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Nuclear $\sigma$ terms and scalar-isoscalar WIMP-nucleus interactions from lattice QCD

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It has been argued that the leading scalar-isoscalar weakly interacting massive particle (WIMP)-nucleus interactions receive parametrically enhanced contributions in the context of nuclear effective field theories [1]. These contributions arise from meson-exchange currents (MECs) and potentially modify the impulse approximation estimates of these interactions by 10%–60%. We point out that these MECs also contribute to the quark mass dependence of nuclear binding energies, that is, nuclear $\sigma$ terms. In this work, we use recent lattice QCD calculations of the binding energies of the deuteron, $^3$He and $^4$He at pion masses near 500 and 800 MeV, combined with the experimentally determined binding energies at the physical point, to provide approximate determinations of the $\sigma$ terms for these light nuclei. For each nucleus, we find that the deviation of the corresponding nuclear $\sigma$ term from the single-nucleon estimate is at the few-percent level, in conflict with the conjectured enhancement. As a consequence, lattice QCD calculations currently indicate that the cross sections for scalar-isoscalar WIMP-nucleus interactions arising from fundamental WIMP interactions with quarks do not suffer from significant uncertainties due to enhanced MECs.

I. INTRODUCTION

Nuclei play a central role in laboratory searches for weakly interacting massive particles (WIMPs), prime candidates for dark matter, which occur naturally in theoretical frameworks beyond the Standard Model, such as supersymmetry. WIMPs passing through the local environment will scatter from nuclei in low-momentum transfer processes given their expected velocity distribution in our solar system ($\langle v \rangle \sim 10^{-3}c$) [2], providing the signature of an isolated recoiling nucleus in a detector. Interpretations of candidate WIMP-nucleus events, along with model predictions for WIMP interaction rates within detectors, depend upon theoretically challenging calculations of the relevant nuclear matrix elements. Typically, the Standard Model fields involved in the WIMP-nucleus interactions are matched onto operators in low-energy effective field theories (EFTs) with well-defined power-counting schemes that describe the dynamics of mesons and baryons, including chiral perturbation theory ($\chi$PT) [3–5], heavy-baryon chiral perturbation theory (HB$\chi$PT) [6–8], and nuclear chiral EFTs [9–18]. Nuclear matrix elements of these hadronic-level operators are then calculated with nuclear many-body techniques to provide the WIMP-nucleus interactions and their associated couplings. At present, it remains challenging to perform both stages of the calculation. The first stage requires solving QCD, which is beginning to be accomplished with the numerical technique of lattice QCD at the physical light-quark masses. The second stage requires forming matrix elements of the hadronic-level operators in a nonrelativistic interacting quantum many-body system [1,19–23]. At some point in the future, lattice QCD calculations will be able to determine such nuclear matrix elements in light nuclei by a direct evaluation, obviating the need for either matching step. However, for the foreseeable future, the less direct approach in which lattice QCD is used to constrain couplings in nuclear chiral EFTs is necessary.

An important feature of nuclei is that, to a large extent, they behave as a collection of nonrelativistic nucleons dominated by two-body interactions, with the three-body and higher interactions strongly suppressed. Such a hierarchy of forces is understood within the framework of low-energy chiral nuclear forces. As such, many low-energy nuclear observables are dominated by the contributions from individual nucleons (the impulse approximation), which would yield the naive estimate

$$g_{Z,N} = g_p Z + g_n N,$$

for a WIMP-nucleus coupling, where $g_{p,n}$ are the appropriate WIMP couplings to protons and neutrons, respectively, and $N$ and $Z$ are the neutron and proton numbers. Typically, nuclear interactions [for instance, meson-exchange currents (MECs)] are expected to correct the impulse approximation result at the few-percent level. However, it has been argued that scalar WIMP interactions with nuclei might violate this hierarchy due to a parametric enhancement of MECs involving the up and down quarks in the low-energy chiral EFT, providing a correction to the impulse approximation, and hence a correction to Eq. (1), at the 10%–60% level [1]. It is important to note...
that such contributions could have a dependence upon $Z$ and $N$ that is substantially different from that given in Eq. (1) and thereby could provide significantly more freedom in the relative contributions for different nuclei. The conjectured enhancement of isoscalar MECs would imply that our knowledge of WIMP-nucleon cross sections is significantly less certain than often assumed and may differ substantially among different nuclear targets. Currently, the most stringent constraints on spin-independent WIMP-nucleon interactions come from the XENON Collaboration [24] (see Ref. [25]) using the impulse approximation (see also limits from CDMS [26], CRESST-II [27], ZEPLIN-III [28], EDELWEISS-II [29], WARP [30], TEXONO [31], and CDEX [32]). For instance, the XENON Collaboration finds that the WIMP-nucleon cross section is less than $2 \times 10^{-45}$ cm$^2$ for a WIMP with a mass of $M_Z = 55$ GeV at the 90% confidence level [25]. However, the DAMA [33] and CoGeNT [34] experiments claim evidence of a light WIMP in the $\sim 10$ GeV range, including statistically significant annual modulation (for an example of analyses of the consistency of the experimental results, see Ref. [35]).

Given the different nuclei used as targets in these experiments, it is possible that an enhanced nuclear contribution with different dependence on $N$ and $Z$ could decrease the tension among these different results [1]. In this work, we use recent lattice QCD calculations of the mass of the deuteron, $^3$He and $^4$He at pion masses of $m_\pi \sim 510$ [36,37] and $\sim 806$ MeV [38], combined with the known masses in nature, to provide constraints on the matrix elements of the light-quark scalar-isoscalar interaction in these light nuclei directly from QCD via their nuclear $\sigma$ terms. We find no evidence for enhanced deviations from the single-nucleon estimate for these interactions, instead finding them to be at the few-percent level, consistent with nuclear effects in other observables.

The lattice QCD based approach of Ref. [39] does not find a bound deuteron at the SU(3)-symmetric point, in seeming disagreement with Refs. [36–38]. Solving the Schrödinger equation with the energy-dependent “potentials” defined in the QCD modeling method recovers the scattering phase shifts of QCD with the same level of rigor as the Lüscher method only at the energy-eigenvalues of the lattice calculation that gave rise to the potentials. Away from those energy eigenvalues, the HAL QCD modeling method produces results that are not those of QCD. However, it should be stressed that at the energy eigenvalues of the lattice calculation, the HAL QCD modeling method does provide a phase shift and binding energy at the same level of rigor as that obtained with Lüscher’s method. HAL QCD trades the energy dependence of the potentials for nonlocality and then approximates the nonlocality by locality and has performed calculations showing that the energy dependence is weak compared with statistical uncertainties in their calculations. Further, they fit these now local potentials to a finite number of Gaussian profiles, along with Yukawa interactions that are softened at short distance. While one would expect that applying this method to the deuteron binding energy calculated in sufficiently large lattice volumes would result in a deuteron binding energy that is close to an energy eigenvalue, they have not presented the eigenvalues. Since solving the discrepancy is not the subject of this work, only the results obtained with Lüscher’s method are used.

II. NUCLEAR $\sigma$ TERMS AND SCALAR WIMP-NUCLEUS INTERACTIONS

Over a significant range of WIMP masses, the strength of their interactions with hadrons is tightly constrained by experimental searches. However, in the absence of definitive observations in multiple processes, the structure of the interaction between WIMPs and Standard Model fields remains unknown. Restricting our discussion to scalar interactions between WIMPs and dimension-three quark bilinears, the leading-order (LO) interactions of spin-$1/2$ WIMPs, $\chi$, in an expansion in derivatives, can be written as

\[
\mathcal{L} = G_F \bar{\chi} \chi \sum_q a_S^{(q)} \bar{q} q
\]

\[
= G_F \bar{\chi} \chi \bar{u} u + \bar{d} d + \bar{s} s + \bar{c} c + \bar{b} b + \bar{\tau} \tau
\]

\[
= \frac{G_F}{2} \bar{\chi} \chi [\bar{u} u + \bar{d} d + \bar{s} s] + \langle \bar{q} q \rangle \langle \bar{q} q \rangle + \langle \bar{q} q \rangle \bar{q} q + \bar{q} q \langle \bar{q} q \rangle + \bar{q} q \langle \bar{q} q \rangle + \bar{q} q \langle \bar{q} q \rangle
\]

\[
+ 2 a_s^{(1)} s \bar{s} s + \ldots,
\]

where $\bar{q} q = \bar{u} u + \bar{d} d$ and $G_F$ is Fermi’s weak coupling constant, here used to normalize the (scale-dependent) couplings of the WIMP(s) to the quarks, $a_S^{(q)}$, with respect to a weak-scale interaction strength. For two light flavors, $a_S = \text{diag}(a_S^{(u)}, a_S^{(d)})$, and for three light flavors, $a_S = \text{diag}(a_S^{(u)}, a_S^{(d)}, a_S^{(s)})$. The generalization to WIMPs with arbitrary spin, but with scalar interactions with the quarks, is obvious. The part of the operator involving Standard Model fields transforms in the same way as the light-quark mass terms in the QCD Lagrangian density, and consequently, its leading hadronic matrix elements are known. Based upon the spontaneously broken $SU(2)_L \otimes SU(2)_R$ chiral symmetry of QCD, the interaction Lagrangian density in Eq. (2) matches onto

\[
\mathcal{L} \rightarrow G_F \bar{\chi} \chi \left( \frac{1}{4} \langle \bar{q} q \rangle \langle \bar{q} q \rangle \right) \text{Tr}(a_S \Sigma^\dagger + a_S^\dagger \Sigma)
\]

\[
+ \frac{1}{4} \langle N | \bar{q} q | N \rangle \langle N | \text{Tr}(a_S \Sigma^\dagger + a_S^\dagger \Sigma) \rangle
\]

\[
- \frac{1}{4} \langle N | \bar{q} \tau^3 q | N \rangle \langle N | \text{Tr}(a_S \Sigma^\dagger + a_S^\dagger \Sigma) \rangle - 4 N^\dagger a_S \bar{s} s N + \ldots,
\]

at the chiral symmetry breaking scale $\Lambda$, which describes the single-hadron matrix elements and the associated interactions at LO in the chiral expansion. $\Sigma$ is the exponentiated pion field, and $N$ is the nucleon field.

In many cases, these are the most relevant operators, but there are dark matter candidates for which scalar-isoscalar gluonic operators are important, such as technibaryon dark matter [40–42].
\[
\Sigma = \exp \left( \frac{2i}{f_\pi} M \right),
\]

\[
M = \left( \begin{array}{cc}
\pi^0 / \sqrt{2} & \pi^+ \\
-\pi^0 / \sqrt{2} & \pi^-
\end{array} \right),
\]

\[
N = \left( \begin{array}{c}
p \\
n
\end{array} \right).
\]

\[f_\pi = 132 \text{ MeV}\] is the pion decay constant, \(a_{S,\xi} = \frac{1}{2}(\xi^\dagger a_{S,\xi}^\dagger + \xi a_{S,\xi}^\dagger)\) with \(\xi = \sqrt{\Sigma}\), and the ellipsis denotes higher-order interactions including those involving more than one nucleon. Expanding Eq. (3) in the number of pion fields (neglecting the shift in the WIMP mass induced by the chiral condensate), the LO contributions to the interactions are

\[
\mathcal{L} \to G_F \bar{\chi} \chi \left( -\frac{a_S^{(u)} + a_S^{(d)}}{f_\pi^2} \langle 0|\bar{q}q|0\rangle \left( \frac{1}{2} (\pi^0)^2 + \pi^+ \pi^- \right) + \frac{1}{2} (a_S^{(u)} + a_S^{(d)}) \langle 0|\bar{q}\tau^3 q|N\rangle N^\dagger N + \frac{1}{2} (a_S^{(u)} - a_S^{(d)}) \langle 0|\bar{q}\tau^3 q|N\rangle N^\dagger \tau^3 N + \cdots \right). \tag{5}
\]

Matching onto the multinucleon interactions is complicated by the fact that contributions from pion-exchange interactions and from local four-nucleon operators are of the same order in the chiral expansion, and the coefficients of the latter are not directly related to multinucleon matrix elements at any order in the chiral expansion. For instance, the four-nucleon operators involving one insertion of the light-quark mass matrix are of the form \([13-15]\]

\[
\mathcal{L}^{N^4-m_q} = D_{S,1}(N^\dagger N)^2 \text{Tr}[m_q \Sigma^\dagger + m_q \Sigma] + D_{S,2}N^\dagger NN^\dagger m_{q,\xi^+} N + D_{T,1}(N^\dagger \sigma^a N)^2 \text{Tr}[m_q \Sigma^\dagger + m_q \Sigma] + D_{T,2}N^\dagger \sigma^a NN^\dagger \sigma^a m_{q,\xi^+} N. \tag{6}
\]

in the low-energy EFT, where \(m_{q,\xi^+} = \frac{1}{2} (\xi^\dagger m_q \xi^\dagger + \xi m_q \xi^\dagger)\) and \(\sigma^a\) are the Pauli matrices. Hence WIMP–two-nucleon interactions are of the form

\[
\mathcal{L}^{N^2-m_q} = -G_F \bar{\chi} \chi \left( D_{S,1}(N^\dagger N)^2 \text{Tr}[a_S \Sigma^\dagger + a_S \Sigma] + D_{S,2}N^\dagger NN^\dagger a_S N + D_{T,1}(N^\dagger \sigma^a N)^2 \times \text{Tr}[a_S \Sigma^\dagger + a_S \Sigma] + D_{T,2}N^\dagger \sigma^a NN^\dagger \sigma^a a_S N \right). \tag{7}
\]

The importance of the various contributions to the scalar-isoscalar matrix elements can be estimated using power counting arguments. The second and third terms in Eq. (5) provide the leading (order \(Q^0\), where \(Q\) denotes the small ratio of scales in the effective theory) scalar interactions of the WIMP and the nucleon that generate the impulse approximation for WIMP-nucleus interactions [see Fig. 1 (left)]. In a nucleus, the first term in Eq. (5) gives rise to an MEC between two nucleons, as shown in Fig. 1 (middle), that naively contributes at order \(1/Q^2\) in the chiral expansion due to the nonderivative interaction of the pions, which is two orders lower than the contribution from the impulse approximation. This term is the origin of the enhancement suggested in Ref. [1]. The isoscalar interactions with the strange and heavier quarks do not contribute to the nonderivative interaction with pions and, as such, are not expected to be enhanced in WIMP-nucleus interactions. To determine the WIMP-nucleus interactions quantitatively, nuclear matrix elements of these operators need to be calculated.

Ideally, one would simply determine the matrix element of the Lagrangian density in Eq. (2) in the ground state of a given nucleus, at the relevant momentum transfer, without performing the intermediate matchings in Eqs. (3) and (5). This would sum the contributions from the hadronic EFT to

\[
\begin{array}{|c|c|c|c|}
\hline
m_z (\text{MeV}) & B_d (\text{MeV}) & B_{^3\text{He}} (\text{MeV}) & B_{^4\text{He}} (\text{MeV}) \\
\hline
138.0 \text{ (experiment, isospin averaged)} & 2.22 & 7.718 & 28.3 \\
390.3(2.6) \text{ [56,57]} & 11(5)(12) & \cdots & \cdots \\
510(2) \text{ [36,37]} & 11.5(1.1)(0.6) & 20.3(4.0)(2.0) & 43(12)(8) \\
806.7(8.9) \text{ [38]} & 19.7(3.1)(4.1) & 66(6)(6) & 110(20)(15) \\
\hline
\end{array}
\]

TABLE I. The binding energies of the deuteron, \(^3\text{He}\) and \(^4\text{He}\) at pion masses of \(m_\pi \sim 390\text{ [56,57]}\), \(-510\text{ [36,37]}\), and \(-806\text{ MeV [38]}\) calculated with lattice QCD, along with their values at the physical point (the uncertainties in the experimental measurements are negligible).
all orders in perturbation theory and provide the necessary matrix elements directly from QCD. While such formidable calculations cannot currently be accomplished, the forward matrix element of the scalar-isoscalar operator can be determined in light nuclei, albeit with significant uncertainties, by combining recent lattice QCD calculations of the binding energies with the corresponding experimental values. The mass of the ground state of a nucleus with \( Z \) protons and \( N \) neutrons, denoted by \( |Z, N(\text{gs})\rangle \), is \( E_{Z,N}^{(\text{gs})} = E_{Z,N}^{(\text{gs})} + \sigma_{Z,N} \), where

\[
\sigma_{Z,N} = \langle Z, N(\text{gs})| \bar{u}u + \bar{d}d|Z, N(\text{gs})\rangle
\]  

is the nuclear \( \sigma \) term and \( E_{Z,N}^{(\text{gs})} \) is the energy of the nuclear ground state in the limit of massless up and down quarks (assuming that the nucleus is bound in this limit). With isospin symmetry, \( m_u = m_d = \bar{m} \), the nuclear \( \sigma \) term becomes

\[
\sigma_{Z,N} = \bar{m}\langle Z, N(\text{gs})| \bar{u}u + \bar{d}d|Z, N(\text{gs})\rangle
\]

\[
= \bar{m} \frac{d}{dm} E_{Z,N}^{(\text{gs})}
\]

\[
= [1 + \mathcal{O}(m_Z^2)] \frac{m_x}{2} \frac{d}{dm} E_{Z,N}^{(\text{gs})},
\]

where we have used the leading contribution to the Gell-Mann–Oakes–Renner (GMOR) relation \[4,43\],

\[
-2\bar{m}\langle 0| \bar{u}u + \bar{d}d|0\rangle = m_x^2 f_L^2 [1 + \mathcal{O}(m_Z^2)],
\]

to relate the quark and pion masses. The relation between the pion mass and the average light-quark mass has been precisely determined with lattice QCD \[44,45\]. The linear relation between \( m_x^2 \) and \( \bar{m} \) is found to hold to better than 10% over a large range of pion masses, even for the heavy pion masses that we consider \[44,45\]. We use this linear relationship in constructing nucleon and nuclear \( \sigma \) terms, \( \sigma_{Z,N} = \frac{m_x}{2} \frac{d}{dm} E_{Z,N}^{(\text{gs})} \), and assign a conservative 10% uncertainty in order to account for the nonlinearity in the GMOR relation (note that this uncertainty later cancels when we take the ratio of \( \sigma \) terms below).

Writing the mass of the nucleus as \( E_{Z,N}^{(\text{gs})} = A M_N - B_{Z,N} \), where \( A = Z + N \), \( M_N \) is the isospin-averaged nucleon mass and \( B_{Z,N} \) is the total binding energy of the nucleus, the (two-flavor) nuclear \( \sigma \) term can be written as

\[
\sigma_{Z,N} = A \sigma_N + \sigma_{B_{Z,N}} = A \sigma_N - \frac{m_x}{2} \frac{d}{dm} B_{Z,N},
\]

where

\[
\sigma_N = \bar{m}\langle N| \bar{u}u + \bar{d}d|N\rangle = \bar{m} \frac{d}{dm} M_N = \frac{m_x}{2} \frac{d}{dm} M_N,
\]

is the nucleon \( \sigma \) term and \( |N\rangle \) is the single-nucleon state. The first term in Eq. (11) is the noninteracting single-nucleon contribution to the nuclear \( \sigma \) term, while the second term corresponds to the corrections due to

FIG. 2 (color online). The deuteron (left panel), \(^3\)He (middle panel), and \(^4\)He (right panel) binding energies from \( n_f = 2 + 1 \) lattice QCD calculations, along with the experimental values. The inner and outer uncertainty bars correspond to the statistical and total (statistical combined with systematic) uncertainties, respectively.
interactions between the nucleons, including the possibly enhanced contributions from MECs. It is useful to define the ratio

$$\delta \sigma_{Z,N} = \frac{1}{A \sigma_N} \frac{m_\pi}{2 \, d \, m_\pi} B_{Z,N}$$

(13)

to quantify the deviations from the impulse approximation. In addition to representing deviations of nuclear \( \sigma \) terms from the impulse approximation, this quantity also describes the deviation of the scalar-isoscalar WIMP-nucleus scattering matrix element from the impulse approximation at zero momentum transfer.

$$\delta \sigma_{Z,N} = \frac{\langle Z, N(\mathrm{gs}) | \bar{u} u + \bar{d} d | Z, N(\mathrm{gs}) \rangle}{A \langle \bar{u} u + \bar{d} d \rangle} - 1.$$  

(14)

### III. LIGHT NUCLEI FROM LATTICE QCD AND THEIR \( \sigma \) TERMS

Lattice QCD has evolved to the stage where the binding energies of the lightest nuclei and hypernuclei have been determined at a small number of relatively heavy pion masses in the limit of isospin symmetry. Further, the mass of the nucleon has been explored extensively over a large range of light-quark masses, with calculations now being performed at the physical value of the pion mass. These sets of calculations, along with the experimental values of the masses of the light nuclei, are sufficient to arrive at a first QCD determination of the nuclear \( \sigma \) terms for these nuclei at a small number of pion masses. This work provides an estimate of the modifications to the impulse approximation for scalar-isoscalar WIMP-nucleus interactions in light nuclei. In particular, these results can be used to explore the conjectured enhancement of MEC contributions to these interactions and to investigate the size of the uncertainties introduced by the use of the impulse approximation in phenomenological analyses. It is important to mention that the EFT description of the quark-mass dependence of the nuclear forces has been developed in Refs. [46–50], and estimates of nuclear \( \sigma \) terms have been made in Refs. [51–55]. We make use of EFT below in assessing a systematic uncertainty due to extrapolation in the light-quark masses.

The binding energies of the deuteron, \(^3\)He and \(^4\)He at pion masses of \( m_\pi \sim 390, 510, \) and \( 806 \) GeV calculated with lattice QCD [36–38,56,57] are presented in Table I, along with their values at the physical point, and are shown in Fig. 2. The binding energies per nucleon are shown in Fig. 3. The lattice QCD calculations were performed with clover-improved discretizations of the quark fields. The \( m_\pi \sim 806 \) MeV calculations were performed with a lattice spacing of \( b \sim 0.15 \) fm (determined at this mass) [38]. The \( m_\pi \sim 390 \) MeV calculations\(^3\) used an anisotropic discretization and were performed with a spatial lattice spacing \( b \sim 0.12 \) fm (when determined by extrapolation to the physical quark masses) [56,57]. Finally, the \( m_\pi \sim 510 \) MeV calculations were performed with \( b \sim 0.09 \) fm [36,57]. Each set of calculations was performed in multiple lattice volumes to distinguish continuum states from bound states, but none of the calculations were extrapolated to the continuum limit, leading to a small additional uncertainty [the binding energies are expected to have uncertainties of \( O(b^2) \)] not shown in Table I that we neglect in our analysis. In each set of calculations, the strange-quark mass was tuned to be approximately its physical value. The small systematic uncertainty in the nuclear \( \sigma \) terms due to the inexact tuning of the strange-quark mass is also neglected in this analysis. Further details of these sets of calculations can be found in the original references [36–38,56,57], and we do not repeat them here.

To estimate the derivative of the nuclear binding energies with respect to the pion mass, as required in Eq. (11) to determine the nuclear \( \sigma \) terms, the finite differences between the binding energies at adjacent pion masses

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\(^3\)Since the calculations with \( m_\pi \sim 390 \) MeV did not determine the binding energies of the \( A = 3, 4 \) systems, and extracted the binding energy of the deuteron with large uncertainties, we do not use these results in our analysis of the sigma terms.
are formed. The extracted nuclear $\sigma$ terms are given in Table II and are shown in Fig. 4; they take their values at the average pion mass of the interval. This procedure assumes that there is negligible curvature between adjacent pion masses; while purely linear behavior is not expected, strong mass dependence is also not expected, as determined from EFT and potential-model phenomenology [46–55] (see below for more discussion on this point). To determine the fractional contribution of the nuclear interactions to the nuclear $\sigma$ terms, $\delta\sigma_{Z,N}$, the contribution from the binding energy is divided by the contribution from noninteracting nucleons, $A\sigma_N$. The single-nucleon mass is known as a function of the pion mass from a broad range of lattice QCD calculations, as summarized and analyzed recently in Refs. [58,59]. Empirically, it is found that the results of lattice QCD calculations of the nucleon mass are reasonably well reproduced by a linear dependence upon the mass of the pion, $M_N = a_0 + a_1 m_\pi$ [58,59], where $a_0 = 802(13)$ MeV is the value in the chiral limit, and $a_1 = 0.991(27)$, naively in conflict with expectations from HB$\chi$PT, which does not allow for a term linear in the pion mass. Recent analysis of lattice calculations near the physical pion mass indicate that the NLO expressions of HB$\chi$PT can also be fit to the lattice results (for instance, see Refs. [60–63]) and provide a nucleon $\sigma$ term at the physical pion mass that is somewhat less than that estimate that follows from the empirical linear relation. Nevertheless, we conclude that the nucleon $\sigma$ term is $\sigma_N = a_1 m_\pi/2$ to a good

FIG. 4 (color online). The nuclear contributions to the deuteron (left panel), $^3$He (middle panel) and $^4$He (right panel) $\sigma$ terms from nuclear interactions. The inner and outer shaded regions correspond to the statistical and total (statistical combined with systematic) uncertainties, respectively.

FIG. 5 (color online). Percentage modifications to the impulse approximation contribution to the deuteron (left panel), $^3$He (middle panel) and $^4$He (right panel) $\sigma$ terms.

FIG. 6 (color online). Fractional contribution to the nuclear $\sigma$ terms from nuclear interactions for an average pion mass of $\langle m_\pi \rangle \sim 325$ MeV (left panel) and $\langle m_\pi \rangle \sim 658$ MeV (right panel).
approximation in the region of heavier pion masses where we extract the nuclear $\sigma$ terms. The fractional contribution of nuclear interactions to the nuclear $\sigma$ terms of the deuteron, $^3$He and $^4$He are shown in Fig. 5. For each nucleus, the nuclear interactions modify the $\sigma$ term by less than 10% of the impulse approximation contribution for both pion masses considered and by less than 2% at the lighter pion mass, as can be seen in Fig. 6.

Given the heavy quark masses used in the existing lattice QCD calculations of the binding energies of the light nuclei, it is worth considering the systematic uncertainty due to extrapolation in the light-quark masses. Using the EFT formalism for determining the pion-mass dependence of the deuteron binding energy developed in Refs. [46,48,50], the coefficient of the four-nucleon contact interaction (with an insertion of the quark mass

\[ \sigma = \sigma_0 + \delta \sigma_1 \]

has been fit to pass through the central value of the lattice QCD datum from Ref. [36]. Fluctuations in the value of this low-energy constant that are consistent with naive dimensional analysis have been considered. The resulting band is shown in Fig. 7. Calculating the deuteron sigma term from the enveloping curves (dashed lines), we find that $|\delta \sigma_{1,1}| < 0.01$, which suggests that the systematic uncertainty in the binding energy due to extrapolation translates to at most a 1% modification of the impulse approximation. We also display boundary curves (solid lines) for a 5% effect ($|\delta \sigma_{1,1}| < 0.05$). One sees that the smooth, monotonic behavior of the deuteron binding energy as a function of the pion mass that is suggested by the lattice data (together with experiment) is consistent with what is expected in the vicinity of the physical point from EFT. Therefore, the likelihood of curvature significant enough to negate the conclusions found in this paper is very small.

A similar analysis can be carried through for $^3$He and $^4$He [51], which also suggests smooth, monotonic behavior in the quark masses in the vicinity of the physical point.

Deviations of the nuclear $\sigma$ terms from the sum of single-nucleon contributions due to the nuclear forces are the same as the deviations in the forward scalar-isoscalar WIMP-nucleus scattering amplitudes from the sum of single-nucleon amplitudes. As such, the results presented in this work are inconsistent with the conjectured enhanced MEC contributions [1] to WIMP-nucleus scattering at the 10%—60% level. Current lattice QCD calculations indicate that deviations in the scalar-isoscalar WIMP-nucleus interaction from interactions between the nucleons are at the percent level, consistent with the typical size of MEC contributions.

**IV. DISCUSSION**

Detecting and understanding dark matter is one of the great challenges of our time. WIMPs endowed with weak-scale interactions arising from relatively straightforward extensions to the Standard Model are natural candidates for dark matter. A significant experimental effort is ongoing to search for WIMPs, and one of the main techniques is to search for nuclear recoils from WIMP-nucleus elastic collisions. Calculations of the cross section for a WIMP-nucleus collision typically involve determining the WIMP-hadron couplings in a hadronic theory (with well-defined small expansion parameters) from the fundamental

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4The empirical fit can be augmented to include higher-order polynomial terms without significantly altering the fitted $\sigma$ term. We also note that a very similar, though somewhat less precise, nucleon $\sigma$ term can be extracted using only the lattice calculations for which we have results for nuclei.

5In calculating the light-quark mass dependence of the deuteron binding energy, Refs. [46–48] use the quark-mass dependences of single-hadron properties, such as $g_A$ and $f_\pi$, determined from $\chi$PT. As a result, the estimated dependences of these quantities on the pion mass used in those works are much stronger than are now known from lattice QCD calculations. Even so, within the uncertainties of the calculations, the deuteron binding energy could vary smoothly with the pion mass, as can be seen in Fig. 11 of Ref. [47] and Fig. 4 of Ref. [48]. In Ref. [50], the pion mass dependence of the single-nucleon properties constrained by lattice QCD calculations were used to constrain behavior of the scattering length in the $^3S_1$ channel, which was found to be consistent with a smooth variation of the deuteron binding energy, but with significant uncertainties.

6Similar considerations follow from an analogous EFT analysis of the singlet channel. While the dineutron is bound at heavy pion masses, a straight-line extrapolation is consistent with unbinding at the physical point, which suggests a very shallow bound state, or a virtual bound state with large negative scattering length as seen in nature.

There may be significant curvature at pion masses below the physical point in the approach to the chiral limit, due to radiation pions [49] or other effects that are nonanalytic in the quark masses.
WIMP-quark and WIMP-gluon interactions and then taking matrix elements of these hadronic-level operators between states in the nucleus. Experimental limits on WIMP interactions with ordinary matter rely upon calculations of WIMP-nucleus cross sections based upon the hadronic-level interactions. It has been conjectured that the scalar-isoscalar WIMP-nucleus cross section could be modified at the 10%–60% level [1] by MECs over naive expectations based upon the WIMP-nucleon interactions alone. Such terms would also scale differently with Z and N than the impulse approximation. The uncertainty that such large MECs would imply would lead to significant modifications to the reported limits on this type of WIMP-nucleon interaction from experiment. In this work, we have combined the recent lattice QCD calculations of the binding energies of the lightest few nuclei to provide a direct QCD evaluation of nuclear σ terms, albeit at unphysical quark masses. These σ terms are related to the amplitude for WIMP-nucleus interactions in the scalar-isoscalar channel. Deviations from the single-nucleon contribution are found to be at the percent level in light nuclei, inconsistent with the conjectured enhancement but consistent with expectations based upon other nuclear observables, such as electromagnetic moments and interactions. Without enhanced MEC contributions in light nuclei, it is likely that these effects are also of their expected size in heavier nuclei, and we expect that the current nuclear calculations [19–23] of WIMP-nucleus scattering cross sections are reliable. Additionally, we note that the phenomenological nuclear σ terms for various nuclei estimated in Refs. [51–55] are also supported by our extractions. None of the lattice QCD calculations of the light nuclei have been extrapolated to the continuum, and as such, there are residual lattice-spacing uncertainties in the results we have presented; however, these systematic deviations from QCD are estimated to be small, and in particular much smaller than other sources of uncertainty. Continuum extrapolated calculations will be performed in the near future.

The conclusions of this work were obtained by assuming that the nuclear binding energies vary smoothly with the light-quark masses. In the two-nucleon sector, a comparison between this dependence constrained by EFT for light pions and the results of lattice QCD at heavier pion masses, outside the range of applicability of the EFT, is consistent with this assumption. While there is no indication that this assumption should not hold for the nuclei considered in this work, it does introduce a systematic uncertainty into our results that, while we have attempted to quantify its impact, remains to be eliminated by future lattice QCD calculations. Our use of naive dimensional analysis to determine the range of coefficients in the EFT provides only an estimate of their values, and the true values could be somewhat larger or somewhat smaller than predicted by NDA; there are well-known examples of both in nature. Therefore, it is possible that the estimate of the systematic uncertainty that we have assigned to the extrapolation is somewhat larger or somewhat smaller than its actual value. Better quantifying the extrapolation in light-quark masses through improved lattice QCD calculations over a range of light-quark masses and better understanding the theoretical form of the extrapolation are focuses of our ongoing program and also that of others. To help understand the disagreement of our results with the predictions of Ref. [1], we consider the problem using KSW power counting in nuclear EFTs in the $^1S_0$ channel [13–15]. While the $^1S_0$ channel does not exhibit a bound state at the physical point, a bound state develops as the pion mass is increased [36–38,56,57]. More importantly, KSW power-counting permits an analytic analysis of the scattering amplitude and bound state properties. The expansion of the dineutron σ term is analogous to the expansion of the electromagnetic form factors and moments of the deuteron with KSW power counting [64], in which it is straightforward to show that the four-nucleon operators, and more generally, the NLO contributions such as the MEC contributions are suppressed by factors of the deuteron binding momentum divided by the EFT cutoff scale compared with the LO single-nucleon contribution. In electroweak processes, deviations from the single-nucleon contribution are at the percent level, and it is reasonable to expect similar modifications for the scalar current. This suggests that the arguments of Ref. [1] regarding the importance of the MECs fail explicitly in this channel. Unfortunately, the chiral expansion of the nuclear potential à la Weinberg is formally inconsistent [13], and KSW power counting is invalid in the $^3S_1$–$^3D_1$-coupled channels. Therefore, an analysis of the behavior of observables based upon the nuclear potential alone cannot be reliably performed. It was such an analysis that was performed in Ref. [1] in order to arrive at the conjecture of parametrically enhanced MECs for the scalar-isoscalar WIMP-nucleus interactions. Current lattice results show that this analysis does indeed fail in the cases under consideration.

It is clear that a systematic study of the light-quark mass dependence of the lightest nuclei is a valuable program to pursue, not only for intellectual reasons, but also in order to reduce the uncertainty in the response of nuclei to probes other than those provided by the electroweak interactions. An important application of these calculations is the refinement and complete quantification of the nuclear physics component of WIMP-nucleus interactions that are crucial for interpreting the results from many experiments searching for dark matter. We anticipate that lattice QCD calculations of a wider range of WIMP-nucleus interactions in the light nuclei will become possible in the near future as larger computational resources become available. While the scalar-isoscalar interactions in the forward direction arising from fundamental quark interactions with WIMPs can be accessed through the light-quark mass dependence of the masses of the nuclei, the
matrix elements of other operator structures in light nuclei will require forming three-point correlation functions or the use of background-field techniques. The results of these calculations refine the structure of the effective interactions and enable the determination of WIMP-nucleus interactions with reduced uncertainties.

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