Holographic Schwinger Effect and the Geometry of Entanglement

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1103/PhysRevLett.111.211603">http://dx.doi.org/10.1103/PhysRevLett.111.211603</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>American Physical Society</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Thu Mar 30 16:01:21 EDT 2017</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/84960">http://hdl.handle.net/1721.1/84960</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.</td>
</tr>
<tr>
<td>Detailed Terms</td>
<td></td>
</tr>
</tbody>
</table>

Terms of Use:

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

Terms of Use:

Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.
Holographic Schwinger Effect and the Geometry of Entanglement

Julian Sonner*

Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA and L.N.S., Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 2 August 2013; published 20 November 2013)

We show that the recently proposed bulk dual of an entangled pair of a quark and an antiquark corresponds to the Lorentzian continuation of the tunneling instanton describing Schwinger pair creation in the dual field theory. This observation supports and further explains the claim by Jensen and Karch that the bulk dual of an Einstein-Podolsky-Rosen pair is a string with a wormhole on its world sheet. We suggest that this constitutes a holographically dual realization of the creation of a Wheeler wormhole.

DOI: 10.1103/PhysRevLett.111.211603 PACS numbers: 11.25.—w, 03.65.Ud, 23.20.Ra

Introduction.—Maldacena and Susskind, sparked by the firewall debate [1,2], conjectured that two seemingly completely disparate physical phenomena, the entanglement of quantum states on one side, and nontraversable wormholes on the other, are in fact intimately related [3]. This conjecture has been summarized in an equation between the acronyms ER = EPR. Here and throughout the Letter, ER = Einstein-Rosen [4] and EPR = Einstein-Podolsky-Rosen [5]. While the authors of [3] have formulated their conjecture for general entangled pairs, there has recently emerged a specific context in which the ER = EPR conjecture can be made very concrete. It was argued in [6] that—within the anti–de Sitter–conformal field theory (AdS-CFT) duality—an EPR pair made from a quark and an antiquark has a bulk dual described by a string whose world sheet has a nontraversable wormhole of the kind the authors of [3] were considering. In this Letter, we show that the exact geometry put forward as the requisite bulk dual of an EPR pair in [6] arises as the Lorentzian continuation of a Euclidean string instanton that describes the production of entangled pairs via the Schwinger effect in the strongly coupled dual gauge theory (see Fig. 1).

This observation puts the interpretation of the solution in [6] in terms of entanglement beyond reasonable doubt and strongly supports the ER = EPR conjecture in the context of the \( \mathcal{N} = 4 \) SYM theory at large \( N \) and large ’t Hooft coupling \( \lambda \) and its gravity dual. We cannot resist but remark that the picture proposed here is rather reminiscent of the “Wheeler wormhole” idea [7] in that the particle antiparticle pair in a maximally entangled singlet state is connected by a wormhole, created together with the pair itself as a consequence of the Schwinger effect. Of course the wormhole in this context lives on a string world sheet whose metric is merely induced from the surrounding gravity theory, while the more general ER = EPR conjecture is still open (see also [8–10]). We hope that the observations made in this Letter may be of help in exploring the issue further.

Creating the entangled pair of Jensen and Karch with the Schwinger effect at strong coupling.—The Schwinger effect is a phenomenon that occurs in quantum field theory in the presence of a strong applied field. The prototypical context, within which Julian Schwinger first derived the eponymous effect [11], gives the probability of pair production of charged particles within a space-time volume \( V \), of mass \( m \) in an applied field \( E \), as

\[
\gamma = \frac{E^2}{8\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n+1}{n^n} e^{-\left(\pi m^2 n/|E|\right)}, \tag{1}
\]

for particles of spin zero. The result for spin \( j \) is also known—see, for example, the description in [12]. Since the particles so produced are formed from a singlet state (the vacuum), they are necessarily entangled with one another, no matter what the actual nature of the particles, may they be electrons and positrons as in the original case, or quarks and antiquarks and even charged \( W \) bosons, as we shall consider in this Letter. The exponential factor in Eq. (1) strongly suggests a derivation of the effect in terms of an instanton sum, where the multi-instanton contributions are suppressed at weak fields and/or large masses, and we now describe very briefly an approach in which this is made precise.

An enlightening treatment of the Schwinger effect is obtained by considering the world line path integral of a particle in Euclidean signature [12–15]. In this formalism, the pair creation effect can be derived by considering the saddle points of the Euclidean path integral, which are given by cyclotron orbits of the particle, with the \( n \) instanton contribution given by a particle going around the orbit \( n \) times along the trajectory \( \vec{r} = (\cos(2\pi n\tau), \sin(2\pi n\tau), 0, 0) \). Recall that we have cyclotron orbits because an electric field acts like a magnetic field once we continue to Euclidean signature. The interpretation of the \( n \)-covered circle is the creation of \( n \) pairs instead of just one [11,14,16]. By evaluating the fluctuation determinants around the classical saddle point configuration and summing over all \( n \), one arrives at Eq. (1).

Semenoff and Zarembo [12], building on earlier work by [17–19], have used this point of view to give a strong-coupling derivation of the Schwinger effect in \( \mathcal{N} = 4 \) SYM theory at large \( N \) and large ’t Hooft coupling derivation of the Schwinger effect in [12–15]. In this formalism, the pair creation effect can be derived by considering the saddle points of the Euclidean path integral, which are given by cyclotron orbits of the particle, with the \( n \) instanton contribution given by a particle going around the orbit \( n \) times along the trajectory \( \vec{r} = (\cos(2\pi n\tau), \sin(2\pi n\tau), 0, 0) \). Recall that we have cyclotron orbits because an electric field acts like a magnetic field once we continue to Euclidean signature. The interpretation of the \( n \)-covered circle is the creation of \( n \) pairs instead of just one [11,14,16]. By evaluating the fluctuation determinants around the classical saddle point configuration and summing over all \( n \), one arrives at Eq. (1).
SYM theory on its Coulomb branch, where the original gauge group is broken from $U(N + 1)$ to $U(1) \times U(N)$. See also the earlier paper by [19] which first applies the instanton world sheet utilized by [12] to the Schwinger effect. We refer the reader to the original references for details. The crux of their computation is that at strong coupling, where the $\mathcal{N} = 4$ theory is described by strings in $AdS_5 \times S^5$, one can similarly calculate the rate of Schwinger pair production by finding a suitable instanton configuration, in their case, for a string world sheet embedded in $AdS_5 \times S^5$ ending on a D3-brane a distance $r_0$ away from the Poincaré horizon. This gives the rate of pair production of massive $W^\pm$ bosons in the Coulomb branch of the theory. We can reinterpret their computation in terms of a pair-creation process of a quark and an antiquark in a $U(N)$ gauge theory with fundamental quarks. Now the string ends on a flavor brane at $r = r_0$ and one can apply an electric field with respect to the $U(1)_B$ baryon number symmetry of the theory. Alternatively, we could stick with the original interpretation of pair-produced $W$ bosons, and phrase the argument of [6] in terms of such entangled states of $W^\pm$ bosons. To be specific, let us take the $AdS_5 \times S^5$ metric as

$$ds^2 = L^2 \left( r^2 dx_\mu dx^\mu + \frac{dr^2}{r^2} + d\Omega_5^2 \right).$$

(2)

Then, as shown in [12], the relevant configuration giving the $n$-instanton contribution is a world sheet, parametrized by $\tau, \sigma$, whose Euclidean embedding coordinates satisfy the equation

$$X = \frac{\cosh(2\pi n \sigma_0)}{\cosh(2\pi n \sigma)} R \hat{t}, \quad r = r_0 \frac{\tanh(2\pi n \sigma_0)}{\tanh(2\pi n \sigma)}.$$  

(3)

This corrects a trivial typo in Eq. (15) of [12]. Here, $\hat{t}$ parametrizes the circular trajectory introduced above and $\sinh(2\pi n \sigma_0) = 1/(R r_0)$, while $n$ is the analog of the number of orbits of the particle we saw above. The radius $R$ is obtained by extremizing the instanton action with the result [12]

$$R = \frac{1}{2\pi m} \sqrt{\frac{(2\pi m^2)^2}{E} - \lambda},$$

(4)

in terms of the mass $m = (\sqrt{\lambda} r_0/2\pi)$ and applied field $E$. We show a representation of the instanton in Fig. 1(a). Continuing this world sheet configuration to Lorentzian time, one finds that it becomes the locus of the hyperbola

$$-t^2 + x^2 + \frac{1}{r^2} = R^2 + \frac{1}{r_0^2},$$

(5)

evidently a solution corresponding to a quark and an antiquark (or a pair of $W^\pm$ bosons) being accelerated away from each other and approaching the speed of light in asymptotic time [20], as illustrated in Fig. 1(b). In analogy with the (scalar) QED case, one is tempted to conclude that the solution for $n > 1$ corresponds to the instanton saddle relevant for the creation of $n$ pairs, but it is not clear that the standard argument used there [16], applies to strongly-coupled non-Abelian theories. What is clear is that this process contributes to the production rate with a suppression $\propto e^{-nS_1}$, where $S_1$ is the one-instanton result [cf. Eq. (1)].

We are now in a position to state the main argument of this Letter: the analytically continued world sheet instanton is precisely the configuration put forward by [6] as a solution for the dual of an entangled pair of quarks in $\mathcal{N} = 4$ SYM. In order to compare to their solution, we simply have to make the identification $b^2 = R^2 + (1/r_0^2)$ and $z = 1/r$. As the authors of [6] pointed out, the world sheet configuration has brane horizons at $z = b$ connected by a nontraversable Lorentzian wormhole. The causal structure of the world sheet corresponding to pair production is illustrated in Fig. 2 (see caption for further explanation).

In this Letter, we added the new point that the configuration can be viewed as arising from the tunneling
many other contexts, for example, [20–27]. We expect that but brane horizons, double sided or not, have appeared in
associated with the Schwinger effect. Since the brane
D3-D5 system in the presence of a strong applied field is
pointed out in [28,29], that the world sheet horizon of the
entangling (pair-creation) effects. Indeed, it was already
the Schwinger pairs being continually produced or similar
should be associated with the entropy of entanglement of
world sheet or volume geometry can be taken literally,
ration with a two-sided horizon. Furthermore, in all cases
we are aware of, (stationary) brane horizons that are not
induced on the world sheet—or world volume—by an
drawn instanton-circle trajectory in the lower half plane.
The causal structure is, thus, the same as the upper half of the
Penrose diagram of the eternal AdS black hole and inherits its
ER bridge.

FIG. 2 (color online). In the upper half plane, the end points of
the string follow the hyperbola [Eq. (5)] and the world sheet
horizon is induced from the Rindler horizon of the pair shown as
solid black lines. The world sheet of the string extends in the
direction perpendicular to the $x - t$ plane, filling in the space
between the blue and red curves (shaded in gray). The Rindler
horizons project to the world sheet horizons at $r = (R^2 + 1/r_0^2)^{-1/2}$. The full solution arises by analytically con-
tinuing the Euclidean instanton through $t = 0$, which we depict
by drawing the instanton-circle trajectory in the lower half plane.
The causal structure is, thus, the same as the upper half of the
Penrose diagram of the eternal AdS black hole and inherits its
instanton describing Schwinger pair creation in the field
theory under consideration. This makes it obvious that the
state is entangled and strongly supports the claim that
ER $=$ EPR for maximally entangled states in the context
of gauge-gravity duality. Obviously, here we should read
the equals sign as “is dual to”. Furthermore, we would like
to identify the Lorentzian embedding [Eq. (5)] as the
“Wheeler wormhole” created together with the pair.
It would be very interesting to consider extensions to states
with less than maximal entanglement.

Discussion.—This Letter has focused on the Schwinger
process in strongly coupled $\mathcal{N} = 4$ SYM in $d = 3 + 1$,
but brane horizons, double sided or not, have appeared in
many other contexts, for example, [20–27]. We expect that
similar arguments can be made about any brane configu-
ration with a two-sided horizon. Furthermore, in all cases
we are aware of, (stationary) brane horizons that are not
induced on the world sheet—or world volume—by an
actual black hole horizon of the background, but rather
appear due to some external forcing, correspond to driven
nonequilibrium (steady) states [23,28]. The entropy asso-
ciated with such horizons, derived by assuming that the
world sheet or volume geometry can be taken literally,
should be associated with the entropy of entanglement of
the Schwinger pairs being continually produced or similar
entangling (pair-creation) effects. Indeed, it was already
pointed out in [28,29], that the world sheet horizon of the
D3-D5 system in the presence of a strong applied field is
associated with the Schwinger effect. Since the brane
horizon depends on the applied field, one can dial its area
at will by changing the field intensity up and down. Thus,
the brane horizon cannot satisfy the second law of BH
thermodynamics in a conventional sense and so it is
satisfying that we are nevertheless able to put forward an
explanation for the observed entropy, namely that it
represents the entropy of entanglement of the pairs being
produced in the driven steady state. This rate of production
goes to zero when the external forcing is switched off,
which also removes the brane horizon.

It is often stated (see, for example, [30]) that particle
creation at a black hole horizon is subtly different from the
conventional Schwinger effect and also that entanglement
entropy is not enough in order to understand the horizon
entropy of general black holes. In this Letter, we have
argued that the two can be dual to each other in a precise
sense. We add the cautionary note that the metric whose
horizon we associate with Schwinger pair production is not
that of dynamical gravity in itself, but rather an induced
one; and indeed, it has been observed before that the black
hole entropy in induced gravity theories can be entirely
accounted for by a suitable entanglement entropy, as
reviewed, for example, in [31].

We conclude by restating the main point: the solution
of [6], previously found in [20], is the Lorentzian con-
tinuation of a family of world sheet instantons that
describes particle creation via the Schwinger effect in
$\mathcal{N} = 4$ SYM theory. Therefore, the interpretation of
this solution as the bulk dual of a maximally entangled
pair is correct and the AdS dual of an EPR pair is a
geometry with an ER bridge. This is very reminiscent of
an old idea due to Wheeler. At least at strong ’t Hooft
coupling and large $N$, we have been able to explicitly
demonstrate that a particle and an antiparticle, pair pro-
duced in an applied field, are connected by a wormhole
(in the dual geometry), and so this wormhole should be
associated with the entanglement between them. It is
tempting to ask what becomes of this wormhole at weak
’t Hooft coupling $\lambda$, where stringy corrections need to be
taken into account in the bulk. More daunting is the task
of elucidating the fate of the ER bridge under $1/N$ cor-
rections when the dual gravity theory ceases to be classi-
cal and the object connecting the particle and antiparticle
becomes very quantum mechanical. At least in principle,
the framework described in this Letter offers a starting
point to look into these interesting questions.

I would like to thank Mike Crossley, Hong Liu,
Josephine Suh and Wojciech Zurek for discussions and
comments on the draft version, and Gordon Semenoff for
helpful correspondence. I thank Los Alamos National
Laboratory and the Asia Pacific Center for Theoretical
Physics (APCTP) for hospitality during the course of this
work. This work was supported in part by the U.S.
Department of Energy (DOE) under cooperative research
agreement Contract No. DE-FG02-05ER41360.