# Modeling Acceleration Decisions for Freeway Merges

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Modeling Acceleration Decisions for Freeway Merges

Charisma F. Choudhury, Varun Ramanujam, and Moshe E. Ben-Akiva

In uncongested traffic situations, a merge is executed when the available gap is sufficiently large. However, in congested traffic, acceptable gaps for merging are often not readily available, and merging can involve more complex mechanisms. For example, the driver in the target lane may slow down and cooperate with the merging driver, or the merging driver may become impatient and decide to force in, and compel the lag driver in the target lane to slow down. Choices of the merging plan or tactic affect the gap acceptance and acceleration decisions of the driver. A driver who has decided to force in, for instance, is likely to accept smaller gaps and accelerate to facilitate the merge. The chosen tactic at any instant, however, is not distinctly observable from the vehicle trajectory. The model presented in this paper extends previous research in modeling the effect of merging plans in the lane-changing decisions by integrating the acceleration decisions of the driver with the gap acceptance decisions.

A combined model for merging plan choice, gap acceptance, target gap selection, and acceleration decisions of drivers merging from the on-ramp is developed in that regard. Parameters of all components of the models are estimated jointly with detailed vehicle trajectory data collected from Interstate 80 in California. The inclusion of the target gap choice and acceleration behavior components has been supported by a validation case study in which the model has been implemented in MITSIMLab and validated against the observed aggregate traffic data collected from US-101 in California.

Merging locations are major sources of freeway bottlenecks and are therefore important for freeway operations analysis. Microscopic simulation tools have been successfully used in analysis of merging bottlenecks and for designing optimum geometric configurations and control strategies for such locations. In normal situations a merge is executed when there is an acceptable gap in the main line. However, in congested situations acceptable gaps are often not available and more complex merging phenomena may be observed. For example, drivers may merge through courtesy of the lag driver in the target lane or decide to force in and compel the lag driver to slow down. Further, the choice of merging tactic may evolve dynamically. A driver may initially plan to merge through normal gap acceptance and later become impatient and decide to force merge. Choices of the merging plan or tactic at any instant (unobserved in the vehicle trajectory data) affect the gap acceptance and acceleration decisions of the driver at that instant. For example, a driver who perceives that the lag driver in the target lane is yielding to help in the merge is likely to accept smaller gaps and may decelerate to facilitate the merge. Or if a driver perceives that the adjacent gap is not favorable for a normal, courtesy, or forced merge, he or she may abandon trying to merge to that gap and rather target a non-adjacent gap and accelerate or decelerate to better position with respect to the chosen gap.

The necessity to integrate drivers’ lane-changing decisions with their acceleration behavior has been established by recent research (1–5). However, the focus of these models is on modeling freeway main-line acceleration behavior, and the complexities associated with the merging behavior are not addressed in detail. Several models have been developed specifically to model the gap-acceptance (6–9) or the acceleration behavior in merging situations (7, 10). But these models do not explicitly address the effect of merging tactic (normal, courtesy, or forced) and gap acceptance in a single decision framework and ignore the possibilities of transition from one plan to another. Hidas developed a merging model that includes both cooperative and forced merge components, but the cooperative lane change part consists only of modeling the decision of the lag driver (whether to provide courtesy to the merging driver) and not the decision of the merging driver (whether to initiate or execute the courtesy lane change) (11). Moreover, although these models have been calibrated with real traffic data to some extent, no rigorous statistical estimation with detailed trajectory data has been deployed. This was reflected in the findings of the next generation simulation (NGSIM) study on identification and prioritization of core algorithm categories, in which the modeling of congested merging conditions has been identified by users as weak points of traffic microsimulation tools (12). Choudhury et al. (13) developed a lane-changing model for freeway merges using detailed NGSIM trajectory data that combine the choice of the merging plan and gap acceptance decisions of the driver into a single framework (14). But the scope of the model was limited to lateral movement decisions only, and the associated acceleration behavior is not integrated in the framework. Validation results of this model revealed that although the performance of the combined model was substantially better than the previous models (which ignore the effect of merging tactics) in regard to replicating the locations of merges, there were significant deviations in simulated speed in the merging lanes (13). This motivated the current research to extend the previous model and include acceleration and deceleration choices in the decision framework.

The model developed in this study explicitly considers cooperation and competition between target lane drivers. Drivers who have initiated a merge but have yet to complete it accelerate or decelerate to facilitate merging. Drivers who have not initiated a merge target a
favorable gap and accelerate or decelerate to better position themselves to exploit the target gap. The gap acceptance, target gap selection, and acceleration decisions of the driver thus have been integrated in a single framework.

The rest of this paper is organized as follows: the structure of the proposed integrated model and its components is first presented. Next, the data used for model estimation are described and the estimation methodology followed by the estimation results is presented. The paper concludes with the operational validation results of the models in MITSIMLab and the summary of findings.

The focus of this paper is on the additional levels (target gap selection and acceleration behavior). The models on the choice of the merging tactic, gap acceptance for the chosen merging tactic, and the state dependence among the subsequent decisions are detailed in Choudhury et al. (13, 15) and Choudhury (16).

MODEL STRUCTURE

The framework of the combined model is summarized in Figure 1. The model hypothesizes four levels of decision making: choice of merging tactic, gap acceptance for the chosen merging tactic, target gap selection, and accelerations. The choice of merging tactic is a complex process that depends on previous decisions. If the driver has not chosen a courtesy or forced merging plan, the choice of target gap is evaluated. The entire decision process is, however, latent, and only the end action of the driver (lane change or no change and acceleration) is observed. Latent choices are shown as ovals, and observed choices are represented as rectangles.

The merging driver first compares the available lead and lag gaps in the main line with the corresponding minimum acceptable gaps (critical gaps) for normal gap acceptance. Critical gaps are modeled as random variables, their means being functions of explanatory variables. If both the lead and the lag gaps are greater than the critical gap, a lane change can be executed using the existing gaps. If the gaps are not acceptable, the merging driver evaluates the speed, acceleration, and relative position of the through vehicles and tries to evaluate whether or not the lag driver is providing courtesy. If the driver perceives that he or she is receiving courtesy from the lag driver, a courtesy merge is initiated. The immediate completion of the initiated courtesy merge, however, may not be possible because of unacceptable adjacent gaps.

If the anticipated gap is unacceptable, the driver decides whether to force his or her way to the main line, compelling the lag driver to slow down. This decision can depend on the urgency of the merge, driver characteristics (e.g., risk averseness), and traffic conditions. As in the courtesy merge, the immediate completion of the initiated forced merge may not be possible because of unacceptable adjacent gaps.

If the driver does not initiate a courtesy or forced merge, he or she evaluates the backward, adjacent, and forward gaps and targets the gap that is perceived to be the best for merging and accelerates or decelerates accordingly. The acceleration behavior of the driver at any instant is thus conditional on any or all of the following: chosen lane-changing tactic, gap acceptance, and target gap choice.

If the driver does not initiate a courtesy or forced merge, the entire decision process is repeated in the next instant. However, if the driver has initiated a courtesy or forced merge and is adjacent to the same
gap, the subsequent decisions involve only evaluation of the adjacent gaps for completion of the initiated merge. After deciding to initiate a courtesy or forced merge, the choice of merging tactic is not reevaluated unless there is a significant change in neighborhood conditions, for example, the lead or the lag in the main line or both change and the driver is adjacent to a new gap. Other model structures were tested during the model development process (e.g., selection of the target gap before selection of merging tactic, selection of target gap before evaluating decision to force merge) but were not well supported by the data.

Each of the four steps is described below. The choice of the merging plan (state transition) and gap acceptance model structures are outlined briefly but have not been described in detail in this paper.

**Merging Tactic**

The driver first evaluates whether a lane change is possible using the existing adjacent gaps without a courtesy or forced merge. So the initial plan (state) of the driver is always normal.

The driver then evaluates the courtesy merge and forced merge plans sequentially. If the adjacent gaps are not acceptable under normal gap acceptance, the merging driver evaluates the speed, acceleration, and relative position of the through vehicles and decides whether to initiate a courtesy merge. The courtesy or discourtesy of the lag driver is reflected in the anticipated gap. If the lag driver has decided to provide courtesy to a merging vehicle and has started to decelerate, the anticipated gap increases. The anticipated gap of a particular driver also depends on the length of the time horizon during which it is estimated (anticipation time $\tau_n^t$). Differences in perception and planning abilities among drivers are captured by the distribution of the length of the time horizon. The probability of initiating a courtesy merging is thus conditional on anticipation time and previous lane actions ($l_{1:t-1}$) as well as driver-specific characteristics such as aggressiveness ($\upsilon_n$) and can be denoted by $P_n(s_l = C \mid l_{1:t-1}, \upsilon_n, \tau_n^t)$. Candidate variables affecting this probability include the acceleration, speed, and position of the lag vehicle and the type of the lag vehicle (heavy vehicle or not) [see Choudhury et al. for details (13)].

If the current gaps are not acceptable and the driver perceives that a courtesy merge is also infeasible (anticipated gap is not acceptable), the driver chooses whether to initiate a forced merge. That is denoted by $P_n(s_l = F \mid l_{1:t-1}, \upsilon_n, \tau_n^t)$. Candidate variables affecting the decision to initiate a forced merge include status of the merging driver (distance to the end of the merging lane, time elapsed since the driver has been waiting to merge, speed, type of vehicle, etc.), status of the lag vehicle in the main line (speed and acceleration of the lag vehicle, lag vehicle type, etc.), and level of congestion in the main line [see Choudhury et al. for details (13)].

**Gap Acceptance**

The driver completes the merge when the adjacent gaps are acceptable. An available gap is acceptable if it is greater than the critical gap. Critical gaps are modeled as random variables. Their means are functions of explanatory variables such as speed and position of lead and lag vehicles and type of vehicle and are conditional on the merging plan of the driver. The probability of gap acceptance (execution of the merge) can be denoted by $P_n(l_g \mid \upsilon_n, \epsilon_n)$ [see Choudhury et al. for details (13)].

**Target Gap Selection**

If the normal gap is not acceptable and the driver has not initiated a courtesy or forced merge, the driver is likely to look at alternative gaps and select the gap perceived to be the best gap as the target gap.

The alternatives in the target gap choice set include available gaps in the vicinity of the subject vehicle (e.g., the adjacent gap, forward gap, and backward gap shown in Figure 2). The adjacent gap, although not acceptable at the time of the decision, may still be chosen in anticipation that it will be acceptable in the future.

The utilities of the different target gaps to driver $n$ at time $t$ are given by

$$U_{it} = \beta_i X_{it} + \upsilon_n + \epsilon_{it}$$

for $i \in I = \{\text{adjacent, backward, forward}\}$  \hspace{1cm} (1)

where

- $X_{it} = \text{vector of explanatory variables corresponding to gap } i \text{ for individual } n \text{ at time } t$,
- $\beta_i = \text{coefficients of explanatory variables for gap } i$,
- $\upsilon_n = \text{driver-specific random term}$,
- $\epsilon_{it} = \text{random error term associated with gap } i \text{ for individual } n \text{ at time } t$.

The utilities of the different gaps are expected to be affected by driving neighborhood explanatory variables, such as the size of the gap and the gap trend, that is, whether it is expanding or narrowing (captured by the relative speed between the intended lead and lag
vehicles), and the subject’s relative speed with respect to these vehicles.

Assuming a logit error structure (identically and independently distributed random error term \(e_{ni}\)), the probability of selecting target gap \(i\) can be expressed as follows:

\[
P_n(t,|v_i) = \frac{\exp(\{\beta_i X_{nt} + \alpha_i v_i\})}{\sum_{i'} \exp(\{\beta_i X_{nt} + \alpha_i v_{i'}\})}
\]

\(\forall i, i' \in I = \{\text{adjacent, backward, forward}\} \quad (2)\)

### Acceleration

The acceleration behavior is expected to be different depending on the driver’s lane-changing decision, chosen merging tactics (courtesy or forced), and target gap. More specifically, four different types of accelerations are considered:

- **Lane-changing acceleration.** This is the acceleration that the driver applies when the adjacent gap is acceptable and the driver responds to the stimuli from the new lead in target lane.
- **Target gap acceleration.** Target gap acceleration is applied to improve position with respect to lead and lag of target gap.
- **Initiated courtesy merging.** This type of acceleration is applied to improve position with respect to lag in target lane.
- **Initiated forced merging.** A forced merging is initiated when the driver initiates his right-of-way to move to the target lane. The stimulus in this case is therefore from the new lead in the target lane.

Different models describe the acceleration behavior under the various situations. However, to create a consistent set of acceleration behaviors, the stimulus-sensitivity framework proposed in the General Motors (GM) model for the car-following regime is adapted for all of these acceleration models. [Researchers at the GM Research Laboratories introduced the sensitivity-stimulus framework that is the basis for most car-following models to date. Examples include Toledo (3), Ahmed (6), Chandler et al. (17), Herman and Rothery (18), Gazis et al. (19), May and Keller (20), Gazis et al. (21), Ozaki (22), and Ferrari (23)]. Thus, the acceleration that driver \(n\) applies in each regime \(r\) is assumed to be a response to stimuli from the environment:

\[
\text{response}_{in}(t) = \text{sensitivity}_{in}(t) \times \text{stimulus}_{in}(t - \tau^n) \quad (3)
\]

where \(\tau^n\) is the driver’s reaction time.

The driver reacts to different stimuli in the various situations, depending on constraints imposed by the driving neighborhood and on the driver’s merging plan. The various types of acceleration are detailed below.

### Lane-Changing Acceleration

Lane-changing acceleration is applied when the driver accepts the available adjacent gap and executes the lane change. In such situations it is assumed that the driver determines the acceleration by evaluating the relations with the target lane leader (the leader in the lane to which the subject is changing). Because the estimation data collection site was heavily congested, the lane-changing acceleration is assumed to be always car-following acceleration or deceleration with respect to the new leader in the target lane (no free-flow consideration).

In the car-following model, the stimulus is given by the subject’s relative speed with respect to the leader (defined here as the speed of the leader less the speed of the subject vehicle). The sensitivity is a function of explanatory variables.

The expected value of the response to the stimuli is positive (acceleration) for positive leader relative speeds, that is, when the leader is faster than the subject vehicle, and negative (deceleration) for negative leader relative speeds. However, the response to positive and negative stimuli may be different because of the different nature of these situations: the main consideration in the reaction to a negative leader relative speed is safety, whereas the acceleration applied in a positive leader relative speed situation may be affected by speed advantage considerations and by inertia (i.e., human’s tendency to conform with the actions of others). To capture these differences the model allows the coefficients of explanatory variables to be different for positive and negative stimuli.

On the basis of the relative speed of the lead in the target lane, the lane-changing (lc) acceleration can thus be car-following acceleration or deceleration:

\[
a_{lc}^n(t) = \begin{cases} a_{lc,acc}^n & \text{if } \Delta V_{lead}^{lc}(t) \geq 0 \\ a_{lc,dec}^n & \text{otherwise} \end{cases} 
\]

where

\[
a = \text{acceleration}; \quad \Delta V_{lead}^{lc} = V_{lead}^{lc} - V_n; \quad V_{lead}^{lc} = \text{speed of lead at time } t; \quad V_n = \text{speed of subject at time } t; \quad \tau^n = \text{reaction time}, \tau^n \sim N(\mu_\tau, \sigma_\tau).
\]

The general functional form of lane-changing acceleration can be expressed as follows:

\[
a_{lc}^n(t) = s_{lc}^k [X_{lc,t}^n(t)] m_{lc}^{\text{lc}} \left[\Delta V_{lc,lead}(t - \tau^n)\right] + e_{lc,j}^n(t)
\]

\(j \in \text{acceleration, deceleration} \quad (5)\)

where

\[
X_{lc,t}^n(t) = \text{explanatory variables related to lane-changing acceleration};
\]

\(s_{lc}^k[.] = \text{sensitivity function for lane-changing acceleration};
\]

\(m_{lc}^{\text{lc}}[.] = \text{stimulus function lane-changing acceleration};\)

\(e_{lc,j}^n = \text{random error term}.
\]

Assuming that the random error term is normally distributed, \(e_{lc,j}^n \sim N(0, \sigma_{lc,j}^n)\), the probability density function of lane-changing acceleration can be expressed as follows:

\[
f(a_{lc}^n(t)|\tau^n) = \frac{1}{\sigma_{lc,j}^n} \phi \left( \frac{a_{lc}^n(t) - s_{lc}^k [X_{lc,t}^n(t)] m_{lc}^{\text{lc}} \left[\Delta V_{lc,lead}(t - \tau^n)\right]}{\sigma_{lc,j}^n} \right)
\]

(6)

Candidate variables affecting sensitivity function include speed of subject vehicle, spacing with lead vehicle, and roadway conditions (e.g., density).

### Target Gap Acceleration

This model captures the behavior of drivers who are unable to execute a normal lane change and have not initiated a courtesy or forced merge. The drivers apply acceleration to better position themselves with
where

\[
a_{n}^{\prime} = \begin{cases} a_{n}^{\prime} & \text{if } h_{n} \leq h_{a} \\ a_{n}^{\prime} & \text{otherwise} \end{cases} \tag{7}
\]

The constrained acceleration is modeled as car-following acceleration or deceleration based on relative speed and position of the current lead vehicle. The constrained acceleration and deceleration models have a functional form similar to lane-changing acceleration, the current lead being the source of the stimuli. That can be expressed as follows:

\[
f(a_{n}^{\prime}(t)|\delta_{n}^{\prime}) = \frac{1}{\sigma_{d,i}} \phi \left( \frac{a_{n}^{\prime}(t) - s_{d}^{\prime}[X_{n}^{\prime}(t)]m_{d}^{\prime}[\Delta V_{n}^{\prime}(t-\tau_{n}^{\prime})]}{\sigma_{d,i}} \right) \tag{8}
\]

where

\[
X_{n}^{\prime}(t) = \text{explanatory variables related to car-following (cf) acceleration},
\]

\[
s_{d}^{\prime}[,] = \text{sensitivity function } m_{d}^{\prime}[,] = \text{stimulus function, and}
\]

\[
\Delta V_{n}^{\prime} = \text{relative speed of lead vehicle in current lane } \epsilon_{n}^{\prime} \sim N(0, \sigma_{d,i}).
\]

The unconstrained target gap acceleration is in effect when the lead in the current lane does not constrain the subject vehicle. Although the detailed specifications of the various models in this group differ, they all incorporate the common assumption that if unconstrained, the driver targets a desired position with respect to the target gap, which would allow the lane change to be performed [as proposed by Toledo (3)]. This desired position is defined as the point in the current lane relative to the target gap that the driver perceives as optimal in regard to facilitating the lane change. The stimulus the driver reacts to is the difference between the desired position and the vehicle’s current position. The desired positions for different cases are illustrated in Figure 3.

The desired position is defined as follows:

\[
d_{n}^{\prime} = \gamma \Delta X_{n}^{\prime}(t-\tau_{n}^{\prime}) \tag{9}
\]

where

\[
\gamma = \text{coefficient estimated from data and}
\]

\[i = \text{adjacent, backward, forward}.
\]

The corresponding variables are detailed in Figure 4 with an example of a driver who targets the forward gap.

The general functional form of the acceleration can be expressed as follows:

\[
f(a_{n}^{\prime}(t)|\delta_{n}^{\prime}) = \frac{1}{\sigma_{d,i}} \phi \left( \frac{a_{n}^{\prime}(t) - s_{d}^{\prime}[X_{n}^{\prime}(t)]m_{d}^{\prime}[\Delta V_{n}^{\prime}(t-\tau_{n}^{\prime})]}{\sigma_{d,i}} \right)
\]

\[i \in \text{forward, adjacent, backward} \tag{10}
\]

\[\text{Initiated Courtesy Acceleration}\]

The initiated courtesy acceleration is the acceleration that drivers apply if they have initiated a courtesy merge, that is, if the anticipated gaps are acceptable. It has a form similar to adjacent gap acceleration although the specification may differ. Variables likely to affect this type of acceleration include desired and current positions and relative speed of lag.

\[\text{Initiated Forced Acceleration}\]

Initiated forced acceleration is applied if the driver has initiated a forced merge. In such cases the driver has already established the right-of-way and is responding to the stimuli from the leader in the target lane. Initiated forced acceleration has a form similar to lane-changing acceleration. The specification may differ, however.
Variables likely to affect this type of acceleration include speed of subject vehicle, spacing with lead vehicle, and roadway conditions (e.g., density).

**ESTIMATION**

**Data**

The disaggregate data used for estimating the merging model were collected from the eastbound direction of Interstate 80 (I-80) in Emeryville, California (Figure 5). The data were collected and processed as part of FHWA's Next Generation Simulation (NGSIM) project. Vehicles were tracked and digitized for a length of 503 m (the length by which merging needs to be completed being 200 m) using the software NG-Video [NGSIM (24)]. The vehicle trajectory data containing the coordinates of the various vehicles in the section were used to derive the required variables for estimation. The merging drivers entering from the on-ramp to the rightmost lane of the main line were used for estimation. The resulting data set included 17,352 observations at a 1-s time resolution of 540 vehicles.

Speeds in the merging section (the on-ramp and part of Lane 6 as defined in Figure 5) varied from 0 m/s to a maximum of 20.7 m/s, with a mean of 4.2 m/s. There were many stop-and-go situations present in the data set. Densities in Lane 6 (Figure 5) ranged from 0 veh/km/lane to 126.7 veh/km/lane with an average of 61.9 veh/km/lane. Of the merging vehicles in the data set, 1.4% were heavy vehicles (trucks in this case). Detailed analysis of the data and data processing methodology are presented in the NGSIM I-80 Data Analysis Report (14).

**Likelihood Function**

The trajectory data include second-by-second lane-changing and acceleration decisions of the driver. The only information about the driver and vehicle characteristics is the length of the vehicle. The following are therefore unobserved in the data:

- Merging tactic (normal, courtesy, or forced),
- Target gaps, and
- Driver and vehicle characteristics (represented by aggressiveness, reaction time, and anticipation time in this case).

The joint probability density of a combination of merging tactic ($s$) lane action ($l$), target gap (TG), and acceleration ($a$) observed for driver $n$ at time $t$, conditional on the individual specific variables ($υ_n$, $τ^R_n$, and $τ^A_n$), is given by

$$f_s(a, TG, l, s | v_n, τ^R_n, τ^A_n) = P_s(l | s, v_n, τ^R_n) P_{TG}(TG | s, l, v_n)$$

$$f_s(a, TG, l, s | v_n, τ^R_n, τ^A_n) = P_s(l | s, v_n, τ^R_n) P_{TG}(TG | s, l, v_n)$$

(11)
where \( P_n(TG_n \mid s_n, l_n, \nu_n) \) and \( f_n(s_n \mid l_n, \nu_n, TG_n, \tau^*_n) \) can be calculated by Equations 2, 6, 8, and 10. \( P_n(s_n \mid l_{n-1}, \nu_{n-1}, \tau_{n-1}^*) \) and \( P_n(l_n \mid s_n, \nu_n) \) follow the functional form proposed by Choudhury et al. (13).

Only the lane-changing and acceleration decisions are observed. The marginal probability of these two variables is given by summing the merging tactic and target gap of the joint probability:

\[
f_n(l_n, a_n \mid s_n, \nu_n, \tau^*_n, \tau^*_n) = \sum_{l} \sum_{a} f_n(s_n, l_n, a_n \mid s_n, \nu_n, \tau^*_n, \tau^*_n)
\]

(12)

where (for individual \( n \) at time \( t \))

\( a_n = \) acceleration (observed),

\( TG_n = \) target gap choice,

\( l_n = \) lane action (observed),

\( s_n = \) state (courtesy, forced, normal),

\( \nu_n = \) individual specific random effect, \( \tau^*_n = \) anticipation time,

\( \tau^*_n = \) reaction time, and

\( P_n(s_n \mid l_{n-1}, \nu_{n-1}, \tau_{n-1}^*) \) can be calculated recursively [detailed in Choudhury et al. (15)].

The behavior of driver \( n \) is observed during a sequence of \( T \) consecutive time intervals. The joint probability of the sequence of observations is the product of the probabilities

\[
f_n(l_n, a_n \mid s_n, \nu_n, \tau^*_n, \tau^*_n) = \prod_{t=1}^{T} f_n(l_n, a_n \mid s_n, \nu_n, \tau^*_n, \tau^*_n)
\]

(13)

where \( l \) and \( a \) are the sequences of lane-changing decisions and accelerations, respectively.

The unconditional individual likelihood function is acquired by integrating the conditional probability over the distributions of the individual specific variables:

\[
L_n = \int \frac{f_n(l_t \mid s_t, a_t, \nu_t, \tau^*_t, \tau^*_t) f \left( \tau^*_t \right) f \left( \tau^*_t \right) f(\nu)}{
u} d\tau^* d\tau^*_t d\nu
\]

(14)

where \( f(\tau^*_t) \) and \( f(\tau^*_t) \) are assumed to follow double truncated normal distributions. The standard normal probability density function is \( f(\nu) \).

Assuming that observations of different drivers are independent, the log-likelihood function for all \( N \) individuals observed is given by

\[
L = \sum_{n=1}^{N} \ln(L_n)
\]

(15)

The maximum likelihood estimates of the model parameters are found by maximizing this function.

### Estimation Results

Estimation results for the choice of merging plan indicate that the remaining distance to the end of the merging lane and aggressiveness of the driver have the most significant effect in initiating a courtesy and forced merge. The choice of merging plan was found to have a significant effect on the probability of gap acceptance. Results in general follow the same trend as the previous disjoint lane-changing model and are not discussed here in detail (13).

### Target Gap Selection

The model assumes that the driver evaluates the utility of the adjacent gap, the forward gap, and the backward gap and selects the one that he or she perceives to be the best. The estimated utility of the target gaps are as follows:

\[
U_{\text{adj}}^n(t) = 0.224 \text{effective_gap_size}_{\text{adj}}^n(t) - 0.0179 \text{rel_gap_speed}_{\text{adj}}^n(t) + 2.10 \text{no_lead/lag_dummy}_{\text{adj}}^n(t) + \epsilon_{\text{adj}}^n(t)
\]

\[
U_{\text{fwd}}^n(t) = -0.772 - 0.482 \text{distance_to_gap}_{\text{fwd}}^n(t) + 0.224 \text{effective_gap_size}_{\text{fwd}}^n(t) - 0.0179 \text{gap_rel_speed}_{\text{fwd}}^n(t) + 2.10 \text{no_lead_dummy}_{\text{fwd}}^n(t) + 0.675\nu_n + \epsilon_{\text{fwd}}^n(t)
\]

\[
U_{\text{bk}}^n(t) = -1.23 - 0.482 \text{distance_to_gap}_{\text{bk}}^n(t) + 0.224 \text{effective_gap_size}_{\text{bk}}^n(t) - 0.179 \text{gap_rel_speed}_{\text{bk}}^n(t) + 2.10 \text{no_lag_dummy}_{\text{bk}}^n(t) + 0.239\nu_n + \epsilon_{\text{bk}}^n(t)
\]

where \( \epsilon_{\text{adj}}^n(t), \epsilon_{\text{fwd}}^n(t), \epsilon_{\text{bk}}^n(t) \) are i.i.d. Gumbel distributed random terms. The parameters significant at 95% confidence level are boldfaced.

The effective gap length and the relative gap speed variables describe the size of the gap and its rate of change. The effective gap is defined by the minimum of the length of the gap in question. The utility of a gap increases with the effective length of the gap because the subject vehicle is more likely to be able to merge into a larger gap relative to a smaller gap.

The relative gap speed is defined by the speed of the vehicle at the rear of the gap less the speed of the vehicle that determines the front of the gap. It is an indicator of the anticipated usefulness of the gap. The estimated coefficient of this variable is negative, suggesting that drivers do try to anticipate the likely future situation when choosing their target gap. The no lead/lag dummy variables capture the effect of the absence of the front or back vehicle on the target gap choice. Drivers have a preference for gaps in which these are missing because the merging maneuver to gaps with no lead or lag requires less effort. The variable distance_to_gap is defined by the space headway between the subject vehicle and the target gap. As expected, drivers have lower utilities for gaps that are farther away. The aggressiveness parameter denotes that aggressive drivers prefer the forward and backward gap to the adjacent gap, which is intuitive.

### ACCELERATION

#### Lane-Changing Acceleration

The lane-changing acceleration model captures the behavior of drivers during the time a lane change is performed. The stimuli in this case come from the new lead vehicle (lead vehicle in the main lane). Depending on the relative speed with the new lead, the applied acceleration can be car-following acceleration or deceleration.

The functional forms of the stimulus and sensitivity functions of the car-following model are adopted from Toledo (3) and Ahmed (6),...
who extended the nonlinear GM model (19). The stimulus term is a nonlinear function of the relative leader speed given by

$$m^{k_j} [\Delta V^{lead}_{n} (t - \tau_j^r)] = [\Delta V^{lead}_{n} (t - \tau_j^r)]^{\lambda}$$

where \( \lambda \) is the corresponding parameter.

A positive correlation between the relative leader speed and the acceleration the driver applies is expected a priori. The parameters \( \lambda \) are, therefore, expected to be positive for both acceleration and deceleration.

The estimated car following acceleration model is given by

$$a^{k_{acc}}_n (t) = 0.016V^{lead}_{n} (t)^{0.102} \Delta X_{n} (t)^{0.0176} \Delta V^{lead}_{n} (t - \tau_j^r)^{0.380}$$

$$+ \varepsilon^{k_{acc}}_n (t)$$

$$\varepsilon^{k_{acc}}_n \sim N(0, 0.383^2)$$

where, at time \( t \)

$$\Delta V^{lead}_{n} (t) = V^{lead}_{n} (t) - V_{n} (t),$$

$$V^{lead}_{n} (t) = \text{speed of lead in target lane},$$

$$V_{n} (t) = \text{subject vehicle speed},$$

$$\Delta X_{n} (t) = \text{spacing},$$

$$k_{n} (t) = \text{density in target lane}.$$

The estimated car following deceleration model is given by

$$a^{k_{dec}}_n (t) = -0.000273 \Delta X_{n} (t)^{0.1185} \Delta V^{lead}_{n} (t - \tau_j^r)^{0.199}$$

$$+ \varepsilon^{k_{dec}}_n (t)$$

$$\varepsilon^{k_{dec}}_n \sim N(0, 0.596^2)$$

The effects of different variables on the mean car-following acceleration and deceleration are shown in Figures 6 and 7, respectively. In these figures the following default values are assumed: the subject speed is 4 m/s, space headway is 12 m, density is 60 veh/km/lane, and the relative leader speed is 0.3 (or –0.3) m/s (values are chosen on the basis of observed mean values in the data collection site).

Car-following acceleration increases with the subject speed, the density, and relative leader speed and decreases with the headway spacing. Car-following deceleration increases (in absolute value) with the density and relative leader speed and decreases with the space headway.

The stimulus term in the car-following regime is a function of the relative leader speed. As expected, the parameter associated with this term is positive for both acceleration and deceleration, which implies a positive correlation between the relative leader speed and the acceleration the subject vehicle applies.

The sensitivity terms are positive and negative for car-following acceleration and car-following deceleration, respectively. However, the magnitude of sensitivity to a negative relative leader speed is much larger than the sensitivity to a positive one. That is expected because a negative relative speed stimulus may have safety implications, whereas a positive relative leader speed stimulus only suggests a possible speed advantage to the driver.

The estimated coefficients of the space headways are negative for both acceleration and deceleration car following. For deceleration car following this is expected because the underlying safety concern increases when the spacing is reduced. In the case of acceleration car following, it may be related to a reduced perception of the leader as a stimulus the driver needs to react to. Similar to the sensitivity constants, the magnitude of the coefficient for deceleration is larger than that for acceleration.

The estimated coefficient of the subject’s speed in the acceleration model is positive, which is contrary to what was expected. This suggests that drivers apply higher accelerations at high speeds and high densities relative to lower speeds and densities. A possible explanation may be related to the acceleration capabilities of vehicles, which are higher at high speeds (and gear) relative to low speeds. Mean accelerations applied at high densities are higher relative to lower
in the previous section, and the parameters are also restricted to be the same. The difference is in the source of stimuli, which in this case comes from the lead in the current lane as opposed to the lead in the target lane. This can be expressed as follows:

\[ a_{i}^{acc}(t) = 0.016 V_{n}(t) 0.102 \Delta X_{i}(t)^{0.0105} k_{s}(t) 0.0176 \left( \Delta V_{n}^{front}(t - \tau_{n}^{s}) \right)^{0.380} + e_{i}^{acc}(t) \]

\[ e_{i}^{acc} \sim \mathcal{N}(0, 0.383^{2}) \]

\[ a_{i}^{dec}(t) = -0.00027 H_{9280} \Delta X_{i}(t)^{0.0108} k_{s}(t) 0.00743 \left( -\Delta V_{n}(t - \tau_{n}^{s}) \right)^{0.199} + e_{i}^{dec}(t) \]

\[ e_{i}^{dec} \sim \mathcal{N}(0, 0.596^{2}) \]

where \( \Delta V_{n}^{front}(t) = V_{n}^{front}(t) - V_{n}(t) \) and \( V_{n}^{front}(t) \) = speed of lead in current lane.

**Unconstrained Target Gap Acceleration**

The unconstrained target gap acceleration depends on the target gap, that is, it is different for front, adjacent, and backward gaps.

The stimulus in this case is the distance required to reach the desired position in the current lane that will facilitate merging to the target gap. The desired position for each of the target gaps is explained in Figure 8.

A nonlinear specification is used for the stimulus term:

\[ f^j \left[ D_{j}(t - \tau_{n}^{s}) \right] = \left( D_{j}(t - \tau_{n}^{s}) \right)^{\gamma^j} \]

\[ j \in \text{forward, adjacent, backward} \]

where \( \gamma^j \) is a constant parameter. \( D_{j}(t - \tau_{n}^{s}) \) is the desired position with respect to the forward gap. It is determined as a fraction of the total length of the gap as shown in Figure 3.

The estimated expression for desired position is as follows:

\[ d_{n}^{a} = 0.266 \Delta X_{i}(t - \tau_{n}^{s}) \]

where \( \Delta X_{i}(t) \) is the clear spacing of the corresponding gap.

The sensitivity term is a nonlinear function of the subject speed and the target lane leader relative speed and can be different for different types of gaps. The sensitivity term for the forward gap is as follows:

\[ x^{\text{ford}} \left[ V_{n}^{\text{ford}}(t) \right] = \alpha^{\text{ford}} V_{n}^{\text{ford}}(t) \beta^{\text{ford}} \exp \left[ \lambda^{\text{ford}} \Delta V_{n}^{\text{fordTL}}(t) \right] \]

\[ \exp \left[ \lambda^{\text{fordTL}} \Delta V_{n}^{\text{ford}}(t) \right] \]

where \( \alpha^{\text{ford}}, \beta^{\text{ford}}, \lambda^{\text{ford}}, \) and \( \lambda^{\text{fordTL}} \) are parameters. \( \Delta V_{n}^{\text{fordTL}}(t) \), and \( \Delta V_{n}^{\text{ford}}(t) \) are the positive and negative relative target lane leader speeds, respectively. These are defined as \( \Delta V_{n}^{\text{fordTL}}(t) = \max(0, \Delta V_{n}^{\text{ford}}(t)) \) and \( \Delta V_{n}^{\text{fordTL}}(t) = \min(0, \Delta V_{n}^{\text{ford}}(t)) \).

This formulation allows the sensitivity of the acceleration to the relative speed of the leader in the lane for the situation in which the leader is faster than the subject to be different from the sensitivity in the situation in which the leader is slower than the subject. The exponential form guarantees continuity of the acceleration when the relative target leader speed approaches zero.
The estimated forward gap acceleration can be expressed as follows:

\[ a_{n,\text{fwd}}(t) = 0.491D_{n}^{\text{fwd}}(t - \tau_{n}^{k})^{0.193} \]

\[ * \exp \left( \left[ \Delta V_{n}^{\text{fwd}}(t) \left( 0.0267(\Delta V_{n}^{\text{fwd}}(t) < 0) \right) - 0.683(\Delta V_{n}^{\text{fwd}}(t) > 0) \right] \right) + \epsilon_{n}^{\text{fwd}}(t) \]

\[ \epsilon_{n}^{\text{fwd}}(t) \sim N(0, 0.785^2) \] (24)

The mean forward gap acceleration is positive and increases with the distance to the desired position and with the target lane relative leader speed. The sensitivity of the forward acceleration to these variables is shown in Figure 9a, which makes the default assumptions that the speeds of the subject and lead vehicles are equal and that the distance to the desired position is 5 m.

Positive correlation between the distance to the desired position and the forward acceleration implies that drivers try to keep the duration of time to complete their short-term plan short. Thus, applying a larger acceleration when the distance they need to cover is longer.

The unconstrained backward gap model is similar to the forward gap acceleration model and can be expressed as follows:

\[ a_{n,\text{bck}}(t) = -0.819D_{n}^{\text{bck}}(t - \tau_{n}^{k})^{0.193} \]

\[ * \exp \left( \left[ \Delta V_{n}^{\text{bck}}(t) \left( 0.0267(\Delta V_{n}^{\text{bck}}(t) < 0) \right) - 0.683(\Delta V_{n}^{\text{bck}}(t) > 0) \right] \right) + \epsilon_{n}^{\text{bck}}(t) \]

\[ \epsilon_{n}^{\text{bck}}(t) \sim N(0, 2.13^2) \] (25)

The mean backward gap acceleration is negative and decreases (in absolute value) with the distance to the desired position and with
the relative target lane lead speed. The sensitivity of the backward acceleration to these variables is shown in Figure 9b, which has the default assumptions that the speeds of the subject and lag vehicles are equal and that the distance to the desired position is 10 m.

The adjacent gap acceleration model describes the behavior of drivers who target the currently adjacent gap to change lanes. A driver in the constrained regime would apply car-following behavior in the acceleration and the deceleration regimes. The unconstrained acceleration is aimed at maneuvering the vehicle to an optimal position in regard to being able to accept the available gap. A constant sensitivity term is used in this model, that is, the adjacent gap acceleration is proportional to the difference between the desired and the actual position.

$$\begin{align*}
a_{c}^{\text{adj}}(t) &= 0.120(D_{c}^{m}(t - \tau_{c}^{m})) + e_{c}^{\text{adj}}(t) \\
e_{c}^{\text{adj}}(t) &\sim N(0, 0.437^2)
\end{align*}$$

The adjacent gap acceleration sensitivity constant is expected to be positive. That is, the driver is likely to accelerate if the stimulus term is positive (i.e., the relative desired position is ahead of the vehicle’s current position) and to decelerate if it is negative (i.e., the relative desired position is behind the vehicle’s current position). This is illustrated in Figure 9c.

**Initiated Courtesy Acceleration**

In the initiated courtesy state the driver accelerates to facilitate merging to the adjacent gap and is modeled as adjacent gap acceleration. Statistical tests indicated that the model parameters are not significantly different from those of normal adjacent gap acceleration.

**Initiated Forced Acceleration**

In the initiated forced state the driver has already established his or her right-of-way and responds to the stimuli from the leader in the target lane. Initiated forced acceleration therefore has the same functional form and parameters as the lane-changing acceleration model. Model parameters different from the lane-changing acceleration were tested but not statistically supported by the data.
VALIDATION

Methodology and Data

The new lane-changing model was implemented in the microscopic traffic simulation model, MITSIMLab (25) and tested for validation using the NGSIM High Level Validation Plan (26). Aggregate data from a different NGSIM site, US-101, in California has been used for validation (27). The validation process is comparative. In this study, the goodness-of-fit statistics of the new model are compared with the base MITSIMLab model that uses the combined gap acceptance model (13) but does not have an integrated acceleration component [uses the general freeway acceleration developed by Toledo (3)]. Because the focus of the research was improving the speed and acceleration simulation of merging drivers, this measure was selected as a validation criterion.

The US-101 data set was collected in a southbound section of US Highway 101 in Los Angeles, California. The collection site was approximately 2,100 ft in length, with five main-line and one auxiliary lane connecting to the Ventura on-ramp and the Lankershim off-ramp (Figure 10). This site has an auxiliary lane after the on-ramp, which was not the case for the I-80 site. In total 45 min of data (7:50–8:35 a.m.) were available. Based on the trajectory data, “synthetic” sensor data were created in three locations (Figure 10). This sensor data replicated approximately 2,100 ft (640 m) were recorded by sensors positioned in these locations. Aggregate calibration was performed first with part of the data (7:50 to 8:20 a.m.) to adjust the driving behavior parameters of other components of MITSIMLab. These parameters included the desired speed distribution, probability of the lag driver yielding to show courtesy, constant terms, and standard deviations of the lane-changing and acceleration models. Sensor data from 8:20 to 8:35 a.m. in the northbound direction (not used for aggregate calibration) were used for validation.

Validation Results

Comparisons of the goodness-of-fit measures of the benchmark model and the new model are presented below.

<table>
<thead>
<tr>
<th></th>
<th>Previous Model</th>
<th>New Model</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE (m/s)</td>
<td>8.77</td>
<td>8.28</td>
<td>5.64</td>
</tr>
<tr>
<td>RMSPE</td>
<td>0.19</td>
<td>0.17</td>
<td>10.12</td>
</tr>
</tbody>
</table>

As evident from the root mean square error (RMSE) and root mean square percentage error (RMSPE), the new model with integrated acceleration performed much better compared with the benchmark model.

CONCLUSION

The detailed structure, estimation results, and validation results of an integrated gap acceptance and acceleration model for freeway merges have been presented in this paper. The model accounts for the effects of the unobserved merging mechanism in the observed driving behavior. Parameters of the models are estimated with detailed vehicle trajectory data collected from I-80 in California. The effect of unobserved driver and vehicle characteristics on the lane-changing process was captured by driver-specific random terms included in different model components. The superiority of this framework was demonstrated by a validation case study in which the performance of the new model was compared against a benchmark model that does not have an integrated gap acceptance and acceleration framework. Note that although the model was developed using trajectory data from a very congested merging location, the site used for validation was less congested and had a different geometric configuration. The validation exercise thus gives some indication of the transferability of the model in other situations. Although these results are promising, further validation studies with varying levels of congestion and geometric characteristics and further level of detail need to be performed to confirm the findings.

The presented model involves only the decisions of the merging driver, the lag driver behavior (whether to yield or move to an inner lane) is not modeled explicitly. In the estimation this is treated as an external variable. In the validation study, the probability that a lag driver will change to an inner lane was addressed by the main-line lane-changing model. The probability that the lag driver will yield to provide courtesy to the merging driver was calibrated along with other MITSIMLab parameters. However, in reality, whether a main-line driver decides to show courtesy to the merging vehicles or makes a lane change can be a function of many factors such as type of merging vehicle, urgency of merge, and type of through vehicle. This may help in improving the validation results and needs to be explored in a detailed study.

Also, in the courtesy merging, the effect of the lag vehicle changing to an inner lane to create gaps for a merging vehicle has not been differentiated from a normal change in the size of the gap (i.e., in such cases the created gap is treated as a new adjacent gap and the entire decision process is repeated). The validity of this assumption needs to be explicitly tested by statistical comparison.

The preliminary validation results demonstrate the improvement in the simulation capability after integration of the acceleration component using aggregate data. Although this is sufficient as a measure of practical interest, a detailed validation study using disaggregate data from different sites will strengthen this finding. Moreover, for practical application of the model another important aspect is the trade-off between the complexity and the accuracy of the model development. In the MITSIMLab implementation, the proposed model did not result in any significant increase in running time of the simulator. The model is currently being validated independently by two commercial developers, Paramics (28) and VISSIM (29), as part of the NGSIM project. Results of the commercial validation are expected to provide a clearer indication of the practical implications of the model improvements.
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