 \to \pi^0\pi^0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal yield syst. uncertainty</td>
<td>3.4%</td>
</tr>
<tr>
<td>( \pi^0 ) efficiency</td>
<td>6.0%</td>
</tr>
<tr>
<td>(</td>
<td>\cos \theta_Z</td>
</tr>
<tr>
<td>Neutrals resolution</td>
<td>1.0%</td>
</tr>
<tr>
<td>( m_{\text{ES}} ) and ( \Delta E ) shape</td>
<td>0.5%</td>
</tr>
<tr>
<td>Number of ( BB ) pairs</td>
<td>1.1%</td>
</tr>
<tr>
<td>Total systematic uncertainty</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

V. RESULTS FOR \( \Delta \alpha_{\pi\pi} \) AND \( \alpha \)

We combine our results for \( B(B^0 \to \pi^0\pi^0) \) with the branching fractions \( B(B^0 \to \pi^+\pi^-) = (5.5\pm0.4\pm0.3) \times 10^{-6} \) and \( B(B^+ \to \pi^+\pi^0) = (5.02\pm0.46\pm0.29) \times 10^{-6} \) previously measured by BABAR [6,15] to evaluate the constraints on both the penguin contribution to \( \alpha \) and on the CKM angle \( \alpha \) itself. Constraints are evaluated by scanning the parameters \( |\Delta \alpha_{\pi\pi}| \) and \( \alpha \), and then calculating the \( \chi^2 \) for the five amplitudes \( \{A^{+0}, A^{-+}, A^{00}, A^{+-}, A^{00}\} \) from our measurements and the isospin-triangle relations [10]. Each \( \chi^2 \) value is converted to a confidence level, which is shown in Fig. 11 for \( \Delta \alpha_{\pi\pi} \) and \( \alpha \). The \( \alpha \) plot exhibits six clear peaks, a result of the eightfold trigonometric ambiguity in the extraction of \( \alpha \) and the fact that two pairs of peaks are nearly merged. The upper bound on \( |\Delta \alpha_{\pi\pi}| \) is 43° at the 90% C.L., and the range [23°,67°] in \( \alpha \) is excluded at the 90% C.L. The point...
TABLE VIII. Summary of dominant contributions to the values of $\alpha$ at the 68% C.L.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma(B(B^0 \rightarrow K_S^0 \pi^0))$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$ efficiency</td>
<td>3.0</td>
</tr>
<tr>
<td>$K_S^0$ efficiency</td>
<td>0.5</td>
</tr>
<tr>
<td>Selection criteria</td>
<td>1.5</td>
</tr>
<tr>
<td>PDF-parameters precision</td>
<td>0.22</td>
</tr>
<tr>
<td>Shape of signal PDFs</td>
<td>0.45</td>
</tr>
<tr>
<td>$B\bar{B}$ background</td>
<td>0.47</td>
</tr>
<tr>
<td>Correlations</td>
<td>0.40</td>
</tr>
<tr>
<td>Resolution function</td>
<td>0.49</td>
</tr>
<tr>
<td>Number of $B\bar{B}$ pairs</td>
<td>1.1</td>
</tr>
<tr>
<td>Total</td>
<td>3.7</td>
</tr>
</tbody>
</table>

FIG. 10 (color online). Plots of (a) $m_{\text{miss}}$, (b) $m_B$, (c) $L_2/L_0$, (d) $\cos \theta_B$, and (e) $\Delta t$ distributions for background events in the $B^0 \rightarrow K_S^0 \pi^0$ sample. The points with error bars represent the data, and the lines show the shapes of signal PDFs as obtained from the ML fit.

$\alpha = 0$, which corresponds to no CP violation, and the values of $\alpha$ near 0 or $\pi$ can be excluded with additional physics input [6,41]. If we consider only the solution preferred in the SM [42], $\alpha$ lies in the range $[71^\circ, 109^\circ]$ at the 68% C.L. This is consistent with the more restrictive constraints on $\alpha$ obtained from analysis of the $B \rightarrow \rho \rho$ system [43], as well as those from $B^0 \rightarrow (\rho \pi)^0$ [44] and $B^0 \rightarrow \alpha \pi$ [45].

VI. CONCLUSIONS

We measure the CP-asymmetry parameters

\[
S_{\pi^- \pi^-} = -0.68 \pm 0.10 \pm 0.03, \\
C_{\pi^- \pi^-} = -0.25 \pm 0.08 \pm 0.02, \\
A_{K^- \pi^+} = -0.107 \pm 0.016^{+0.006}_{-0.004}, \\
C_{\pi^0 \pi^0} = -0.43 \pm 0.26 \pm 0.05, \\
\]

and CP-averaged branching fractions

\[
B(B^0 \rightarrow \pi^0 \pi^0) = (1.83 \pm 0.21 \pm 0.13) \times 10^{-6}, \\
B(B^0 \rightarrow K^0 \pi^0) = (10.1 \pm 0.6 \pm 0.4) \times 10^{-6}. 
\]

We find a 68% C.L. region for $\alpha$ of $[71^\circ, 109^\circ]$ and exclude values in the range $[23^\circ, 67^\circ]$ at the 90% C.L. We observe direct CP violation in $B^0 \rightarrow K^- \pi^+$ with a significance of $6.1\sigma$ and in $B^0 \rightarrow \pi^+ \pi^-$ with a significance of $6.7\sigma$, including systematic uncertainties. Ignoring color-suppressed tree amplitudes, the charge asymmetries in


\( K^+ \pi^- \) and \( K^+ \pi^0 \) should be equal [21], which is not supported by recent BABAR and Belle data [5,6,46]. These results might indicate a large color-suppressed amplitude, an enhanced electroweak penguin, or possibly new-physics effects [47].

Our result for \( \mathcal{B}(B^0 \rightarrow K^0 \pi^0) \) is consistent with the sum-rule prediction [21,22] \( \mathcal{B}(K^0 \pi^0) = \frac{1}{2}(\mathcal{B}(K^+ \pi^-) + 2\mathcal{B}(K^0 \pi^0) - 2\mathcal{B}(K^0 \pi^0)) = (8.4 \pm 0.8) \times 10^{-6} \) obtained using the currently published results [6,15–18] for the three \( B \rightarrow K \pi \) rates on the right-hand side of this equation and the lifetimes \( \tau_+ \) and \( \tau_0 \) of the charged and neutral \( B \) mesons.

The results presented here supersede those of our prior publications [5–7].

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[28] The use of charge-conjugate modes is implied throughout this paper unless otherwise noted.
[34] We use the shorthand notation $\rho$ and $a_1$ to refer to the $\rho(770)$ and $a_1(1260)$, respectively.
[36] The function is $f(x) \approx x\sqrt{1-x^2}\exp[-\zeta(1-x^2)]$, where the slope $\zeta$ is a fit parameter and $x = m_{ES}/E_b$; H. Albrecht et al. (ARGUS Collaboration), Z. Phys. C 48, 543 (1990).