**Metamaterial broadband angular selectivity**

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<td>As Published</td>
<td><a href="http://dx.doi.org/10.1103/PhysRevB.90.125422">http://dx.doi.org/10.1103/PhysRevB.90.125422</a></td>
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<tr>
<td>Publisher</td>
<td>American Physical Society</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Sun Apr 24 16:54:33 EDT 2016</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/89661">http://hdl.handle.net/1721.1/89661</a></td>
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Light selection based purely on the direction of propagation has long been a scientific challenge [1–3]. Narrow-band angularly selective materials can be achieved by metamaterials [4] or photonic crystals [5]; however, optimally, an angularly selective material system should work over a broadband spectrum. Such a system could play a crucial role in many applications, such as directional control of electromagnetic wave emitters and detectors, high efficiency solar energy conversion [6,7], privacy protection [8], and high signal-to-noise-ratio detectors.

Recent work by Shen et al. [9] has shown that one can utilize the characteristic Brewster modes to achieve broadband angular selectivity. The key concept in this work was to tailor the overlap of the band gaps of multiple one-dimensionally periodic isotropic photonic crystals, each with a different periodicity, such that the band gaps cover the entire visible spectrum, while visible light propagating at the Brewster angle of the material system does not experience any reflections. Unfortunately, for an isotropic-isotropic bilayer system, the Brewster angle is determined solely by the two dielectric constants of these materials; hence, it is fixed once the materials are given. Furthermore, among naturally occurring materials, one does not have much flexibility in choosing materials that have the precisely needed dielectric constants, and therefore the available range of the Brewster angles is limited. For example, the Brewster angle at the interface of two dielectric media (in the lower index isotropic material) is always larger than 45°. In many of the applications mentioned above, it is crucial for the material system to have an arbitrary selective angle, instead of only angles larger than 45°. Furthermore, the ability to control light would be even better if the selective angle could be tuned to a broad range of angles; and, by increasing the number of stacks, the angular transmission window can be made as narrow as desired. As a proof of principle, we realize the idea experimentally in the microwave regime. The angular selectivity and tunability we report here can have various applications such as in directional control of electromagnetic emitters and detectors.

DOI: 10.1103/PhysRevB.90.125422

PACS number(s): 07.57.-c, 42.70.Qs, 42.25.Bs
leads to a change in the index of the anisotropic \((A)\) layers have the same index. Right panel: the change in incident angle is experienced by ordinary waves, \(\tilde{\epsilon}_{\theta}\) dielectric constant of the anisotropic layers where \(\theta\) matches the isotropic layer and \(\tilde{n}_{\theta}\) is the refractive index experienced by the \(z\) component of the electric field, and \(\tilde{n}_{\theta}^{2} = \frac{\epsilon_{x}^{2}}{\epsilon_{y}} = \frac{\epsilon_{y}^{2}}{\epsilon_{x}}\) is the refractive index experienced by the \(x\) and \(y\) components of the electric field.

At normal incidence, for \(p\)-polarized light, the effective dielectric constant of the anisotropic layers \(n_{e}(0)^{2} = \tilde{n}_{e}^{2} = \epsilon_{x}\) matches the isotropic layer \(\epsilon_{\text{iso}}\); therefore no photonic band gap exists, and all the normal incident light gets transmitted \([R_{p} = 0\) in Eq. (1)]. On the other hand, when the incident light is no longer normal to the surface, the \(p\)-polarized light has \(E_{z} \neq 0\), and experiences an index contrast \(n_{e}^{p}(\theta) = \sqrt{\epsilon_{\text{iso}}} \neq n_{g}^{p} = n_{e}(\theta)\) (Fig. 1). As a result, a photonic band gap will open. Furthermore, we notice that as \(\theta\) gets larger, the \(\tilde{n}_{e}\) term in Eq. (4) becomes more important, hence the size of the band gap increases as the propagation angle deviates from the normal direction. The band gap causes reflection of the \(p\)-polarized incident light, while the \(s\)-polarized light still has \(E_{z} = 0\), so it remains as an ordinary wave experiencing no index contrast \(n_{s}^{p} = n_{g}^{p}\), hence, \(s\)-polarized light will be transmitted at all angles.

The method described above provides an idealistic way of creating an angular photonic band gap. However, in practice it is hard to find a low-loss anisotropic material, as well as an isotropic material whose dielectric constants exactly match that of the anisotropic material. In our design, we use a metamaterial to replace the anisotropic layers in Fig. 1, as shown in Fig. 2(a). Each metamaterial layer consists of several high-index \((\epsilon_{1} = 10)\) and low-index \((\epsilon_{2} = \epsilon_{\text{air}} = 1)\) material layers. We assume that each layer has a homogeneous and isotropic permittivity and permeability. When the high-index layers are sufficiently thin compared with the wavelength, the effective medium theorem allows us to treat the whole system as a single anisotropic medium with the effective dielectric index

\[
n_{e}(\theta) = \tilde{n}_{e},
\]

where \(\theta\) is the angle between the \(z\) axis [Fig. 2(a)] and \(\vec{k}\) in the material. \(n_{e}(\theta)\) is the effective refractive index experienced by extraordinary waves, \(n_{e}(\theta)\) is the effective refractive index experienced by ordinary waves, \(\tilde{n}_{e}^{2} = \frac{\epsilon_{x}^{2}}{\epsilon_{y}}\) is the refractive index experienced by the \(z\) component of the electric field, and \(\tilde{n}_{e}^{2} = \frac{\epsilon_{x}^{2}}{\epsilon_{y}} = \frac{\epsilon_{y}^{2}}{\epsilon_{x}}\) is the refractive index experienced by the \(x\) and \(y\) components of the electric field.

FIG. 1. (Color online) Effective index for \(p\)-polarized light in an isotropic \((A)\) -anisotropic \((B)\) multilayer system. Left panel: all the layers have the same index. Right panel: the change in incident angle leads to a change in the index of the anisotropic \((B)\) material layers.

FIG. 2. (Color online) Metamaterial behavior. (a) Schematic illustration of a stack of isotropic-anisotropic photonic crystals. Layer \(A\) is an isotropic medium; layer \(B\) is an effective anisotropic medium consisting of two different isotropic media with dielectric constants \(\epsilon_{1}\) and \(\epsilon_{2}\). (b) \(p\)-polarized transmission spectrum for a 30-bilayer structure with \([\epsilon_{\text{iso}}, \epsilon_{1}, \epsilon_{2}] = [2.25, 10, 1]\), and \(r = 6.5\). The unit of \(y\) axis---\(x\) is the periodicity of the structure. (c) \(p\)-polarized transmission spectrum of three stacks of 30-bilayer structures described in part (b). The periodicities of these stacks form a geometric series \(a_{1} = a_{1} q^{i-1}\) with \(q = 1.0234\), where \(a_{1}\) is the periodicity of the \(i\)th stack.
The dielectric permittivity tensor \( \{\epsilon_x, \epsilon_y, \epsilon_z\} \) [14]:

\[
\epsilon_A = \epsilon_Y = \frac{\epsilon_1 + r \epsilon_2}{1 + r}, \quad (6)
\]

\[
\frac{1}{\epsilon_z} = \frac{1}{1 + r} \left( \frac{1}{\epsilon_1} + \frac{r}{\epsilon_2} \right), \quad (7)
\]

where \( r \) is the ratio of the thickness of the two materials \( \epsilon_1 \) and \( \epsilon_2 \; r = \frac{d_1}{d_2} \).

For example, in order to achieve the normal incidence angular selectivity, we need the dielectric permittivity tensor of the anisotropic material to satisfy Eq. (3). For the isotropic material \( (A) \) layers, we need to choose \( \epsilon_{iso} \) that lies between \( \epsilon_1 \) and \( \epsilon_2 \). For definiteness, we choose a practical value of \( \epsilon_{iso} = 2.25 \) (common polymers). From Eqs. (6) and (7), with material properties \( \epsilon_1 = 10 \) and \( \epsilon_2 = 1 \), and the constraint \( \epsilon_s = \epsilon_x = \epsilon_{iso} = 2.25 \), we can solve for \( r \), obtaining \( r = 6.5 \).

Using the parameters calculated above and with a 30-bilayer structure, the transmission spectrum of \( p \)-polarized light at various incident angles is calculated using the transfer matrix method [15], and the result is plotted in Fig. 2(b). In Fig. 2(b) the wavelength regime plotted is much larger than \( d_2 \); in such a regime, the light interacts with layer \( B \) as if it is a homogeneous medium, and experiences an effective anisotropic dielectric permittivity.

One can enhance the bandwidth of the angular photonic band gap by stacking more bilayers with different periodicities [16, 17]. In Figs. 2(c) and 2(d), we present the stacking effect on the transmission spectrum for \( p \)-polarized light. When we have a sufficient number of stacks, a broadband angular selectivity (bandwidth \( >30\% \)) at normal incidence can be achieved.

In general, the Brewster angle for isotropic-anisotropic photonic crystals depends on \( \epsilon_x, \epsilon_z, \) and \( \epsilon_{iso} \) [Eq. (2)]. In our metamaterial system, it depends strongly on \( r \). Substituting Eqs. (6) and (7) into Eq. (2), we get

\[
\theta_B (r) = \arctan \left[ \frac{\epsilon_1 \epsilon_2 (\epsilon_1' + r \epsilon_2' - 1 - r)}{(1 + r) \epsilon_1' \epsilon_2' - \epsilon_2' - \epsilon_1' r} \right], \quad (8)
\]

where \( \epsilon_1' = \frac{\epsilon_1}{\epsilon_{iso}} \) and \( \epsilon_2' = \frac{\epsilon_2}{\epsilon_{iso}} \). From Eq. (2), we can see that in order to have a nontrivial Brewster angle, we need \( \epsilon_{iso} \) to be larger than \( \max\{\epsilon_x, \epsilon_z\} \) or smaller than \( \min\{\epsilon_x, \epsilon_z\} \); otherwise there will be no Brewster angle.

The result in Eq. (8) shows that it is possible to adjust the Brewster angle by changing the ratio \( r = \frac{d_1}{d_2} \), or by changing the spacing distance \( d_2 \) when everything else is fixed. In Fig. 3, we show the photonic band diagrams [18] of a simple anisotropic-isotropic quarter-wave stack. The band diagrams (explained in the caption) are calculated with preconditioned conjugate-gradient minimization of the block Rayleigh quotient in a plane-wave basis, using a freely available software package [19]. The dielectric tensor of the anisotropic material in each band diagram is calculated by Eq. (6) with \( r = 6.5, \ r = 9, \ r = 11, \) and \( r = 30 \), respectively. In the photonic band diagram, modes with propagation direction forming an angle \( \theta \) with respect to the \( z \) axis in Fig. 1 (in the isotropic layers) lie on a straight line represented by \( \omega = k_c c / (\sqrt{\epsilon_{iso}} \sin \theta) \); for general propagation angle \( \theta \), this line will extend both through the regions of the extended modes, as well as through the band-gap regions. However, for \( p \)-polarized light, at the Brewster angle \( \theta_B \), the extended modes exist regardless of \( \omega \) (dashed line in Fig. 3). It is clear that the Brewster angle increases as we increase \( r \). When \( r \to \infty \), the Brewster angle (defined in the isotropic layer) approaches \( \theta_B = \arctan \frac{\omega}{\epsilon_{iso}} \) [15]. Note that if \( \epsilon_2 \) is some soft elastic material [such as poly(dimethyl siloxane) (PDMS) or air], one can simply vary \( r \) easily by changing the distance \( d_2 \) in real time, and hence varying the Brewster angle accordingly. The dependence of the Brewster angle on \( r \) is presented in Fig. 4(d). At small \( r \), there is either no Brewster angle or the light cannot be coupled into the Brewster angle from air. As \( r \) gets larger, we see a rapid increase in the Brewster angle, which eventually plateaus, approaching the isotropic-isotropic limit, \( \theta_B = \arctan \frac{\omega}{\epsilon_{iso}} \). Such tunability of the Brewster angle does not exist in conventional (nonmetamaterial) isotropic-isotropic or isotropic-anisotropic photonic crystals, where the Brewster angle depends solely on the materials’ dielectric properties.

Similar to what we obtained in Fig. 2, we can enhance the bandwidth of angular selectivity by adding stacks with different periodicities. The transmission spectra of metamaterial systems with \( n = n = 30 \) [see Fig. 2(a)] and different \( r \)’s were calculated with the transfer matrix method [15] and plotted in Figs. 4(a), 4(b), and 4(c), respectively. Note that due to the inherent properties of Eq. (1), the angular selective window gets narrower as the Brewster angle increases.
FIG. 4. (Color online) Angular selectivity at oblique angles. Same materials and structure as illustrated in Fig. 2, \( n = m = 30, \) and \( q = 1.0373 \) but with different \( r. \) In (a), (b), and (c) we used \( \epsilon_1 = 10, \epsilon_2 = 1, \) and \( \epsilon_{iso} = 2.25. \) (a) \( r = 9 \) and \( \theta_B = 24^\circ. \) (b) \( r = 11 \) and \( \theta_B = 38^\circ. \) (c) \( r = 30 \) and \( \theta_B = 50^\circ. \) (d) Dependence of the Brewster angle (coupled in from air) on \( r \) for various values of \( \epsilon_1 \) and \( \epsilon_2. \) Solid and dashed lines correspond, respectively, to \( \epsilon_2 \) for air and \( \epsilon_2 \) for PDMS.

To show the feasibility of the above-mentioned method, we present an experimental realization in the microwave regime. Since our goal here is only to demonstrate the concept, we kept the experimental setup simple. We implemented the geometry design in Fig. 2, we can choose material system consisting of SiO2 \((\epsilon_1 = 2),\) poly(methyl methacrylate) \((\epsilon_{iso} = 2.25),\) and Ta2O5 \((\epsilon_2 = 4.33)\) to realize the generalized Brewster angle in heterostructured photonic crystals. Due to the limited choice of natural anisotropic materials, the method of using metamaterials to create an effective anisotropic medium is proposed. With the proposed material system, we demonstrate the possibility of tuning the selective angle in a wide range, simply by modifying the design of the material system with a fixed material composition. As a proof of principle, a simple experimental demonstration in the microwave regime was reported.

FIG. 5. (Color online) Experimental verification. (a) A schematic illustration of the experimental setup. (b) Photo of the fabricated sample. (c), (d) Comparison between \( p \)-polarized transmission spectrum of transfer matrix method and the experimental measurements.
normal incidence angular selectivity (with angular window less than 20°) in the entire visible spectrum (400–700 nm). We would like to point out that in optical regime, although the material loss is almost negligible since the material system is thin, the disorder of the layer thickness due to fabrication uncertainty might affect the performance. Another potentially interesting feature would be to explore the dynamical tuning of the selective angle.

We thank Dr. Ling Lu for the valuable discussion. This work was partially supported by the Army Research Office through the ISN under Contract No. W911NF-13-D0001, and Chinese National Science Foundation (CNSF) under Contract No. 61131002. The fabrication part of the effort, as well as (partially) M.S. were supported by the MIT S3TEC Energy Research Frontier Center of the Department of Energy under Grant No. DE-SC0001299.