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Inductor Design Methods with Low-permeability RF Core Materials

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Abstract—This paper presents a design procedure for inductors based on low-permeability magnetic materials, for use in very high frequency (VHF) power conversion. The proposed procedure offers an easy and fast way to compare different magnetic materials based on Steinmetz parameters and quickly select the best among them, to estimate the achievable inductor quality factor and size, and to design the inductor. Approximations used in the proposed methods are discussed. Geometry optimization of magnetic-core inductors is also investigated. The proposed design procedure and methods are verified by experiments.

I. BACKGROUND

There is a growing interest in switched-mode power electronics capable of efficient operation at very high switching frequencies (e.g., 10-100 MHz) [1]. Power electronics operating at such frequencies include resonant inverters [2]–[11] (e.g., for heating, plasma generation, imaging, and communications) and resonant dc-dc converters [2], [4], [12]–[21] (which utilize high frequency operation to achieve small size and fast transient response.) These designs utilize magnetic components operating at high frequencies, and often under large flux swings. These magnetic components should have a high quality factor to achieve high efficiency power conversion. Unfortunately, most high-permeability magnetic materials exhibit unacceptably high losses at frequencies above a few megahertz. There are some low-permeability materials (e.g., relative permeabilities in the range of 4-40) that can be used effectively at moderate flux swings at frequencies up to many tens of megahertz [22]–[24]. However, working with such low-permeability materials - and the ungapped core structures they are typically available in - presents somewhat different constraints and challenges than with typical high-permeability low-frequency materials [25]. Because of VHF operation and the low-permeability characteristics of such materials, the operating flux density is limited by core loss rather than saturation, and a gap is not necessary to prevent the core from saturating in many applications. Without a gap, the core loss begins to dominate the total loss and copper loss can be ignored in many cases. The performance of a VHF magnetic-core inductor thus depends heavily on the loss characteristics of the magnetic material. Moreover, there appears to be a lack of good design procedures for selecting among low-permeability magnetic materials and available core sizes.

This paper, which expands upon an earlier conference paper [26], investigates a design procedure for inductors using low-permeability magnetic materials. This method is based on knowledge of the material loss characteristics, such as collected in [22], [24], and is particularly suited for VHF inductor designs. With the methods developed here, different magnetic materials are compared fairly and conveniently, and both the achievable quality factor and size of a magnetic-core inductor can be evaluated before the final design.

Section II of the paper introduces the inductor design considerations and questions to be addressed. Section III illustrates the inductor design procedure and methods employed in it. Section IV shows some experimental results to verify the design procedure. Section V concludes the paper. In Appendices A and B, we check an important assumption behind our methods as well as investigate geometry optimization problems of magnetic-core inductors.

II. INDUCTOR DESIGN CONSIDERATIONS AND QUESTIONS

In this paper, we only consider inductor designs under a limited set of conditions in order to make the problem tractable. Nevertheless, these conditions are both very reasonable and practical for inductors at very high frequencies. The limited conditions we address are as follows:

1) Use of ungapped cores made of low-permeability materials.
2) Single-layer, foil wound designs in the skin depth limit on toroidal core shapes. A toroidal inductor design keeps most of flux inside the core, thus reducing EMI/EMC problems. A foil winding design can further reduce the copper loss compared to a wire-wound one [27].
3) Design based on knowledge of Steinmetz parameters for materials of interest. Such parameters are often not published or readily available for these materials, but can be obtained using methods such as that of [22], [24], [28].
4) Design assuming sinusoidal excitation at one frequency. In VHF resonant inverters or converters, inductors often have approximately sinusoidal current at a single frequency. Note that consideration of variable frequency operation, dc currents, and multiple frequency components greatly increases complexity of both loss calculation and design [29]–[34].

Fig. 1 shows an inductor has been designed and fabricated under the above conditions and replaced the original coreless resonant inductor \( L_0 \) in a 30 MHz \( \phi_2 \) inverter [3], [23], [24]. The magnetic-core inductor provides a substantial volumetric advantage over that achievable with a coreless design in this application [23], [24].
Given a selection of available cores in different low-permeability materials, and a design specification including inductance $L$, current amplitude $I_{pk}$, frequency $f_s$, we answer three important questions about design of VHF inductors under the above conditions:

1) Which magnetic material from an available set will yield maximum quality factor $Q_L$ for a given size?

2) Given the ability to continuously scale core size, what material will yield the smallest size for a given quality factor $Q_L$?

3) For an achievable quality factor $Q_L$ and inductor size, how should we design the inductor with the selected best magnetic material to meet design specifications?

These questions are addressed in the next section.

III. INDUCTOR DESIGN PROCEDURE AND METHODS

A. Inductor Design Procedure

Fig. 2 provides a high-level illustration of the proposed design procedure. First, select design specifications from the system requirements. Second, select the best magnetic material from a set of low-permeability materials with known Steinmetz parameters. In the third and fourth steps, we estimate the achievable quality factor $Q_L$ and size of the inductor with the best available material. If the results are satisfactory, we design the inductor. If not, it means the design requirements can’t be satisfied even with the best available magnetic material, and one must revise the inductor design requirements. A key feature of this design procedure is that magnetic materials are compared first and the best material is selected before completing any individual design, greatly reducing design time and effort. Some important information such as the maximum quality factor $Q_L$, and the smallest possible size can be acquired before the final design. By this procedure, we design an inductor only with one size and one material instead of investigating numerous (perhaps thousands) of combinations to meet the design specifications.

(1) to (3) are used often in our design procedure. In VHF power conversion, ac losses (conductor/copper and core losses) usually dominate and we thus ignore dc losses (conductor loss) here. In (1) and (2), we use the quality factor $Q_L$ to evaluate the ac losses of an inductor at a single frequency. $R_{ac}$ is the equivalent total ac resistance of a magnetic-core inductor including copper loss and core loss, $R_{cu}$ is the equivalent resistance owing to copper loss, and $R_{co}$ is an equivalent resistance owing to core loss. The Steinmetz equation is an empirical means to estimate loss characteristics of magnetic materials [35], [36]. It has many extensions [29]–[34], [36]–[38], but we only consider the formulation for sinusoidal drive at a single frequency here. In (3), $B_{pk}$ is the peak amplitude of average (sinusoidal) flux density inside the material and $P_V$ is power loss per unit core volume. $K$ and $\beta$ have been calculated

$1^*$Use of average flux density in the core simplifies the calculations. For typical core sizes, this approximation is well justified, and the error of this approximation should be lower than 10% as shown in Appendix A. (Flux nonuniformity for other cases is treated in [39], [40], for example.)
for several commercial low-permeability rf magnetic materials from 20 MHz to 70 MHz in [22], [24], [28].

\[ Q_L = \frac{\omega L}{R_{ac}} \]  
\[ R_{ac} = R_{cu} + R_{co} \]  
\[ P_V = KB_{pk}^2 \]  

B. Method to Select Among Magnetic Materials

In the second step of the design procedure, we identify the best material. We begin with a coreless inductor to make a comparison among different design options (including magnetic materials) for a given \( L, I_{pk}, f_s \), minimum \( Q_L \) and maximum size limitation. Ignoring the “single-turn” inductance associated with the circumferential current component around the core [27], the number of turns \( N_{air} \) for a coreless inductor can be calculated from (4) [27]:

\[ N_{air} \approx \sqrt{\frac{2\pi L}{h\mu_0 \ln \left( \frac{d_i}{d_o} \right)}} \]  

\( d_o, d_i \) and \( h \) are the outside diameter, inside diameter and height of the coreless inductor. The approximate average flux density \( B_{pk-air} \) inside the core is calculated by (5):

\[ B_{pk-air} = \frac{\mu_0 N_{air} I_{pk}}{0.5\pi (d_i + d_o)} \]  

Likewise, the number of turns \( N \) and average flux density \( B_{pk} \) of a magnetic-core inductor are calculated by (6) and (7) 2:

\[ N \approx \sqrt{\frac{2\pi L}{h\mu_0 \mu_r \ln \left( \frac{d_i}{d_o} \right)}} \]  

\[ B_{pk} = \frac{\mu_0 \mu_r N I_{pk}}{0.5\pi (d_i + d_o)} = \mu_r^{0.5} B_{pk-air} \]  

For a given \( L \) and specified dimensions in (7), average flux density \( B_{pk} \) inside the core may be different for each magnetic material, which is one of the reasons we can’t compare their loss characteristics for different magnetic materials directly at the same flux density level. However, we propose here a method by which direct comparisons can be made: \( B_{pk} \) of each magnetic material can be normalized to the coreless inductor flux density \( B_{pk-air} \) by its relative permeability \( \mu_r \). For a given design specification, all magnetic materials will have the same normalized flux density, which is equal to \( \mu_r^{-0.5} B_{pk} \). Given a set of Steinmetz parameters, we can draw the curves of \( P_V \) vs. \( \mu_r^{-0.5} B_{pk} \) for all available magnetic materials. We compare \( P_V \) of these materials at \( \mu_r^{-0.5} B_{pk} = B_{pk-air} \) and decide which material has the smallest core loss for the given design specification.

An example is shown in Fig. 3, in which we consider a design of a magnetic-core inductor at \( I_{pk} = 2 \text{ A} \) and \( f_s = 30 \text{ MHz} \) with \( L = 200 \text{ nH} \) and maximum size \( d_o = 12.7 \text{ mm}, d_i = 6.3 \text{ mm} \) and \( h = 6.3 \text{ mm} \) 3. Beginning with a coreless inductor, we calculate \( B_{pk-air} = 1.3 \text{ mT} \) from (4) and (5). Using data from [22], [24], loss curves of \( P_V \) vs. \( \mu_r^{-0.5} B_{pk} \) are plotted for the materials N40, P, M3 and 67 4. We compare their \( P_V \) at \( B_{pk-air} \) and find that N40 material has the smallest core loss (614 mw/cm³). If we ignore the copper loss, the magnetic-core inductor with N40 material will achieve the highest \( Q_L \) for given design specifications. We can also observe in Fig. 3 that N40 is better than the other magnetic materials and 67 is worse than the others over a wide range of flux density. This will help us to design a magnetic-core inductor if its operating current level is unknown or very wide.

So far, we still don’t know if the magnetic-core inductor with the best material is better than a coreless inductor of the same size. There is no core loss and Steinmetz parameters for a coreless inductor. But we can still compare its copper loss to core losses of other magnetic materials on the same graph. To accommodate the coreless design, we define \( P_{V-air} \) at \( B_{pk-air} \) as the power loss per unit volume for a coreless inductor and calculate it by (8):

\[ P_{V-air} = \frac{R_{cu-air} I_{pk}^2}{2V} \]  

where \( R_{cu-air} \) is the copper resistance of a coreless inductor and \( V \) is the volume of the coreless inductor. \( R_{cu-air} \) (or the copper resistance of a magnetic-core inductor \( R_{cu} \)) depends heavily on a coreless or magnetic-core inductor winding design pattern. One could find the ac resistance of a coreless inductor by constructing and measuring it or simulating it.

3Examples in the paper are confined to 30 MHz. However, the same design procedure can be applied to a broader range of frequencies.

4-17 material in [22], [24] has a very low relative permeability and low core loss characteristics. Compared to its core loss, the copper loss of -17 material can’t be ignored. As a special case, -17 is not considered here. However, the methods introduced in this paper can still be applied for -17 material with special considerations of its copper loss.
using computational techniques. Alternatively, the resistance can be estimated for different design variants:

1) In [22], [24], the windings are made of an equal-width foil-like conductor, and $R_{cu-single-turn}$ is the ac copper resistance of a single turn inductor:

$$R_{cu} \approx N^2 \frac{\rho_{cu}}{\delta_{cu}} \left( \frac{2h}{d_i} + \frac{d_o}{d_i} - 1 \right)$$  \hspace{1cm} (9)

2) In [22], [24], $R_{cu}$ can alternatively be estimated from the foil width, length and skin depth:

$$R_{cu} \approx \frac{\rho_{cu} l_{cu}}{\delta_{cu} w_{cu}}$$  \hspace{1cm} (10)

3) In [27], the windings are made of foil-like conductor tapered to conform to the shape of the toroid:

$$R_{cu} \approx N^2 \frac{\rho_{cu}}{\delta_{cu}} \left( \frac{h}{d_i} + \frac{h}{d_o} + 2 \ln \frac{d_o}{d_i} \right)$$  \hspace{1cm} (11)

For example, the loss characteristics of a coreless inductor estimated by (9) is included in Fig. 4. We can see N40 is the only magnetic material which has lower loss than the coreless inductor. Thus the magnetic-core inductor built with N40 may have a higher quality factor $Q_L$ than the coreless inductor. The magnetic-core inductor built with other materials (e.g. M3, P and 67) will not be better than the coreless inductor and need not be considered in the following steps. Here, we can see that this comparison lets us exclude most of available magnetic materials in the pool from the design, saving time and effort.

From previous measurements in [22], [24], the core loss ($R_{co}$) usually dominates the total loss of an ungapped VHF magnetic-core inductor. However, this statement should be checked to make sure that it is still correct for an individual design. By a similar method, we can define the copper loss per unit volume $P_{V-cu}$ of a magnetic-core inductor and mark it on the graph of $P_V$ vs. $\mu_r^{0.5} B_{pk}$. From (4) and (6),

$$N = \mu_r^{0.5} N_{air}$$  \hspace{1cm} (12)

$$R_{cu} = N^2 R_{cu-single-turn} = \mu_r^{-1} R_{cu-air}$$  \hspace{1cm} (13)

$$P_{V-cu} = R_{cu} \frac{1}{2V} I_{pk}^2 = \mu_r^{-1} R_{cu-air} \frac{1}{2V} I_{pk}^2 = \mu_r^{-1} P_{V-air}$$  \hspace{1cm} (14)

$P_{V-air}$ can be calculated from (8). The copper loss characteristic of a magnetic-core inductor for the example specifications built in N40 magnetic material is marked in Fig. 5. In this example, the copper loss of the magnetic-core inductor is much smaller than its core loss (an order of magnitude or more $^5$).

C. $Q_L$ Estimation with Given Maximum Inductor Size

In the third step of the design procedure in Fig. 2, we estimate the highest quality factor that can be achieved. If we ignore the copper loss comparing to the core loss of the magnetic-core inductor, the quality factor $Q_L$ can be estimated by (15) and (16):

$$R_{co} \approx \frac{\text{Total Core Loss}}{0.5 I_{pk}^2} = \frac{P_V V}{0.5 I_{pk}^2}$$  \hspace{1cm} (15)

$$Q_L \approx \frac{\omega L}{R_{co}} = \omega L \frac{0.5 I_{pk}^2}{P_V V}$$  \hspace{1cm} (16)

E.g., for the magnetic-core inductor built in N40, $P_V = 614$ mW/cm$^3$ at $B_{pk-air} = 1.3$ mT, and $Q_L \approx 198$ by (16). In this example $R_{co} \approx 0.19 \Omega$ and $R_{cu} \approx 0.03 \Omega$, where $R_{co} \gg R_{cu}$. $Q_L$ can also be estimated by (2), in which copper loss is included, and $Q_L \approx 171$ by (2).

$^5$We note that the simple copper loss calculations of (9) in a cored inductor design may have up to 30% error [22], [24], but this degree of accuracy is sufficient for our present purposes.
D. Size Estimation with Given Minimum $Q_L$

In this subsection, we illustrate the third step in our inductor design procedure, estimating minimum achievable inductor size for a required quality factor. Because the method introduced in this subsection is not as simple and direct as the method in Section III-B, we begin this subsection with a general description of the method. Then we derive equations needed in our method for inductor size estimation. As we have done in Section III-B, step by step design examples are given to aid understanding of the method.

We again begin with a coreless inductor design, calculate its size and compare the size of a magnetic-core inductor with it. In our method, we define the scaling factor $\lambda$ as the dimension ratio of a magnetic-core inductor and the coreless inductor for given $L$, $Q_L$, $f_s$ and $I_{pk}$, and we assume that the relative ratio of the 3 dimensions is kept constant during the scaling. Thus, we scale each dimension ($x$, $y$, $z$) describing the shape of the coreless inductor by a factor $\lambda$ to get the corresponding dimension of a magnetic-core inductor: the coreless inductor thus has $\lambda = 1$, and the magnetic-core inductor with the minimum $\lambda$ has the smallest size.

Our method has four main steps:

1) Given $L$, $I_{pk}$, $f_s$ and minimum required $Q_L$, design a coreless inductor and get its dimension parameters $d_o$, $d_i$, $h$.
2) Calculate $B_{pk-air}$ of the coreless inductor, compare its $P_{V-air}$ to $P_V$ of other magnetic materials at $B_{pk-air}$ on the graph of $P_V$ vs. $\mu_r^{-0.5}B_{pk}$, and decide the possible best materials for the inductor design.
3) Calculate the scaling factor $\lambda$ for the possible best materials.
4) Check the flux density $B_{pk}'$, core loss $P_{V'}$, and copper loss $P_{V'-cu}$ of the magnetic-core inductor after scaling on the graph of $P_{V'}$ vs. $\mu_r^{-0.5}B_{pk}$.

1) Step I, Calculate Coreless Design: From (4), the quality factor $Q_L$ of a coreless inductor can be calculated by (17):

$$Q_L = \frac{\omega L}{R_{cu-single-turn}} = \frac{\mu_0 f_s}{R_{cu-single-turn}} h \ln \left( \frac{d_o}{d_i} \right)$$

$R_{cu-single-turn}$ can be estimated by (9) or (11).

If we assume $d_i = 0.5d_o$, we can solve the dimension parameters $d_o$, $d_i$, $h$ of a coreless inductor from (9)(11), and (17) for given $f_s$, $Q_L$, and ratio of $h$ and $d_o$. In Appendix B, we show that this assumption is very reasonable because letting $d_i = 0.5d_o$ yields an inductor with nearly optimum $Q_L$ and thus the smallest size.

2) Step II, Evaluate Magnetic Materials: After calculating the dimensions of the coreless inductor, its $B_{pk-air}$ and $P_{V-air}$ can be calculated by (5) and (8). $P_V$ at $B_{pk-air}$ of all the magnetic materials can be found from the graph of $P_V$ vs. $\mu_r^{-0.5}B_{pk}$. For example, we consider the design of a coreless inductor with $L = 200$ nH, $I_{pk} = 2$ A, $f_s = 30$ MHz, and $Q_L = 116$. We define this coreless inductor as having $\lambda = 1$. Its dimensions are $d_o = 12.7$ mm, $d_i = 6.3$ mm and $h = 6.3$ mm. The question we seek to answer is: if we build an inductor with magnetic materials, how small it could be while achieving the specified minimum $Q_L$. We first calculate $B_{pk-air} = 1.3$ mT and $P_{V-air} = 1073$ mW/cm$^3$ and then find $P_V$ for each magnetic material in Fig. 4. N40 is the only magnetic material which has $P_V$ smaller than $P_{V-air}$, thus the magnetic-core inductor made in N40 is the only possible design with the size smaller than the coreless inductor ($\lambda < 1$) at the same $Q_L$. Magnetic-core inductors made by other materials will have larger sizes than the coreless inductor and are not considered here. This conclusion is further proved in (26). Just as in Section III-B, we can see here our method in this subsection again helps us to exclude many available magnetic materials in the pool from the complicated problem of inductor size scaling.

3) Step III, Scaling: Here we introduce how to perform the scaling. Before beginning the derivation, we define the following parameters:

1) $V$, $d_o$, $d_i$, $h$, $N$, $B_{pk}$, $P_V$, $P_{V-cu}$, $R_{co}$ and $R_{cu}$ are the volume, outside diameter, inside diameter, height, number of turns, average peak ac flux density, core loss density, copper loss density, equivalent core resistance, and copper resistance of a magnetic-core inductor before scaling - i.e., having the same size as the coreless inductor ($\lambda = 1$).
2) $V'$, $d_o'$, $d_i'$, $h'$, $N'$, $B'_{pk}$, $P'_{V}$, $P'_{V-cu}$, $R'_{co}$ and $R'_{cu}$ are the same definitions of the magnetic-core inductor after scaling.
3) $V$, $d_o$, $d_i$, $h$, $N_{air}$, $B_{pk-air}$, $P_{V-air}$ and $R_{cu-air}$ are the similar definitions of the coreless inductor before scaling ($\lambda = 1$).

From the definition above,

$$\lambda = \frac{d_o'}{d_o} = \frac{d_i'}{d_i} = \frac{h'}{h} \quad \lambda = \frac{d_o}{d_i}$$

$$V' = \lambda^3 V$$

Thus: similar to (6),

$$N' = \sqrt{\frac{2\pi L}{\lambda h \mu_0 \mu_r \ln \left( \frac{d_o}{d_i} \right)}} = \lambda^{-0.5} N$$

Similar to (7) and from (20),

$$B'_{pk} = \frac{\mu_0 h \mu_r N' f_{pk}}{0.5\pi (d_i' + d_o')} = \lambda^{-1.5} B_{pk}$$

$$P'_{V'} = K B'_{pk} = \lambda^{-1.5} P_V$$

From (9) and (11), we observe that $R_{cu-single-turn}$ is constant during scaling. This is because the effective conductor thickness is the skin depth (invariant to scaling). This results in constant “ohms per square”, making the total single-turn resistance invariant to scaling. From (12) and (20),

$$R'_{cu} = N'^2 R_{cu-single-turn} = \lambda^{-1} \mu_r^{-1} R_{cu-air}$$

Similar to (14), and from (19) and (23):

$$P'_{V'-cu} = \frac{R'_{cu} I_{pk}^2}{2V'} = \lambda^{-4} \mu_r^{-1} P_{V-air}$$

For constant $Q_L$, the total loss is the same for both the coreless inductor and the magnetic-core inductor, thus from (19), (22) and (24):

$$P'_{V} + P'_{V'-cu} V' = P_{V-air} V$$
and then find \( P_{V} \), there is more than one possible material which can be used. Materials: geometry and scaling that achieves the smallest size at the design with completion of this last step, we now have an inductor which has \( P_{V} \). In the example, \( P_{V} \) is 13990 mW/cm\(^3\) and represents the best design choice.

We check the flux density \( B_{pk} \), core loss \( P_{V} \), and copper loss \( P_{V' - cu} \) of the scaled magnetic-core inductor built with N40 material on the graph of \( P_{V} \) vs. \( \mu_r^{-0.5}B_{pk} \) by (22), (24) and (27). In the example, the scaled design operates at a normalized flux density \( \mu_r^{-0.5}B_{pk} = 5.0 \) mT, \( P_{V} = 9621 \) mW/cm\(^3\), and \( P_{V' - cu} = 6819 \) mW/cm\(^3\) as illustrated in Fig. 8. We can see that the core loss of N40 is the lowest among the materials at this normalized flux density of the N40 design. If we build a magnetic-core inductor with other materials with the same size after scaling, the inductor will have the smallest scaling factor for the specified minimum \( Q_L \) and represents the best design choice.
have a lower quality factor and must have a bigger size to satisfy the design requirement for minimum quality factor; this confirms our conclusion that the magnetic-core inductor built with N40 will have the smallest size.

E. Inductor Design with the Best Magnetic Material

Having satisfied quality factor $Q_L$ and inductor size requirements, the inductor can be designed with the selected best magnetic material (N40). To provide a complete answer for the previous design example, we summarize the results of each step in Fig. 2:

1. We give the design requirements: $L = 200 \text{ nH}$, $I_{pk} = 2 \text{ A}$, $f_s = 30 \text{ MHz}$, minimum $Q_L = 116$ and maximum size of $d_o = 12.7 \text{ mm}$, $d_i = 6.3 \text{ mm}$ and $h = 6.3 \text{ mm}$.

2. Given available magnetic materials (67, P, M3 and N40) and their Steinmetz parameters, we determine that N40 is the best material for design.

3. Given the maximum size, estimate the highest $Q_L$ of a magnetic-core inductor with N40 material (about $Q_L = 171$).

4. Given the minimum $Q_L$, estimate the scaling factor $\lambda = 0.17$ and the minimum size $d_o = 2.2 \text{ mm}$, $d_i = 1.1 \text{ mm}$ and $h = 1.1 \text{ mm}$ calculated by (18).

5. We check the results in the third and fourth steps and see if they satisfy the design requirements.

6. If we prefer a coreless inductor with the highest $Q_L$ at the maximum size, the inductor will have a number of turns $N = 4$ calculated by (6), an inductance $L = 199 \text{ nH}$, and a core size of $d_o = 12.7 \text{ mm}$, $d_i = 6.3 \text{ mm}$ and $h = 6.3 \text{ mm}$. Its quality factor $Q_L$ has been estimated in Section III-C as 198 neglecting copper loss and 171 including copper loss. If we prefer a cored inductor with the minimum size at the minimum allowed $Q_L$ of 116, the inductor will have a number of turns $N' = 10$ calculated by (20) and the core size is $d_o = 2.2 \text{ mm}$, $d_i = 1.1 \text{ mm}$ and $h = 1.1 \text{ mm}$.

Compared to a coreless design, the magnetic-core inductor with N40 material will have 47% higher quality factor $Q_L$ for the same maximum size or 83% size reduction for the same minimum quality factor.

F. Relationship between Quality Factor $Q_L$ and Inductor Size

The method to calculate the scaling factor $\lambda$ proposed in Section III-D has two shortcomings: the first is that we may need to rely on some numerical methods to solve the nonlinear equation (26), and the second is that it doesn’t reveal the relationship of a magnetic-core inductor’s quality factor and its size in a direct and intuitively understandable way. In this subsection, we study the inductor’s quality factor as the function of its scaling factor $\lambda$.

We again begin our derivations with a coreless inductor. We define quality factor $Q_{L0}$ with given maximum size ($\lambda = 1$) and calculate it by the following equation:

$$Q_{L0} = \frac{\omega L}{R_{cu-air}} = \frac{\mu_0 f}{R_{cu-single-turn}} \ln \left( \frac{d_o}{d_i} \right)$$

Because $R_{cu-single-turn}$ is constant during the scaling as described before, the quality factor after scaling can be calculated by the following equation:

$$Q_L(\lambda) = \frac{\omega L}{R_{cu-air}} = \frac{\mu_0 f}{R_{cu-single-turn}} \ln \left( \frac{d_o\lambda}{d_i\lambda} \right) = \lambda Q_{L0}$$

From (29), we can see there is a linear functional relationship between quality factor $Q_L$ and scaling factor $\lambda$ for a coreless inductor. This general result for coreless design is well known; see the classic scaling rules for coreless solenoids, for example [1], [41]–[44]. For a magnetic-core inductor, from (23),

$$R_{cu} = \lambda^{-1} \mu_r^{-1} R_{cu-air} = \frac{\omega L}{\lambda \mu_r Q_{L0}}$$

From (19) and (22),

$$R_{co} = \frac{P_{V'} V'}{0.5 I_{pk}^2} = \frac{\lambda^3 - 1.5 \beta P_V V}{0.5 I_{pk}^2} = \frac{\lambda^3 - 1.5 \beta}{\lambda} \frac{P_V}{P_{V-air}} \frac{V'}{0.5 I_{pk}^2}$$

$$= \frac{\lambda^3 - 1.5 \beta}{\lambda} \frac{P_V}{P_{V-air}} R_{cu-air} = \frac{\lambda^3 - 1.5 \beta}{\lambda} \frac{P_V}{P_{V-air}} \frac{\omega L}{Q_{L0}}$$

From (30) and (31),

$$Q_L(\lambda) = \frac{\omega L}{R_{cu} + R_{co}} = \frac{\lambda Q_{L0}}{1 + \lambda^3 - 1.5 \beta \frac{P_V}{P_{V-air}}}$$

Starting from a baseline design, from (29) and (32), we can plot $Q_L$ vs. $\lambda$ for magnetic-core inductors made in all available magnetic materials, as well as for a coreless inductor. An example is shown in Fig. 9. The baseline design specifications are $L = 200 \text{ nH}$, $I_{pk} = 2 \text{ A}$, $f_s = 30 \text{ MHz}$, and $Q_L = 116$. At $\lambda = 1$, the coreless inductor has dimensions $d_o = 12.7 \text{ mm}$, $d_i = 6.3 \text{ mm}$ and $h = 6.3 \text{ mm}$. For a given $Q_L$, we can use results such as in Fig. 9 to find its scaling factor $\lambda$ and further decide the inductor size for every magnetic material using (18). Moreover, for a given size (scaling factor $\lambda$), we can find the quality factor $Q_L$ that is achievable for each material.

While this approach to explore inductor scaling and sizing is informative, it still has some shortcomings. The first one
is that we have to generate a new graph by (29) and (32) for each baseline design of interest. The second one is that we can’t a priori exclude some magnetic materials from consideration as we did in Section III-D. If we have dozens of magnetic materials, the graph of \( Q_L \) vs. \( \lambda \) will become very complicated. The third one is that all these materials are compared on the graph at a fixed current level and not over a wide operating range. Nevertheless the methods illustrated here provide a better view into design tradeoffs than one otherwise available.

IV. EXPERIMENTAL VERIFICATION

We carried out several experiments to verify the design procedure illustrated in this paper. Firstly, we want to verify the design steps 2 and 3 in Fig. 2. That is, given available magnetic materials and design requirements (inductance \( L \), current amplitude \( I_{pk} \), and frequency \( f_s \)), we want to determine the best material to yield maximum quality factor \( Q_L \) for a given size, and estimate the highest \( Q_L \) that can be achieved at that size. Design parameters for the example application are repeated here: \( d_o = 12.7 \text{ mm}, d_i = 6.3 \text{ mm}, h = 6.3 \text{ mm}, L = 200 \text{ nH}, I_{pk} = 2 \text{ A, and } f_s = 30 \text{ MHz.} \)

As predicted in our design procedure, N40 is the best material and the magnetic-core inductor with N40 has quality factor \( Q_L = 171 \). We designed and fabricated a magnetic-core inductor with copper foil and N40 core to satisfy the design specifications, and measured its inductance and quality factor \( Q_L \). The results are listed and compared in Table II. The cores in M3 and P materials have \( d_i \) larger than the design specification, e.g. \( \frac{d_i}{d_o} = 0.62 \text{ vs. } 0.50 \). The core loss estimation error due to this is small loss less than 4% higher as shown in Fig. 11 in Appendix B. The core in 67 material has a \( d_i \) larger and \( h \) lower than the design specifications, and its volume is thus 71.4% of the specified one. The core loss estimation error due to this is only 3.1% higher calculated by (48) where \( \beta = 2.18 \) [22]. We can see the measurement results fit very well with the predicted values and the magnetic-core inductor with N40 material is the best design compared to others as we have predicted in our design procedure. To some extent, these results also verify the results of approximations and optimizations shown in Appendices A and B.

Secondly, we verified the design step 4 illustrated in Section III-D. That is, given \( L, I_{pk}, f_s \) and the minimum \( Q_L \), determine the best material for design and estimate the minimum size achievable for that \( Q_L \) requirement. This experiment is much more difficult than the first one because limited availability of core sizes. If we design a magnetic-core inductor with N40 material which has the scaling factor \( \lambda = 0.17 \) as calculated in Section III-D, the inductor after scaling has 10 turns and dimensions \( d_o' = 2.16 \text{ mm, } d_i' = 1.07 \text{ mm and } h' = 1.07 \text{ mm.} \) The winding of copper foil has a width of less than 0.34 mm! It is very hard to wind such a narrow copper foil on this tiny core by hand. The magnetic-core inductor with P material in Section III-D5 has a higher scaling factor \( \lambda \) and thus a larger core size after scaling. So for simplicity, we verified the design of P material instead of N40. The design parameters are repeated here: \( L = 200 \text{ nH, } I_{pk} = 0.5 \text{ A, } f_s = 30 \text{ MHz and } Q_L = 116 \). The scaling factor \( \lambda = 0.77 \) calculated by (26) and shown in Table I. The core dimensions after scaling are \( d_o' = 9.78 \text{ mm, } d_i' = 4.85 \text{ mm and } h' = 4.85 \text{ mm.} \) The available core with the closest size has dimensions \( d_o'' = 9.63 \text{ mm, } d_i'' = 4.66 \text{ mm and } h'' = 3.21 \text{ mm.} \) We designed and fabricated a 3-turn magnetic-core inductor with P material and measured its inductance \( L \) and quality factor \( Q_L \). The results are shown in Table III. We can see the measurement results fit very well with the predicted value (for the actual size) and (26) is thus verified.

Thirdly, we verified (32) and the relationship of \( Q_L \) vs. \( \lambda \) shown in Fig. 9. The design parameters are repeated here: \( L = 200 \text{ nH, } I_{pk} = 2 \text{ A and } f_s = 30 \text{ MHz.} \) At \( \lambda = 1 \), the coreless inductor has the dimensions \( d_o = 12.7 \text{ mm, } d_i = 6.3 \text{ mm} \) and \( Q_L = 171 \). The scaling factor \( \lambda = 2 \) gives a \( Q_L \) of 71.4% of the specified one.
TABLE III
MAGNETIC-CORE INDUCTOR DESIGNED AT L = 200 NH, I_{pk} = 0.5 A AND f_s = 30 MHZ WITH THE SCALING FACTOR \lambda = 0.77.

<table>
<thead>
<tr>
<th>Material</th>
<th>(P)</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>11-220-P</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Measured</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_o) (mm)</td>
<td>4.66</td>
</tr>
<tr>
<td>(h') (mm)</td>
<td>3.21</td>
</tr>
<tr>
<td>(L) (nH)</td>
<td>168</td>
</tr>
<tr>
<td>(Q_L)</td>
<td>110</td>
</tr>
</tbody>
</table>

and \(h = 6.3\) mm. We design a magnetic-core inductor with N40 material and scaling factor \(\lambda = 0.5\). The magnetic-core inductor after scaling thus has dimensions \(d'_o = 6.35\) mm, \(d'_i = 3.15\) mm and \(h' = 3.15\) mm. The closest available core size has the dimensions \(d'_o = 5.84\) mm, \(d'_i = 3.05\) mm and \(h' = 4.06\) mm. We designed and fabricated a 5-turn magnetic-core inductor with N40 materials and measure its inductance \(L\) and inductor quality factor \(Q_L\). The results are shown in Table IV. We can see the measurement results fit very well with predicted value (for the actual size) and thus (26) and Fig. 9 are verified.

TABLE IV
MAGNETIC-CORE INDUCTOR DESIGNED AT L = 200 NH, I_{pk} = 2 A AND \(f_s = 30 MHZ\) WITH THE SCALING FACTOR \(\lambda = 0.5\).

<table>
<thead>
<tr>
<th>Material</th>
<th>N40</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>T231216T</td>
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<tr>
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</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
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<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.05</td>
</tr>
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<td></td>
<td></td>
<td>4.06</td>
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<td></td>
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</tr>
<tr>
<td></td>
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<td>154</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this paper, we propose an inductor design procedure using low permeability magnetic materials. The design procedure is based on the use of Steinmetz parameters and low-permeability ungapped cores. With this procedure, different magnetic materials are compared fairly and fast, and both the quality factor \(Q\) and the size of a magnetic-core inductor can be predicted before the final design. We also compare a magnetic-core inductor design to a coreless inductor design in our design procedure. Some problems, such as optimization of magnetic-core inductors, are also investigated in this paper. The procedure and methods proposed in this paper can help to design magnetic-core inductors with low-permeability rf core materials.

APPENDIX A

APPROXIMATIONS USED IN THE PROPOSED METHODS

In Section III, we average the flux density inside a toroidal core to simplify our calculations. Because the flux density inside a toroidal core is actually not uniform, we need to know if our approximation is reasonable. For a magnetic-core inductor, the average flux density is:

\[
B_{pk} = \frac{\mu_0 \mu_r N I_{pk}}{0.5 \pi (d_o + d_i)} = \frac{I_{pk}}{0.25(d_o + d_i)} \cdot \frac{\mu_0 \mu_r L}{2 \pi h \ln \left( \frac{d_o}{d_i} \right)} \tag{33}
\]

The average total core loss:

\[
P(d_i) = P_V V = KB_{pk}^\beta \left[ \left( \frac{d_o}{2} \right)^2 - \left( \frac{d_i}{2} \right)^2 \right] h = \pi h K \left[ \frac{I_{pk}}{0.25(d_o + d_i)} \right] \frac{\mu_0 \mu_r L}{2 \pi h \ln \left( \frac{d_o}{d_i} \right)} \beta \left[ \left( \frac{d_o}{2} \right)^2 - \left( \frac{d_i}{2} \right)^2 \right] \tag{34}
\]

The total core loss \(P_0\) is calculated without the approximation of uniform flux. From (6),

\[
P_b(r) = \frac{\mu_0 \mu_r N I_{pk}}{2 \pi r} = \frac{I_{pk}}{r} \frac{\mu_0 \mu_r L}{2 \pi h \ln \left( \frac{d_o}{d_i} \right)} \tag{35}
\]

Where \(r\) specifies a radius from the center of the core (\(d_o < r < \frac{d_o}{2}\)).

\[
P_0(d_i) = \int_{\frac{d_o}{2}}^{d_o} P_b V = \int_{\frac{d_o}{2}}^{d_o} KB_{pk}(r)^\beta 2 \pi r h dr = 2 \pi h K \left[ \frac{I_{pk}}{0.25(d_o + d_i)} \right] \frac{\mu_0 \mu_r L}{2 \pi h \ln \left( \frac{d_o}{d_i} \right)} \int_{\frac{d_o}{2}}^{d_o} r^{1 - \beta} dr \tag{36}
\]

If \(\beta \neq 2\),

\[
P_0(d_i) = 2 \pi h K \left[ \frac{I_{pk}}{0.25(d_o + d_i)} \right] \frac{\mu_0 \mu_r L}{2 \pi h \ln \left( \frac{d_o}{d_i} \right)} \left[ \left( \frac{d_o}{2} \right)^{2 - \beta} - \left( \frac{d_i}{2} \right)^{2 - \beta} \right] \tag{37}
\]

If \(\beta = 2\),

\[
P_0(d_i) = \mu_0 \mu_r K L I_{pk}^2 \tag{38}
\]

We compare \(P_0\) and \(P\) and define the error by the following equation:

\[
Error = 1 - \frac{P(d_i)}{P_0(d_i)} \tag{39}
\]

from (34) and (37), if \(\beta \neq 2\),

\[
Error = 1 - (2 - \beta) 2^{2\beta-1} \left( \frac{1 - \left( \frac{d_i}{d_o} \right)^2}{1 + \left( \frac{d_i}{d_o} \right)^2} \right)^{-\beta} \tag{40}
\]

from (34) and (38), if \(\beta = 2\),

\[
Error = 1 - 2 \left( \frac{1 - \left( \frac{d_i}{d_o} \right)^2}{1 + \left( \frac{d_i}{d_o} \right)^2} \right) \ln \left( \frac{d_o}{d_i} \right) \tag{41}
\]

We can see \(Error\) only depends on the magnetic material Steinmetz parameter \(\beta\) and the dimension ratio of \(d_i\) to \(d_o\). \(Error\) is plotted in Fig. 10 for different \(\frac{d_i}{d_o}\) and \(\beta\). We can see it is below 10% if \(\beta\) is less than 3 and \(\frac{d_i}{d_o} > 0.52\) which is typical for most of available commercial magnetic cores. We assume an optimum \(d_i \approx 0.5 d_o\) later. So the error of this approximation should be lower than 10%, which is reasonable.
If $\beta = 2$, core loss $P$ is constant, independent of dimensions. If $\beta \neq 2$, similar to (37),

$$
P(h, d_o, d_i) = \frac{2\pi h K}{2 - \beta} \left[ \ln \left( \frac{d_o}{d_i} \right) \right]^{\frac{\beta}{2}} \left[ \frac{2 - \beta}{2\pi h \ln \left( \frac{d_o}{d_i} \right)} \right]^{\frac{\beta}{2}} \left( \frac{d_o}{d_i} \right)^{2 - \beta} - \left( \frac{d_i}{2} \right)^{2 - \beta}
$$

$$
h = \frac{4V}{\pi (d_o^2 - d_i^2)}
$$
Eliminating \( h \), we find:

\[
P(V, d_o, d_i) = \frac{8KV}{(2-\beta)(d_o^2-d_i^2)} \left[ I_{pk} \frac{\mu_0 \mu_r L (d_o^2-d_i^2)}{8V \ln \left( \frac{d_o}{d_i} \right)} \right]^\beta \left[ \left( \frac{d_o}{2} \right)^{2-\beta} - \left( \frac{d_i}{2} \right)^{2-\beta} \right]
\]

\[P(V, d_o, d_i) = \frac{8KV}{2-\beta} \left( I_{pk} \frac{\mu_0 \mu_r L}{2V} \right)^\beta \left[ \frac{1 - \left( \frac{d_i}{d_o} \right)^{2-\beta}}{1 - \left( \frac{d_i}{d_o} \right)^{2-\beta}} \right]^\beta \]

\[
\frac{P(V, d_i / d_o)}{P(V, 0.5)} = \left[ \frac{1 - \left( \frac{d_i}{d_o} \right)^{2-\beta}}{1 - \left( \frac{d_i}{d_o} \right)^{2-\beta}} \right]^\beta
\]

From (47), we know the total loss \( P \) only depends on the volume \( V \) and the ratio of \( d_i \) and \( d_o \). If \( d_i \) constant, we can get:

\[
P(V, d_i / d_o) \propto V^{1-0.5\beta}
\]

Because \( \beta \geq 2 \) in most materials, the total core loss \( P \) decreases as the inductor’s volume \( V \) is increased. Larger volume will always help reduce core loss and improve quality factor \( Q_L \) in a magnetic-core inductor (so long as core loss is dominant).

If we normalize \( P \) by \( P(d_i = 0.5d_o) \):

\[
P(V, 0.5) = \frac{2KV}{2-\beta} \left( I_{pk} \frac{\mu_0 \mu_r L}{2V} \right)^\beta \left[ \frac{1 - 2^{\beta-2}}{1 - 2^{\beta-2}} \right]^\beta
\]

\[
\left[ \frac{1 - 2^{\beta-2}}{1 - 2^{\beta-2}} \right] \approx \frac{2KV}{2-\beta} \left( I_{pk} \frac{\mu_0 \mu_r L}{2V} \right)^\beta \left[ \frac{1 - 2^{\beta-2}}{0.75} \right]
\]

For a constant volume \( V \),

\[
\frac{P(V, d_i / d_o)}{P(V, 0.5)} = \left[ \frac{1 - \left( \frac{d_i}{d_o} \right)^{2-\beta}}{1 - \left( \frac{d_i}{d_o} \right)^{2-\beta}} \right]^\beta
\]

Notice that in (50) the normalized \( P \) only depends on the material’s Steinmetz parameter \( \beta \) and the ratio of \( d_i \) to \( d_o \), independent of \( V \) and other individual dimension parameters (e.g., \( d_o \), \( d_i \), or \( h \)). Fig. 12 plots the normalized \( P \) vs. \( d_i/d_o \). For \( d_i > 0.3d_o \), the total loss \( P \) is flat and almost constant. Thus we can conclude that, to a first approximation, the total loss \( P \) only depends on the inductor’s volume \( V \), and is independent of its dimensions \( d_i \), \( d_o \), and \( h \) when \( d_i > 0.3d_o \). In other words, for a given total core loss \( P \), there is only one solution for the volume \( V \) that will achieve it, but there are many solutions for the dimensions \( d_i \), \( d_o \), and \( h \) that can be used. Moreover, from this result we can conclude that we needn’t concern ourselves with the optimum geometry changing with absolute scale (in terms of core loss), so may confidently use the results of Section III in finding an optimum inductor design.

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