Higgs couplings can be measured at a linear collider with high precision. We estimate the uncertainties of such measurements, including theoretical errors. Based on these results we show an extrapolation for a combined analysis at a linear collider and a high-luminosity LHC.

A new particle compatible with a Higgs boson \(^1\) has been discovered by ATLAS \(^2\) and CMS \(^3\). While the existence of a narrow light resonance has been established beyond reasonable doubt, the endeavor to study its properties has only begun \(^1\).

Soon, the LHC energy will be increased to up to 14 TeV. Detector and machine upgrades can give us an integrated luminosity of up to 3000 fb\(^{-1}\) at this energy, defining a high-luminosity LHC (HL-LHC) scenario. A linear collider (ILC) first accumulating 250 fb\(^{-1}\) of data at 250 GeV (LC250) \(^5\) \(^6\), and upgradable to 500 fb\(^{-1}\) at 500 GeV (LC500) \(^5\), can be viewed as a dedicated Higgs factory. Further steps in center-of-mass energy above 1 TeV are possible \(^7\).

We study the determination of the Higgs boson couplings at a linear collider running at energies up to 500 GeV and combine its expected results with extrapolated measurements at the HL-LHC.

Analysis setup — Higgs couplings are defined as prefactors of the respective Lagrangian terms coupling the Higgs field to other Standard Model particles. They are defined relative to the tree-level couplings predicted by the Standard Model \(^8\) \(^9\),

\[
g_{x \rightarrow H} \equiv g_x = (1 + \Delta_x) \, g^\text{SM}_x . \tag{1}
\]

The loop-induced Higgs-photon (Higgs-gluon) coupling then reads

\[
g_{\gamma \gamma H} \equiv g_{\gamma} = (1 + \Delta^\text{SM}_\gamma + \Delta_\gamma) \, g^\text{SM}_\gamma . \tag{2}
\]

A modification of the underlying tree-level couplings induces \(\Delta^\text{SM}_\gamma\). The remaining \(\Delta_\gamma\) characterizes genuine non-Standard Model contributions. Equivalent parameters \(\kappa_x \equiv 1 + \Delta_x\) have been introduced in Ref. \(^10\), without disentangling modified tree-level couplings and new states in the loop-induced couplings.

All extrapolated HL-LHC measurements are based on the previous detailed studies \(^8\) \(^11\). They include a subjet analysis for associated \(VH\) production, but not the subjet analysis in the \(t\bar{t}H\) channel \(^12\). The \(VH\) channel is crucial for the determination of \(\Delta_0\) \(^8\) \(^13\).

All statistical errors are scaled to the increased integrated luminosity. The statistical component of experimental uncertainties on background rates, which are determined from data, will improve correspondingly. The increase of statistics will also improve the statistical component of the systematic errors. On the other hand, experimental conditions (pile-up) will become significantly more difficult for some of the crucial channels, like weak-boson-fusion Higgs production and hadronic Higgs decays. Therefore, the same performance of particle identification and \(b\)-tagging as for lower instantaneous luminosity is assumed, i.e., the relative errors for experimental systematics used in the previous studies are not changed. Theory errors on the cross sections and on the Higgs branching ratios are added linearly and included via the profile likelihood \(\text{Rfit}\) scheme \(^14\) \(^15\). Because Higgs analyses start to depend more on exclusive jet observables \(^16\), for which collinear factorization may no longer hold, rendering the application of fixed-order QCD corrections difficult, we refrain from postulating an improved theory uncertainty.

All linear collider measurements used in this study are taken from Refs. \(^17\) \(^19\), with the exception of the additional measurement of the \(W\)-fusion process with a \(H \rightarrow b\bar{b}\) decay at 250 GeV \(^20\). The expected error on the luminosity measurement of 0.3\% \(^21\) is added to each measurement (100\% correlated), but has no effect on the end result. As the ILC will likely first run at 250 GeV and then at 500 GeV, the latter results will include all 250 GeV measurements. Also for the ILC each observable, with the exception of the inclusive Higgs-strahlung cross section measurement, is the product of cross section times branching ratio, so we apply the same \(\text{Rfit}\) procedure as for the HL-LHC.

For the linear collider production cross sections we assume an error of 0.5\% for \(ZH\) and \(\nu\bar{\nu}H\) production and of 1\% for \(t\bar{t}H\) production. As a consequence, the theoretical errors for HL-LHC measurements are dominated by the cross sections, whereas at the ILC the error on the branching ratio is the limiting factor.

Higgs width — the main task of Higgs analyses is the precise simultaneous determination of the individual couplings \(^22\). Hadron collider experiments cannot directly measure a narrow Higgs width, so they cannot simultaneously constrain the couplings and new contributions to the total width. In addition, the Higgs decay to charm quarks is challenging, but important, because it contributes to the total width at the per-cent level. There-
Figure 1: Expected precision for Higgs coupling measurements at the HL-LHC, ILC at 250 GeV and their combination. For the latter we also show the fit including \( \Delta_c \). The inner bars for HL-LHC denote a scenario with improved experimental systematic uncertainties.

Therefore, we assume

\[
\Gamma_{\text{tot}} = \sum_{\text{obs}} \Gamma_x(g_x) + 2\text{nd generation} < 2 \text{ GeV} . \tag{3}
\]

The upper limit of 2 GeV takes into account that a larger width would become visible in the mass measurement. The second generation is linked to the third generation via \( g_{c} = \frac{m_{c}}{m_{t}} g_{t}^{\text{SM}}(1 + \Delta_t) \). The leptonic muon Yukawa might be observable at the LHC in weak boson fusion or inclusive searches, depending on the available luminosity [23].

At the ILC the situation is very different: the total width is not needed. A difference in the interpretation of our results we need to keep in mind: while electroweak corrections are not expected to interfere at the level of precision of our HL-LHC analysis, at the ILC the individual measurement of Higgs couplings will most likely require an appropriate ultraviolet completion [24]. In this largely experimentally driven study we assume the existence of such a picture.

At a linear collider the errors on Higgs branching ratios \( BR_x \) or particle widths \( \Gamma_x \) are crucial [25]. As theory errors on the latter we assume 4% for decays into quarks, 2% for gluons, and 1% for all other decays [8]. Translated into branching ratios this corresponds for example to an error around 2% on the branching ratio into bottom quarks. Further improvements on these values in the future are possible, but we decided to remain conservative.

The error on the branching ratios follows from simple error propagation, where theory errors are added linearly,

\[
\delta BR_x = \sum_k \left| \frac{\partial}{\partial \Gamma_k} BR_x \right| \delta \Gamma_k .
\]

\[
= \frac{1}{\Gamma_{\text{tot}}} \left( BR_x \sum_k \delta \Gamma_k + (1 - 2BR_x) \delta \Gamma_x \right) . \tag{5}
\]

Higgs couplings — the result of an individual and simultaneous determination of the Higgs couplings are shown in Fig. [1]. For the LHC, we need to make an assumption about the width, shown in Eq. [3]. At LC250 the inclusive \( ZH \) rate gives direct access to \( \Delta_Z \) at the percent level. No assumption about the width is needed.

The simplest model for modified Higgs couplings is a global factor \( \Delta_H \), which arises through a Higgs portal [20].
or in simple strongly interacting extensions \cite{27}. In Fig. 1 we see that we can measure this single parameter at the HL-LHC with an error around 4%. A further increase in statistics would not improve this error as this determination is limited by the theoretical error. Reducing the luminosity error from 5% to 2% and neglecting all other systematic errors, the accuracy improves only by about 20% as shown by the inner bars, corroborating the theory limitation. The LC250 with its smaller theoretical error improves the determination to about 1% with a further decrease in the error for the LC500 as shown in Fig. 2. The combination of measurements from HL-LHC with LC250 or LC500 is dominated by the precision of the linear collider. All numerical values in Figs. 1 and 2 are also reproduced in Table I.

The (more) model independent determination of individual Higgs couplings is also shown in Fig. 1. At the HL-LHC alone the most precisely measured couplings are those to the weak bosons at about 8%. The measurement of the quark Yukawas is challenging, as reflected in the larger errors. Unless one of the ingredients to this analysis significantly changes, LHC will measure Higgs couplings to Standard Model particles at an excellent 8–15% level, assuming no significant contributions from new physics channels *.

For easier comparison in Fig. 1 and Fig. 2 the constraint on the Higgs width and the link between the second and third generation was applied also for LC250 and LC500. However, once linear collider data enter the picture, these two assumptions can be removed. This introduces $\Delta_c$ as an additional parameter. The impact of the additional parameter on the results is illustrated only in the combined analyses.

Compared to the individual results for the HL-LHC or the LC250, a significant improvement of the joint analysis is clearly visible. For example, $\Delta_W$ is improved in the combined analysis; according to Eq. (2) this improves the measurement of $\Delta^{SM}_t$, so $\Delta_t$ can be determined at the 5% level. When adding $\Delta_c$, the precision decreases slightly, shown in Fig. 1 as expected from adding an additional parameter to a system while using the same observables. The limitation of the LC250 coupling determination comes from the total width, which can be measured to 10% \cite{20} due to the small cross section for $W$-fusion at this center-of-mass energy. As a matter of fact, at the LC250 all couplings with the exception of $\Delta_b$ are limited by statistics.

Once the ILC energy is increased to 500 GeV, more decay channels can be observed in $W$-fusion production. The results of the coupling measurement are shown in Fig. 2. Compared to the LC250 setup the precision on the tree-level couplings is improved by roughly a factor 2 and $\Delta_t$ is now measured directly. The overall picture of the combined HL-LHC and LC500 analysis does not change qualitatively: in particular the loop-induced couplings benefit significantly when compared to individual linear collider and HL-LHC analyses.

Table I: Errors on Higgs couplings for different collider scenarios, as shown in Figs. 1 and 2

<table>
<thead>
<tr>
<th>$\Delta_H$</th>
<th>$\Delta_L$</th>
<th>$\Delta_{c+}$</th>
<th>$\Delta_{c-}$</th>
<th>$\Delta_{x+}$</th>
<th>$\Delta_{x-}$</th>
<th>$\Delta_{y+}$</th>
<th>$\Delta_{y-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HL-LHC +</td>
<td>HL-LHC +</td>
<td>LC250</td>
<td>LHC + LC250</td>
<td>LHC + LC250</td>
<td>LHC + LC500</td>
<td>LHC + LC500</td>
<td></td>
</tr>
<tr>
<td>improved</td>
<td>improved</td>
<td>$\Delta_c$</td>
<td>$\Delta_c$</td>
<td>$\Delta_c$</td>
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<td>$\Delta_H$</td>
<td>$\Delta_L$</td>
<td>$\Delta_{c+}$</td>
<td>$\Delta_{c-}$</td>
<td>$\Delta_{x+}$</td>
<td>$\Delta_{x-}$</td>
<td>$\Delta_{y+}$</td>
<td>$\Delta_{y-}$</td>
</tr>
</tbody>
</table>

* For this specific LHC scenario we estimate the effect of flat theory errors \cite{13,15} compared to Gaussian theory errors. For the same width we find that a Gaussian ansatz underestimates the errors bars on the $\Delta_j$ by a relative 10% to 20%.
plings like $\Delta_x$. The study of exotic Higgs decays is not any longer limited by the ability to identify the corresponding final state.

A combined analysis of HL-LHC and ILC measurements will make the determination of the couplings more precise than at any single machine. HL-LHC and ILC form a dream team to study the properties of the Higgs boson and to establish the properties of the Higgs boson in and beyond the Standard Model with high precision.

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[22] for an estimate of expected effects see e.g. R. S. Gupta, H. Rzehak and J. D. Wells, Phys. Rev. D 86, 095001 (2012).


