Abstract

We show that children in the Tsimane’, a farming-foraging group in the Bolivian rain forest, learn number words along a similar developmental trajectory to children from industrialized countries. Tsimane’ children successively acquire the first three or four number words, before fully learning how counting works. However, their learning is substantially delayed relative to children from the United States, Russia, and Japan. The presence of a similar developmental trajectory likely indicates that the incremental stages of numerical knowledge—but not their timing—reflect a fundamental property of number concept acquisition which is relatively independent of language, culture, age, and early education.

Introduction

Children’s acquisition of the natural numbers is an exemplar of conceptual change, where an essentially novel system of concepts is constructed by young learners (Carey, 2009). Children begin acquiring number words by memorizing the counting sequence, “one,” “two,” “three,” etc., without knowledge of each word’s cardinal meaning (Fuson, 1988). Children then progress through a stereotypical series of subset-knower levels, successively learning the meaning of “one,” then “two,” then “three” and sometimes “four” (Wynn, 1990, 1992; Sarnecka & Lee, 2009; Lee & Sarnecka, 2010a, 2010b). After learning the meaning of “three” or “four,”—typically at around age 3:6 in the US—children undergo an apparent conceptual shift, and rapidly acquire the meanings of many higher number words all at once. At this stage, children become cardinal principle (CP) knowers, and they can accurately use counting to determine cardinality (Wynn, 1990, 1992). In becoming CP-knowers, Carey (2009) argues children undergo a drastic conceptual change, shifting from an object-based core system for representing the small cardinalities to a richer, constructed system capable of representing any exact cardinality via counting (for alternatives, see...
R. Gelman & Gallistel, 1978; Leslie, Gelman, & Gallistel, 2008). Piantadosi, Tenenbaum, and Goodman (2012) provide a fully-implemented computational model along similar lines to Carey (2009), demonstrating that this developmental progression can be explained by statistical inference over a rich space of hypotheses, operating in response to observed data. In this model, learners transition between different structured hypotheses (compositional functions) that manipulate sets of objects, as they observe data from parental utterances. Both Carey’s and Piantadosi et al.’s accounts treat number as a fundamental conceptual innovation—likely reflecting deep and uniquely human cognitive processes that eventually come to support our rich adult conceptual systems. For interesting critiques and discussion of these theories, see Davidson, Eng, and Barner (2012), Rips, Asmuth, and Bloomfield (2006, 2008), and Rips, Bloomfield, and Asmuth (2008).

To our knowledge, all previous studies of these stages of numerical learning have focused on children from industrialized countries, primarily the United States. Studies of adult numerical knowledge in vastly different cultures, have revealed that languages need not express natural number concepts at all (Frank, Everett, Fedorenko, & Gibson, 2008; cf. Gordon, 2004), or have provided evidence for universal and distinct systems of numerical representation (Pica, Lemer, Izard, & Dehaene, 2004). For learning number words, typical middle-class children in the United States are in a very particular educational setup including a society that emphasizes numeracy and education from a young age, the presence of educational television and toys, and parents who are themselves very highly educated.

Here, we examine the trajectory of number learning in the Tsimane’, an indigenous farming-foraging group in the Bolivian rain-forest (see Huanca, 2008, for a cultural overview). The Tsimane’ number roughly 8000 and live in villages in the Department of Beni in lowland Bolivia, near the town of San Borja. The Tsimane’ have primarily been studied with respect to health (Foster et al., 2005; McDade et al., 2005; Godoy et al., 2006; Gurven, Kaplan, & Supa, 2007), economics (Kirby et al., 2002; Godoy, Jacobson, & Wilkie, 1998; Godoy & Jacobson, 1999; Henrich et al., 2001; Reyes-Garcia et al., 2003; Godoy et al., 2010, and indigenous knowledge (Reyes-Garcia et al., 2003; Reyes-Garcia et al., 2005; Huanca, 1999). For Tsimane’ children, educational advancement—particularly early education—is not prioritized, in contrast to most industrialized countries. The types of educational toys and television common in the US are not available to Tsimane’ children. Indeed, many adults have no formal education, and poor or no knowledge of basic arithmetic (Piantadosi, Jara-Ettinger, & Gibson, forthcoming). The Tsimane’ therefore provide an ideal test case for determining what number word learning looks like away from many features of industrialized culture.

We are primarily interested in whether Tsimane’ children progress through the same stages of subset knower levels as US children. As we show, Tsimane’ children acquire number words substantially later than their counterparts in industrialized cultures. Because of this delay, their stages of knowledge are especially interesting because their number acquisition happens with more mature cognitive machinery. Several distinct learning trajectories are possible, each potentially quite informative about the underlying representational and learning mechanisms. For instance, children might come to understand cardinality earlier (in the count list), after learning only one or two number-word meanings rather than three or four. This might suggest that maturational constraints are what prevent learners from making
the CP-transition—a three-knower only becomes a CP-knower when their representational machinery has developed or aged enough to support the transition. In older children, this development would have already happened, so learners would make the inductive leap without going through all the stages observed in younger children. An alternative pattern is that learners might persist up until five- or six-knowers, or higher. Carey (2009), for instance, suggests that it is meaningful that children make the CP-transition around three or four, right where their ability to track sets is limited. If this is true—the CP-transition happens when children run out of representational resources for smaller sets—and older children have greater representational capacities for small sets, they should make the transition later in the count list. Finally, one could imagine a wholly more variable developmental trajectory—perhaps children without the educational resources of industrialized countries acquire number words in a more haphazard order or show very little systematicity. This could happen for certain kinds of learners if there’s nothing really “special” about the earlier cardinalities that lead to their early acquisition, or perhaps if parents do not emphasize reciting the counting list in order.

An alternative to such age effects is that in learning number, the same developmental stages occur regardless of age. In this sense, number-word learning could proceed in much the same way as general vocabulary learning, where learners seem to go through similar stages of growth regardless of their age (Snedeker, Geren, & Shafto, 2007, 2012). For number learning, the incremental stages of numerical knowledge might be an inherent part of learning number words. Contemporary accounts of knower-levels predict that knower-levels should occur for any learners of number, likely due representational or inferential considerations of the learning problem (e.g. Piantadosi et al., 2012; Carey, 2009). If similar stages of development do not occur for Tsimane’ children—either at the same or later ages—then the predominant theories must miss an important aspect of how the learning works; conversely, replication of knower-levels in a very different culture and language would indicate that the stages of number knowledge are truly fundamental to learning number.

Give-N with Tsimane’ children

Research was conducted in the summer of 2012 while in San Borja, Bolivia. We collaborated closely with R. Godoy and T. Huanca from the Centro Boliviano de Investigación y de Desarrollo Socio Integral (CBIDS), who provided translators, logistical coordination, and expertise in Tsimane’ culture. Our studies were approved by the Gran Consejo Tsimane’ (Tsimane’ grand council), as well as institutional IRBs.

We recruited 92 children aged 3 to 12 to perform a simple variant of Wynn (1992)’s Give-N task (42 males and 50 females). Simultaneously with the child studies, adults from the village were tested on other numerical tasks.

Methods

Tsimane’ children were tested in each village’s schoolhouse, which is also used for community gatherings. Because the arrival of researchers is rare and interesting to the Tsimane’, it proved difficult to separate children for individual testing (a general feature of working with the Tsimane’). Tests were therefore not private, as participants were typically surrounded by other children, their parents, and others from the village. At the start of
testing, a translator explained the task and that people should not provide help to the
participants since we are interested in what the children know. Each child participant was
then asked to move \( N \) coins from one half-sheet of white paper to another, with \( N \) ranging
through a random order\(^1\) of 1, 2, 3, \ldots, 8. Occasionally, children would receive help from
others (“pick up one more!” or “that’s right!”). These instances were noted in the data,
and the target quantity was asked for again at the end of testing. In this case, the response
without any help was used as the child’s response. In a few pilot instances, children were
also asked multiple times for target quantities; for instance, if a child correctly counted
1, 2, and 4 but not 3. These additional trials were run out of curiosity on only several
children, so here we only report and analyze the first unaided response children made to
each cardinality. Due to the time and social constraints of testing in Tsimane’ villages,
children were only asked for each numerosity one time, other than the occasional repeated
trials. This kept the individual subject time very short (approximately 2 minutes) and
prevented these children who are generally unaccustomed to such testing situations from
becoming bored. Child participants were compensated with both educational materials
(pens, pads, and erasers) and small toys (toy rings, bracelets, and airplanes). While we
almost always observed children pick up the coins one at a time, we did not typically
observe children count out loud, except for some who performed perfectly or near-perfectly
on the task (i.e. CP-knowers).

Sophisticated and elegant techniques have been developed for classifying children
into knower levels (Sarnecka & Lee, 2009; Lee & Sarnecka, 2010b, 2010a; Negen, Sarnecka,
& Lee, 2011). Our primary analysis, however, is concerned with testing if the data we
observe plausibly can be described in a knower-level theory, rather than finding the best
classification for each child. Here, we therefore construct a classification rule for knower-
levels and determine if our ability to classify children according to this rule out-performs
statistically-matched null data with no inherent knower-levels. We use a rule similar to
Wynn (1992) that formalizes the types of knowledge an \( N \)-knower should have\(^2\), namely
correctness for the first \( N \) cardinalities, and exclusivity such that these cardinalities are not
given for higher words:

1. A child is a CP-knower if they make at most one mistake on the 8 trials.
2. A child is an \( N \)-knower if they are correct on the first \( N \) responses, they never
give \( N \) for any number words higher than the \( N \)’th, and they get at most one other answer
correct.
3. A child is a 0-knower if they get at most one answer correct, assuming they are
not classified under (2).
4. Otherwise a child cannot be classified into a knower-level.

Because 8 (the entire set) is frequently returned by children, queries to “eight” are not
included in the analysis when applying rules (2) and (3). It is important to emphasize
that there are many possible behavioral patterns that fail to be classified as a knower level

\(^1\)We also include in our analysis 11 pilot subjects who performed the task always starting with 1, and
then a random order of the other digits.

\(^2\)We also used the tool developed by Negen et al. (2011), building off of Lee and Sarnecka (2010b, 2010a),
who constructed a Bayesian data analysis model for categorizing children into knower-levels. For our data,
use of the spreadsheet from Negen et al. (2011) gave somewhat uninterpretable results, likely because of
our unusually small number of samples per subject and a different baseline distribution of responses among
Tsimane’ children than is assumed in their model.
under this scheme. For instance, children who correctly responded to more than one non-
sequential number word would not be categorized as any “standard” developmental stage. 
Our classification scheme therefore implements fairly strong criteria about what we expect 
children to look like according to previous developmental studies; the strictness of our rule 
means it would be easy for us to fail to find these patterns if Tsimane’ children truly do 
behave differently.

Results

Results from the Give-N task are shown in Tables 1 and 2. In these tables, each row 
shows a single participant with their responses to each of the eight cardinalities. While 
the cardinalities were queried in random order, they appear in sorted order in this table. 
Missing data points (e.g. places where children were helped on both a trial and the repeat, 
or experimenter errors) are noted with a dash. For visualization, this table colors correct 
responses blue. Each row also shows the child’s age, sex, and number of years of education. 
Table 1 shows children who could readily be classified into 0-, 1-, 2-, 3-, 4-, or CP-knower 
levels, while Table 2 shows those who were classified as something else, either 5- and 6-
knowers, or unclassifiable under our rules (“X”). We note, however, that what we call CP-
knowers here are only children who were very accurate on the task—we did not explicitly 
test their knowledge of the relationship between counting and cardinality.

We find that 76% of children are assigned to a standard subset-knower level (1-knower 
through 4-knower or CP-knower). This can be interpreted as roughly the percentage of 
children who can be “fit” into a knower-level classification. As shown in Table 1, there do appear to be knower-level stages, with children successively figuring out the number words up to four and then transitioning to knowing all the word meanings. Of the remaining children, 2% are classified as 5-knowers, 0.0% as 6-knowers, and 22% are unclassifiable under our rules. However, examination of the data reveals that many of the children who are counted as “unclassifiable” may actually have a knower-level, and fail to be classified 
only due to the strictness of our rules. Similarly, the 5- and 6-knowers each have only made 
two off-by-one errors in counting, likely indicating that they are in truth CP knowers.

It is important to compare these classification rates to a statistical baseline in order to 
determine (i) if the percentage of children classified as standard knower-levels is statistically 
meaningful, and (ii) if the subset knower-levels beyond 4-knowers are statistically meaning-
ful. To address this, we performed a permutation test that shuffled the responses to each 
number word across children (see Davison & Hinkley, 1997, for an overview of permutation 
tests). This keeps the distribution of responses to each word exactly the same as in our real 
data, but scrambles particular responses across children. Therefore, the ability to classify 
such shuffled data into “knower-levels” corresponds to a matched baseline classification ac-
curacy when the process generating the data really does not have knower-levels. For (i), 
we find that we are able to classify children at above chance levels. In shuffled samples, 
32% of children are assigned a standard knower-level (0- through 4-knower, or CP-knower). 
Our finding of 76% classification is significantly and substantially above this rate in the 
permutation test ($p < 0.0001$). Thus in real data, we are better able to group kids into 
knower levels than in statistically matched null data with no inherent knower-levels. Note 
that this analysis counts all of the children in Table 2 as unclassifiable, which is likely a very 
conservative assumption. For (ii), we find that neither the 2% rate of 5-knowers or the 0%
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| CP            | F   | 9   | 3    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| CP            | M   | 10  | 4    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| CP            | M   | 11  | 2    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| CP            | M   | 11  | 2    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| CP            | M   | 11  | 4    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| CP            | M   | 11  | 5    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| CP            | M   | 12  | 2    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| CP            | M   | 6   | 1    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
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| CP            | M   | 8   | 2    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |
| CP            | M   | 9   | 3    | 1  | 2 | 3 | 4 | 5 | 6 | 7 |

Table 1: Behavioral responses of children classified into knower-levels, along with their sex, age, and education. Blue denotes a correct response. Children who could not be classified by these rules are shown in Table 2.
rate of 6-knowers is significantly different from their expected values of 0.6% ($p = 0.11$) and 0.06% ($p = 0.95$) respectively. This means that our data provides no statistical evidence in support of 5- or 6-knower levels.

It is also informative to perform the same permutation tests for (i) and (ii), but only on the non-CP-knowers. Success on this statistical test would ensure that our significant results are not driven by some children performing very well (the CP-knowers) and some performing less well. For this, we find that while 69% of the non-CP-knowers are classified into standard knower-levels, only 54% are classified as such in the permutation test, a substantial and significant ($p = 0.0001$) difference. Thus, even among just the non-CP-knowers, we are better able to “fit” children into a knower-level theory than would be expected by chance. As above, we can examine the significance of 5- and 6-knowers in this restricted sample. Here we find more 5-knowers than would be expected (3% vs. 0.1%, $p = 0.004$) and no support for 6-knowers (0% vs. 0.1%, $p = 0.99$). While this analysis provides some support for 5-knowers in the sample, we note that this pattern is driven by only two children (Table 2), both of which could plausibly be CP-knowers who happened to make two errors.

A key feature of the knower-level theory is that children successively learn the number words. We can test this successive learning by determining how likely it is for children to demonstrate knowledge of a number $n$, but fail on $n-1$. High percentages would suggest that many number words are not learned in counting order. In fact, as examination of Tables 1 and 2 suggests, we find that it is relatively rare for children to succeed on $n$ and fail on $n-1$: this occurred on only 9% of the time, as compared to an expected value of

Table 2: Children that were not classified as standard knower levels by our rules, approximately 22% of children run. Note that most of these children are very similar to some knower level, but with more than one mistake or higher cardinality correct. This suggests that many of these children may truly have a knower level, but are mis-classified due to noise and the strictness of our rules.

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20% in permuted samples ($p < 0.0001$). This analysis held also when looking only at non-CP-knowers (11% vs. 16%, $p < 0.0001$), suggesting that Tsimane’ children in the sample learned number words successively.

In general, our analysis shows that the Tsimane’ pattern largely as would be expected under the knower-level account. Our data provides little support for the existence of higher subset-knower levels (five- and six-knowers). We additionally note that the data presented in Tables 1 and 2 intuitively supports a knower-level hypothesis, with almost all children patterning like subset- or CP-knowers. In Table 2, there are children who, for instance, look like CP knowers but make more than one mistake, or who are like $N$-knowers, but appear to have partial knowledge of the next several cardinalities. This table also reveals several children who do not fit well into a knower-level framework—for instance, a child who gets 2, 3, 4, 5 correct but not 1. We believe such instances are likely noise, rather than genuine variability in the developmental trajectory.

A striking feature of these results is that children appear to go through these stages much later than children in industrialized countries. Figure 1 compares our data from the Tsimane’ to children in the US, Japan, and Russia. Data from these other countries comes from a variety of knower-level studies\(^3\) (Negen & Sarnecka, 2009, in press; Sarnecka & Carey, 2008; Sarnecka & Gelman, 2004; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007; Sarnecka & Lee, 2009; Slusser & Sarnecka, 2009, 2011; Slusser, Sarnecka, Cheung,

\(^3\)We are grateful to Meghan Goldman and Barbara Sarnecka for compiling and sharing these data.

Figure 1. The age ranges of different subset-knower levels from a variety of previous studies (US, Japanese, and Russian), compared to Tsimane’ children. The solid ranges show the mean ages plus and minus one standard deviation; the dotted lines show the minimum and maximum ages for each level. This demonstrates shows a substantial delay in the course of development, on the order of 2 ~ 6 years, for Tsimane’ children.
Table 3: Results of a mixed effect logistic regression predicting accuracy from demographic variables, including random effects of child, queried set size, and village. Note that age and education have not been standardized, and so the coefficients can be interpreted as the change in log-odds of a correct response for each additional year along these variables. Sex is dummy-coded.

& Barner, 2009). This figure shows the age range for 0- through CP-knowers (“C”) for each group. Here, the solid ranges denote the means plus or minus one standard deviation, illustrating the range of typical values (rather than the confidence in the estimated mean). The dotted lines indicate the minimum and maximum age for each level. This plot shows that Tsimane’ children are substantially delayed relative to these other countries, in some cases essentially starting learning number only by the time other children are finished, with a delay between stages ranging from about 2 to 6 years. Note the substantially higher variability (standard deviations) for when children learn number in Tsimane’, which is likely attributable to lower levels of education and numeracy in the Tsimane’ culture, relative to the US, Russia or Japan. The presence of a delay is perhaps not surprising, given findings showing that children in low-SES groups of the United States are also delayed in number learning (Sarnecka, Goldman, & Slusser, forthcoming).

Next, we present a logistic regression analysis, to reveal how the demographic predictors—age, sex, and years of education—influence numerical knowledge. Table 3 shows the results of a mixed effect logistic regression model (A. Gelman & Hill, 2007), with random effects of target cardinality, child, and village. While age and education are substantially collinear ($R = 0.81$), we find that the full model is better than the reduced model without age ($p = 0.007$) and better than the reduced model without education ($p = 0.03$). This indicates that age and education are both significant predictors of number knowledge. To intuitively interpret the size of these coefficients, the coefficients indicate that with zero education, the age at which children achieve 50% on the Give-N task for numbers 1 through 8 (intuitively, the point of a “four-knower” or approximately the timing of the CP transition) is at about 3.258/0.49 ≈ 6.6 years old. Similarly, we can compare the coefficients for age and education: in terms of performance on the task, a year of education is worth roughly about 0.71/0.48 = 1.46 years of life without education. Interestingly, in our sample, all children who were classified as CP-knowers had at least one year of education, indicating that progressing beyond subset-knower levels might be difficult from a Tsimane’ child’s typical input, without education.

In order to investigate children’s delayed learning, we additionally examined children’s
knowledge of the counting routine. We attempted to elicit counting out loud in three of
the villages visited. While counting is a skill that US children are already able to do by
the time they start learning number words at age 2 or 3 (Fuson, 1988), we were generally
unsuccessful in eliciting counting among the Tsimane’ children in this study who were not
CP-knowers: $13/38 = 34\%$ of non-CP-knowers refused to count out loud, $9/38 = 24\%$
counted correctly only to 4 or fewer, and the rest counted to a number greater than 4
$(16/38 = 42\%)$. We additionally asked parents from two villages if they taught children to
count before school, and $19/24 = 79\%$ reported that they had not, although our cultural
experts said that normally parents do. Non-CP-knowers’ refusal to count is consistent with
lack of knowledge of the counting routine, and indeed our subjective sense was that most
subset-knowers did not typically know the words for numbers greater than “three” or “four.”
However, psychology paradigms are unusual to indigenous populations so children’s refusal
to count might be due to other factors such as shyness or confusion.

Discussion

Our results demonstrate that children in a starkly different culture still go through
the same developmental stages in number learning as is observed in industrialized countries.
However, these stages happen at a much later age. Importantly for developmental theories,
the fact that the same developmental trajectory can occur at such later ages argues strongly
against any kind of maturational account of number learning. Indeed, the stages of number
knowledge are not due only to, for instance, universal cognitive resource limitations of early
childhood, since the same pattern is found in substantially older children. As such, our
findings echo results from Snedeker et al. (2007, 2012), who demonstrate that international
adoptees who learn language later than typical infants still show similar developmental
patterns in vocabulary growth.

Because these successive stages of number knowledge are found in such distinct cul-
tural and educational environments, they should be considered one of the primary data
points for developmental theorizing to explain. Carey (2009) explains these stages by posi-
ting that children’s early representations are supported by a system capable of tracking a
small number of objects (Le Corre & Carey, 2007, 2008). The resource limitations of this
system prevent it from representing more than 3 or 4 objects, so learners must develop a new
system—based on counting—in order to determine higher cardinalities. Piantadosi et al.
(2012)’s account is closely related, caching out resource limitations in long term conceptual
memory in terms of a limited set of representationally primitive operations (composable
functions) available to learners. The key feature of both of these accounts is that the early
stages of numerical knowledge differ in essence from the later representations, and that the
early form is supported by an innate (or very early acquired) system for small set manipu-
lation. In such accounts, the discrete stages of knowledge result from inherent properties
of children’s representational system. This view that the stages of knowledge are driven
by the learner’s core representational system accords with our finding of similar stages in a
substantially distinct culture.

Carey (2009) suggests that it is critically important that children learn to recite the
list of number words in order before making the CP-transition. She argues that this list
provides a placeholder structure that provides a framework for the CP-induction—moving
“one more” in terms of cardinality equates to one further item on the memorized list of
words. An interesting possibility is that children in this population may not learn the counting routine before school—and perhaps not at all without school—explaining their overall inability or unwillingness to count out loud. If Tsimane’ children do not have this linguistic structure in place, they may essentially be acquiring number words primarily in order frequency without regard for their sequential relationship. Because higher number words have lower frequency in general (Sarnecka et al., 2007; Barner, Libenson, Cheung, & Takasaki, 2009), this type of learning would still predict successive acquisition of low number words. Critically, though, such children may have difficulty making the CP-transition if they are lacking the placeholder structure (and the placeholder structure is truly crucial to the key induction). An important direction for future work will be to explore relationship between knowledge of the linguistic structure and learning of the meanings, among populations like the Tsimane’ where such knowledge is likely highly variable.

Generally, these results support the hypothesis that the stages of numerical knowledge result from rational statistical learning, a view made explicit in Piantadosi et al. (2012) but one that would in principle be compatible with possible formalizations of Carey (2009) or Leslie et al. (2008). Under Piantadosi et al. (2012)’s account, for instance, learners go through the subset-knower stages because these stages represent the “best” compromise between having a simple conceptual system (a 0-knower) and perfectly explaining the observed data (a CP-knower). As learners accumulate more data, they justify increasingly complex representations, until discovering a CP-knower system. It is reasonable to expect that older children—or indeed any learners—would make similarly smart inferences about how to relate their representations to observed data and thus traverse the same developmental trajectory. The delay in Tsimane’ is explainable in this account: while we do not have a Tsimane’ corpus to evaluate the frequency of number words, it is likely that among indigenous farming-foraging groups (or hunter-gatherer groups), number words are used much less frequently than in industrialized countries, and mathematical ability appears to be less common (Piantadosi et al., forthcoming). In this case, it should simply take Tsimane’ children longer to acquire the data necessary to transition between subset-knower hypotheses.

A data-driven explanation for the Tsimane’ learner’s delay fits with many studies documenting influences on early numeracy in industrialized countries. For instance, parent-child interaction about numbers is extremely important (LeFevre et al., 2009; Skwarchuk, 2009; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Gunderson & Levine, 2011; Kleemans, Peeters, Segers, & Verhoeven, 2012), and so children of adults who are less numerate or place less value on teaching number would incur substantial delays (see also Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Jordan & Levine, 2009). Numeracy across industrialized countries has also been studied extensively. Broadly, children’s performance on numerical tasks correlate with cross-cultural differences in parental involvement with numeracy (LeFevre, Polyzo, Skwarchuk, Fast, & Sowinski, 2010; LeFevre, Clarke, & Stringer, 2002), and the degree to which early education values mathematics, as compared to other topic areas (Aunio, Ee, Lim, Hautamäki, & Van Luit, 2004; Aunio et al., 2006; Aunio, Aubrey, Godfrey, Pan, & Liu, 2008). In light of these findings, the delay among Tsimane’ is not surprising*.

*We note that in industrialized countries, delayed numerical acquisition substantially hinders later math achievement (Aunio & Niemivirta, 2010; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Jordan, Glutting,
Finally, it is also important to consider other possible influences on Tsimane’ learning. For instance, a possible contributing factor may be the way in which Tsimane marks grammatical number. Sarnecka et al. (2007) argues that the lack of grammatical number in Japanese slows Japanese children’s number learning, although there are other plausible differences between the languages which could cause the delay (Barner et al., 2009). Le Corre, Li, Huang, Gia, and Carey (under review) present evidence from Mandarin showing that despite apparently better input for learning, Mandarin learners are delayed, they argue, due to the Mandarin’s numerical syntax (including classifiers). Tsimane’ maintains a weak singular/plural distinction, with marking on verbs only in certain situations (Sakel, 2011), and noun marking only on human referents or when plurality is focused. Marking is done through the clitic in, which can appear with variable locations and sometimes multiple times in order to emphasize plurality (Sakel, personal communication). It is conceivable that if early number learning is supported primarily by grammatical number, children may be delayed due to the complexity or variability of this system. However, we believe this is unlikely to explain the large difference between Tsimane’ and languages like Japanese (Table 1) in which most utterances lack singular/plural marking. Finally, additional possibilities for the delay must also be considered, including the fact that arithmetic in schools is generally taught in Spanish, not Tsimane’, so children of school age might face a challenge with two languages; indeed, CP-knowers who counted out loud appeared to primarily do so in Spanish. There is additionally the possibility that the delay is in part due to other factors that differ in the Tsimane’, such as documented nutritional and developmental differences (Foster et al., 2005).

Conclusion

These results suggest that the series of number-knower levels reported for children in the US, Japan and Russia is also found in children of the Tsimane’. Like children in industrialized countries, Tsimane’ children learn the first three or four number words before arriving at a fundamental insight that rapidly provides them with higher number word meanings. Interestingly, however, they do so at a much later age, indicating that the pattern observed in number learning does not result from, for instance, maturational effects. Following Carey (2009), these results suggest that the learning trajectory reflects a fundamental developmental process—one that is not an artifact of US education and culture, but likely the result of shared cognitive representations and processes. Thus, our results suggest that this pattern in number learning is a likely developmental universal, to be expected in any place where children must discover how language expresses natural number concepts.

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