Theoretical Kinetics Study of the $\text{O}^{(3P)} + \text{CH}_4/\text{CD}_4$ Hydrogen Abstraction Reaction: The Role of Anharmonicity, Recrossing Effects,
Theoretical Kinetics Study of the O(3P) + CH₄/CD₄ Hydrogen Abstraction Reaction: The Role of Anharmonicity, Recrossing Effects and Quantum Mechanical Tunneling.

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Abstract

Using a recently developed full-dimensional accurate analytical potential energy surface [Gonzalez-Lavado, E.; Corchado J. C.; Espinosa-Garcia, J. J.Chem.Phys. 2014, 140, 064310], we investigate the thermal rate coefficients of the O(3P) + CH₄/CD₄ reactions with ring polymer molecular dynamics (RPMD) and with variational transition state theory with multidimensional tunnelling corrections (VTST/MT). The results of the present calculations are compared with available experimental data for a wide temperature range 200–2500 K. In the classical high-temperature limit, the RPMD results match perfectly the experimental data, while VTST results are smaller by a factor of two. We suggest that this discrepancy is due to the harmonic approximation used in the present VTST calculations which leads to an overestimation of the variational effects. At low temperatures the tunnelling plays an important role, which is captured by both methods, although they both overestimate the experimental values. The analysis of the kinetic isotope effects shows discrepancy between both approaches, with the VTST values smaller by a factor about two at very low temperatures. Unfortunately, no experimental results are available to shed any light on this comparison, which keeps it as an open question.
**Keywords:** Oxygen and methane reaction; Ring polymer molecular dynamics; Variational transition-state theory; Analytical potential energy surface; Rate coefficients; Tunnelling.

**Legend:** RPMD and VTST kinetics calculations on a full-dimensional analytical potential energy surface (PES-2014) for the $\text{O(^3P) + CH}_4 \rightarrow \text{OH} + \text{CH}_3$ reaction reproduce the experiment.
1. Introduction

Ground-state oxygen atom chemistry is of great importance over a wide temperature and energy range. The title reaction, \( \text{O}(^3\text{P}) + \text{CH}_4 \rightarrow \text{OH} + \text{CH}_3 \), is important for hydrocarbon combustion and it is also a prototypical reaction for oxygen–hydrocarbon combustion. At high temperatures (1000–2500 K), because of heavy-light-heavy mass combination, many recrossings of the dividing surface separating reactants from products are expected. At low temperatures (200–300 K), because of light hydrogen atom transfer this reaction is also a good candidate to study quantum tunnelling effects. Moreover, at very high energies (2–5 eV) it is an important step in the mechanism of polymer hydrocarbon decomposition in low Earth orbit (LEO).

The thermal rate coefficients of the reaction with methane, \( \text{O}(^3\text{P}) + \text{CH}_4 \rightarrow \text{OH} + \text{CH}_3 \), have been measured experimentally with a wide variety of methods.\(^1\)-\(^21\) The most recent recommended expression for the rate coefficients for the temperature range 300-2500 K is,\(^21\)

\[
k(T) = 1.15 \times 10^{-15} T^{1.56} \exp(-4270/T) \text{ cm}^3 \text{ molecule}^{-1} \text{ s}^{-1} \quad (1)
\]

although below 400 K, the value of \( k(T) \) is considerably less reliable due to the uncertainties in the reaction stoichiometry,\(^18\) and possibly large tunneling effects. In addition, expression (1) gives low-temperature rate coefficients lower than most of the experimental results, and Cohen\(^18\) proposed an alternative expression of the experimental results given by

\[
k(T) = 2.69 \times 10^{-18} T^{2.3} \exp(-3570/T) \text{ cm}^3 \text{ molecule}^{-1} \text{ s}^{-1} \quad (2)
\]

Equation 2 leads to higher values of the rate coefficient than Eq. 1 at low temperatures (up to 60% higher at 300 K), in better agreement with the experimental values.

The kinetics of this reaction has also received a great theoretical attention.\(^22\)-\(^29\) In 1988 some of us\(^22\) studied the kinetics of this reaction using variational transition state theory (VTST) in curvilinear coordinates with transmission coefficients calculated by the microcanonical optimized multidimensional tunneling approximation\(^22\) (VTST/MT), where a dual-level algorithm was used for the dynamical calculations. Good agreement with experiment was obtained. In that work we developed the first analytical potential energy surface, which was, however, not symmetric with respect to the permutation of the four hydrogens. Based on this surface, Clary\(^23\) performed time-independent quantum scattering calculations, obtaining rate coefficients lower than the experimental values. To improve the deficiencies of the earlier surface, in 2000 our
group\textsuperscript{24} reported a new analytical surface for the title reaction (PES-2000), which was symmetric with respect to any permutation of the four methane hydrogens. Using this surface the thermal rate coefficients of this reaction have been calculated using different approaches: variational transition state theory with multidimensional tunneling (VTST/MT),\textsuperscript{24} multiconfiguration time-dependent Hartree (MCTDH),\textsuperscript{25} quasi-classical trajectory (QCT),\textsuperscript{26} reduced-dimensional quantum dynamics (QD),\textsuperscript{27,28} and more recently ring polymer molecular dynamics (RPMD),\textsuperscript{29} yielded in general good agreement with the experimental data. In this latter paper, authors noted however, that it is possible that this PES-2000 is still not sufficiently accurate. For example, the PES has a barrier height of 13.0 kcal mol\textsuperscript{-1} and an imaginary frequency of 1549 cm\textsuperscript{-1} at the transition state, which is significantly smaller than the ab initio values, 14.0 kcal mol\textsuperscript{-1} and about 2000 cm\textsuperscript{-1}, respectively. So, a thinner barrier might facilitate more facile tunneling. To correct the deficiencies of PES-2000, recently our lab reported the construction of a new analytical surface for the title reaction based \textit{exclusively} in high-level ab initio calculations, denominated as PES-2014.\textsuperscript{30} An advantage of this surface is that not only the energy is obtained analytically, but also the first energy derivatives, i.e., the gradients, which is a very desirable feature in kinetics and dynamics calculations.

In addition, this reaction also represents a theoretical challenge because the approach of O(\(^3\)P) along a CH bond has 3-fold symmetry and leads to a Jahn-Teller conical intersection rather than a saddle point. The conical intersection corresponds to a \(^3\)E state, and breaking the C\(_{3v}\) symmetry splits this into two surfaces, \(^3\)A' and \(^3\)A''. Walch and Dunning\textsuperscript{31} and Schlegel et al.\textsuperscript{32} using ab initio molecular orbital calculations, obtained the optimized geometries and frequencies at the stationary points, and the barrier height and heat of reaction. The saddle point geometry is found to be of C\(_{3}\) symmetry, but close to C\(_{3v}\) and the predicted barrier heights for the two surfaces (\(^3\)A' and \(^3\)A'') present a very small difference (0.2 kcal mol\textsuperscript{-1}).

The title reaction is a hydrogen abstraction reaction, which is affected by quantum effects, such as zero-point energy (ZPE) and tunnelling, and recrossing. Theoretically, the accuracy of the kinetics (and also dynamics) description of a chemical reaction depends mainly on two factors: the kinetics (dynamics) approach and the potential energy surface (PES). Accurate thermal rate coefficients only could be obtained using exact quantum mechanical methods (based on accurate surfaces), which is usually prohibitive for full-dimensional polyatomic systems. Different approaches have been proposed to circumvent this problem, and in the present paper we analyze
two approaches: variational transition state theory with multidimensional tunneling effect (VTST/MT) and ring polymer molecular dynamics (RPMD), which take into account the quantum effects (tunneling and ZPE) and recrossing in different ways. Firstly, at low temperatures, while in the first one (VTST/MT) the tunnelling is included as a semi-classical correction factor to the short-time limit of the classical real-time correlation functions calculated using quantum partition functions, in the second one (RPMD) its path integral nature allows the inclusion of quantum tunneling effect through a connection between RPMD and semi-classical instanton theory and ZPE effect due to rigorous treatment of the quantum Boltzmann operator. Secondly, at high temperatures, while the first one reduces the recrossing by variational optimization of the dividing surface, the RPMD rate theory treats recrossing more explicitly by analysing the long-time limit of the corresponding real-time correlation functions. Note that both VTST and RPMD approaches assume Boltzmann equilibrium of energy levels, i.e. high-pressure limit.

In summary, the main aim of the present paper is to analyse various effects that affect the calculations of the thermal rate coefficients using two dynamics approaches. The article is structured as follows. In Section 2, a brief description of the PES-2014 is presented. In Section 3, we briefly outline some computational details of the kinetics calculations. The results are presented and compared with the available experimental data in Section 4. Finally, Section 5 contains the conclusions.

2. Potential energy surface

The PES-2014 has been exhaustively described in an earlier paper, and here it will be briefly outlined. Basically it consists of four LEP-type (London–Eyring–Polanyi) stretching terms, valence (val) bending terms for each bond angle in methane, quadratic-quartic terms whose aim is to correctly describe the out-of-plane motion of methyl, and a van der Waals term to describe the intermediate complexes in the entrance and exit channels. In addition, it includes a series of switching functions allowing the smooth change from pyramidal CH$_4$ to planar CH$_3$ product. It depends on 41 adjustable parameters, which are fitted to very high-level ab initio calculations, CCSD(T) = FULL/aug-cc-pVQZ. This surface presents a barrier height of 14.1 kcal mol$^{-1}$ and reaction energy of 5.8 kcal mol$^{-1}$, reproducing benchmark calculations. In addition it presents intermediate complexes in the entrance and exit channels. For the
sake of clarity, Figure 1 plots schematically the variations in potential energy along the hydrogen abstraction reaction path for the PES-2014.


a) VTST/MT approach. Using the new PES-2014 surface, the reaction path was calculated starting from the saddle point geometry and going downhill to both reactants and products in mass-weighted Cartesian coordinates, using Page and McIver's method\textsuperscript{36} with a step-size of 0.001 u\textsuperscript{1/2} a\textsubscript{0} obtaining the minimum energy path, MEP.\textsuperscript{37} The Hessian matrix was evaluated at every point along this reaction path. Along this minimum energy path (MEP), the reaction coordinate, s, is defined as the signed distance from the saddle point, with s > 0 referring to the product side. In the rest of paper, the unit of s is a\textsubscript{0}, and all calculations are carried out in mass-scaled coordinates with a reduced mass \( \mu \) equal to 1 u. Thus, distances through the mass-scaled coordinates in a\textsubscript{0} are equivalent to distances through mass-weighted coordinates in u\textsuperscript{1/2} a\textsubscript{0}. We calculate the reaction-path between s = -3.0 and s = +3.0 a\textsubscript{0}; all rate constants are well converged with respect to the gradient step-size, distance between Hessian matrices, and extent of reaction path calculated.

Along the MEP, we calculated the vibrational frequencies after having projected out the motion along the reaction path using curvilinear redundant internal coordinates.\textsuperscript{38,39} These coordinates are a nonlinear function of Cartesian coordinates, and they avoid unphysical imaginary values of the vibrational frequencies over a wide range of the reaction coordinate. With this information, we calculated the ground-state vibrationally adiabatic potential curve as

\[ V_s^G(s) = V_{\text{MEP}}(s) + \int_{0}^{s} \text{int}^G(c) \]  

where s is the reaction coordinate, measured as the mass-weighted distance along the reaction path to the saddle point, for which s = 0, \( V_{\text{MEP}}(s) \) is the classical energy along the MEP with its energy zero at the reactants, and \( \text{int}^G(s) \) is the zero-point energy at s.

Rate coefficients were calculated using canonical variational transition-state theory (CVT),\textsuperscript{40,41} which locates the dividing surface between reactants and products at a point \( s^{*}\text{CVT}(T) \) along the reaction path that minimizes the generalized TST rate coefficients, \( k^{GT}(T,s) \) for a given temperature T. Thermodynamically, this is equivalent
to locating the transition state at the maximum $\Delta G^{GT,0} [T,s^{*\text{CVT}}(T)]$ of the free energy of activation $\Delta G(T,s)$.\textsuperscript{40,41} Thus, the thermal rate coefficient will be given by

$$k^{\text{CVT}}(T) = \sigma \frac{k_B T}{\hbar} K^o \exp\left[\frac{-\Delta G(T, s^{*\text{CVT}})}{k_B T}\right]$$

(4)

with $k_B$ being Boltzmann’s coefficient, $\hbar$ being Planck’s constant, $\sigma$ being the symmetry factor (the number of equivalent reaction paths, which were assumed to be 4), and $K^o$ being the reciprocal of the standard-state concentration, taken as 1 molecule cm$^{-3}$. The analysis of the $V_s G(s)$ curve showed the existence of two maxima, therefore, the rate coefficients were finally calculated using the canonical unified statistical model (CUS).\textsuperscript{41,42}

$$k^{\text{CUS}}(T) = k^{\text{CVT}}(T) R^{\text{CUS}}(T)$$

(5)

with

$$R^{\text{CUS}}(T) = \left[1 + \frac{k^{\text{CVT}}}{k^{\text{max}}} + \frac{k^{\text{CVT}}}{k^{\text{min}}}\right]^{-1}$$

(6)

where $k^{\text{CVT}}$ is calculated at the highest maximum of $\Delta G$, $k^{\text{max}}$ at the second highest maximum, and $k^{\text{min}}$ at the minimum of $\Delta G$, measured from the reactants. $R^{\text{CUS}}(T)$ is so a measure of the recrossing effect.

In calculating electronic partition functions, we included the spin-orbit splitting of O($^3\text{P}$), which is 158.26 and 226.98 cm$^{-1}$ for $^3\text{P}_1$ and $^3\text{P}_0$ relative to $^3\text{P}_2$. We also included the two electronic states for the OH product in the calculation of its electronic partition function, with a 140 cm$^{-1}$ splitting. As a consequence of the electronic degeneracy (Jahn-Teller effect), the potential energy surface for the O($^3\text{P}$) + CH$_4$ reaction splits into two surfaces of respective symmetries $^3\text{A'}$ and $^3\text{A''}$. Since our calculations are only based on the lowest surface, the two surfaces, however, making almost equal contributions to the rate, the electronic partition coefficients of the triplet ground-electronic state generalized transition states are multiplied by 2, yielding an overall value of 6. This is equivalent to using the following total electronic partition function ratio

$$Q_e(T) = \frac{6}{5 + 3e^{-e^{(\text{R})}/RT} + e^{e^{(\text{R})}/RT}}.$$  

(7)

The rotational partition functions were calculated classically and quantum effects in motions orthogonal to the reaction path were included by using quantum-mechanical
vibrational partition functions in the harmonic approximation in redundant internal coordinates.

Finally, we considered the tunnelling contributions. This reaction has a heavy-light-heavy mass combination, and, therefore, a large curvature tunneling (LCT) calculation should be considered. We also considered the microcanonical optimized multidimensional tunneling (µOMT) approach in which, at each total energy, the larger of the small-curvature (SCT) and large-curvature (LCT) tunneling probabilities is taken as the best estimate. In the LCT approximation, tunnelling proceeds along straight-line paths from the entrance valley of the PES to the product valley and perpendicular to the reaction path, and two versions of this method, LCT3 and LCT4 have been developed. A better way to compute tunneling is to optimize the angle between the reaction path and the tunneling path so that the tunneling probability is the highest or the imaginary action is the least. This angle is 0° in the SCT case and 90° in the LCT case. This is (very briefly described) the least-action tunneling (LAT). The LAT method is considered to be the most accurate, but it has not been developed for polyatomic reactions until very recently, and it has been scarcely applied.

All kinetics calculations were performed using the POLYRATE-2010 code developed by Truhlar and co-workers.

b) RPMD approach.

Ring Polymer Molecular Dynamics (RPMD) is a recently proposed full-dimensional approach to treat quantum real-time dynamics of the system based on the classical isomorphism between quantum system and classical ring polymer consisting of n copies (beads) of this system interacting through harmonic interaction. This isomorphism allows to approximate the quantum real-time dynamics using the classical real-time evolution in an extended phase space of the ring polymer. The RPMD rate coefficient is rigorously independent of the choice of the transition state dividing surface used to compute it, a feature that distinguishes it from all transition state theory-based methods.

Application of RPMD to gas-phase bimolecular chemical reactions has demonstrated that it provides systematic and consistent performance across a wide range of system dimensionalities. In all systems considered so far, the RPMD rate coefficient captures almost perfectly the ZPE effect, and is usually within a factor of 2-3 of accurate results at very low temperatures in the deep quantum tunnelling regime.
when compared to rigorous quantum mechanical results available for these systems. Most chemical reactions can be studied using RPMD with only about 1-2 order(s) of magnitude higher computational costs than conventional quasi-classical trajectory (QCT) calculations.

The RPMD calculations were carried out using the RPMDrate code developed by Suleimanov and co-workers,\(^{50}\) the working equations of the RPMD rate theory can be found in Refs. 34,50. The input parameters are summarized in Table 1. We also repeated our calculations at 2500 K with one ring polymer bead, which corresponds to purely classical simulation.\(^{34}\) Note that as in the VTST case all rate coefficients are corrected with the electronic partition function given by Eq. (7).

4. Results and discussion

Table 2 lists the theoretical forward rate coefficients in the temperature range 200-2500 K obtained with the PES-2014 surface, together with experimental rate coefficients\(^{18,21}\) for comparison. Figure 2 shows the corresponding Arrhenius plots. Except for the lowest temperature (200 K), the RPMD rate coefficients are larger than the CUS/LAT and the difference increases with temperature. At 2500 K, the RPMD rate coefficient is higher than the CUS/LAT counterpart by a factor of almost 2. Since RPMD rate theory is exact in the classical high-temperature limit,\(^{34}\) these results suggest that the CUS/LAT method underestimates the rate coefficients, and this is possibly related to approximations used in the TST-based methods.\(^{59,60}\) This point is also confirmed by excellent agreement of the RPMD rate coefficients with experimental values at 400-2500 K.

To deeply analyze the reason behind this discrepancy between CUS/LAT and RPMD at high temperatures, we performed additional calculations at 2500 K using conventional TST (i.e., locating the transition state dividing surface at the saddle point), CVT and CUS implemented in POLYRATE-2010. In both cases tunnelling is included by means of the LAT method, although it is expected to be negligible at such a high temperature. TST and CVT differ in the location of the transition state, but CVT considers that only the variational TST bottleneck controls the rate coefficient. CUS, however, takes into account other relative maxima on the free energy curve and the possibility that the system can be thermalized in one of the minima between two maxima. Thus, the CVT recrossing factor, computed as the ratio between CVT and TST rate coefficients, measures the effect of the displacement of the maxima of the free
energy curve from the saddle point, also known as “variational effect”, while the CUS recrossing factor (CUS rate coefficient over TST rate coefficient) additionally includes the effect of the multiple maxima on recrossing.

We compare these rate coefficients with the RPMD, pure classical and classical TST (locating the dividing surface at the saddle point) counterparts obtained using RPMDrate in Table 3. The classical results by RPMDrate were obtained by setting the number of ring polymer beads to one.34 We first note that at the highest temperature considered in this work, 2500 K, the difference between pure classical and RPMD rate coefficient is only 15% which shows that we achieved the classical high-temperature limit where all quantum mechanical effects become practically negligible. The POLYRATE results confirm this point: the difference between TST calculations with classical and quantum partition functions is also within 15 %. The LAT correction for tunnelling is 1.036 and is negligible, as expected at such high temperature. However, a noticeable discrepancy is observed between the classical TST rate coefficients obtained using these two codes. The classical TST rate coefficient obtained by RPMDrate using full-dimensional classical molecular dynamics is 30% higher than the one obtained using partition functions evaluated with POLYRATE. The difference between these two rate coefficients can only be attributed to effects of anharmonicity, mode-mode coupling, or the approximation of considering the reaction coordinate motion as separable.

From Table 3 we can infer that the CVT and CUS rate coefficients are almost the same, and the recrossing factor, CUS/TST = 0.3248, is almost exclusively due to the variational shift of the maximum of the free energy curve from the saddle point (at 2500 K the transition state is located at s=−0.315 a₀ in the entrance channel). In the previous study by one of us61 the importance of the anharmonicity on the final rate constants was analysed in detail for the benchmark Cl+H₂ reaction. It was found that, when the lowest modes are treated as anharmonic oscillators, the motion along the reaction coordinate leads to a larger decrease of the zero-point energy than when the harmonic approach is used. When the lowest vibrational modes are treated using harmonic approximation, this leads to overprediction of their contribution shifting the free energy maximum from the saddle point and thus overestimating the variational effects.

In the case of the title polyatomic reaction, the anharmonic calculation is not so straightforward, because anharmonicity is not included for curvilinear coordinates or large-curvature tunnelling methods in the POLYRATE-2010 code.49 It is only
implemented using the rectilinear coordinates which exhibit unphysical imaginary values of the vibrational frequencies for the present system. However, to put into evidence this effect at least qualitatively, we have calculated anharmonicity of the stretching modes using the Wentzel-Kramers-Brillouin (WKB) approximation, taking into account the following considerations about the calculation in non-stationary points (i.e. points along the reaction path) and about the proper WKB method.

Computing anharmonic vibrations only on the stationary points will not shed any light on the subject, since we are proposing that the main effect of anharmonicity is that it would reduce variational effects, thus increasing the rate constants. Therefore, we need to compute variational TST rate constants using anharmonic vibrations to see if our assumption is correct. Unfortunately, performing an accurate anharmonic vibrational analysis on non-stationary points is far from being straightforward. On non-stationary points we need to project out from the Hessian matrix the motion in the direction of the reaction path, which is obtained from the gradient. One needs an accurate gradient and Hessian calculation to obtain meaningful results since numerical errors can change the direction that is projected-out. Additionally, numerical errors get amplified when computing 3rd or even 4th order derivatives. On top of it all, some vibrations become so floppy along the reaction path that numerical errors can be bigger than the effect of the vibration itself. Numerical errors enter in an exponential way on the partition functions, causing significant noise on the free energy curve, and rate constants are exponential functions of this free energy. Bumpy free energy curves produce unaccurate rate constants. Therefore, one has to be extremely cautious and try to remove all significant numerical errors in the calculation.

But numerical problems are not the only issue. To the best of our knowledge, no general method has been proposed to successfully deal with anharmonicity for polyatomic systems along a reaction path. For example, Isaacson\textsuperscript{63,64} developed a method using perturbation theory that seems to be promising. However, it has only been applied to three- or four-atoms reactions. Moreover, perturbation theory for degenerate vibrations can be problematic due to the presence of Fermi resonances. Thus, when we tried to apply perturbation theory to our calculations using standard codes we did not get physically meaningful results.

Finally, we evaluated the ground-state vibrational energies by using the WKB approximation as implemented in POLYRATE-2010. This method is designed for stretching modes, and we believe that the most anharmonic vibration is the symmetric stretch of the C-H-O system, that we sometimes call the reactive mode because it is the
mode more related to the hydrogen transfer. However, in its present implementation
this approach is used only for the vibrational ground-state while higher energy
vibrational levels (used in the calculation of the partition functions) are obtained by
harmonic approximations. Therefore, our approach to the problem is only qualitative.
Figure 3 shows the vibrationally adiabatic ground state curve ($V_{aG}$) near the top of the
barrier, where motions are less floppy and the anharmonic methods work best. The
blue line is computed using harmonic vibrations, while the red line is computed using
the anharmonic results. The sudden increase in the vibration of the reactive mode
causes the peak to the left of the saddle point. The presence of this peak shifts the
transition state from the saddle point to a point around $s = -0.3$ bohr at 2000K, and is
much softened when vibrational energy is computed with the WKB method. Therefore,
one would expect that, according to our previous discussion, the harmonic
approximations overestimate the importance of variational effects, and under an
anharmonic approximation one would expect higher rate constants at those
temperatures where variational effects are more significant, i.e., at the higher
temperature range.

Reverting back to Table 2, let’s analyse now the low temperatures regime ($T \leq$
300 K). Both CUS and RPMD methods overestimate the experimental rate coefficients,
although this comparison must be taken with caution due to the experimental
difficulties in the deep tunnelling regime. From the previous studies,\textsuperscript{34,50-58} one can
expect that RPMD method will overestimate the exact quantum mechanical (QM) rates
for an asymmetric reaction by about a factor of no more than 2-3 at low temperatures.
At the same time, for the title reaction on the PES-2000, the RPMD demonstrated
perfect agreement with the rigorous quantum mechanical calculations though, as
discussed in Introduction, the previous PES-2000 should exhibit less tunneling.\textsuperscript{51} We
suggest that rigorous quantum mechanical calculations on PES-2014 will be of
invaluable help in solving this puzzle. We also suggest that the LAT method
overestimates tunnelling for the title reaction at the lowest temperatures considered,
e.g., at 200 K where it gives higher values than RPMD. In fact, when the LCT-4 version
is used, a better agreement with experiment at 300 K is obtained, $1.0 \times 10^{-17}$ cm$^3$
molecule$^{-1}$ s$^{-1}$, while the LCT-3 version overestimates this effect, giving a poor
agreement with experiment, $4.4 \times 10^{-17}$ cm$^3$ molecule$^{-1}$ s$^{-1}$. However, it is worth noting
that both RPMD and CUS/LAT results show a similar behaviour of the curvature of
the Arrhenius plot, which is more pronounced than the $k(T)$ recommended in the
literature (Eq.(2)) due to a large tunnelling contribution at low temperatures (Fig. 2).
As it was noted before, the influence of the semiclassical approaches to estimate tunnelling in the VTST method is very important and it deserves further analysis. For the title reaction at the lowest temperature analyzed, 200 K, LCT-3 gives the higher tunnelling coefficient, $\kappa = 7310.0$, while the LCT-4 version gives the lower coefficient, $\kappa = 929.4$, with the LAT approach in an intermediate situation, $\kappa = 2569.0$. The imaginary action integrals are plotted in Figure 4 (upper panel). The SCT approach gives, obviously, the highest values, related to the lowest tunnelling contributions as was expected in this heavy-light-heavy mass combination. The LAT and LCT-4 present similar behaviour in all energy range, which is also expected since LAT is based on the LCT-4 probabilities. The closeness between these two methods clearly indicates that variational optimization of the tunnelling path has a relatively minor effect, and that the reaction is a large-curvature tunnelling reaction. The LCT-3 approach gives the lowest values of the action integral and consequently the largest transmission factors. As noted elsewhere, this is due to the total neglect of the anharmonicity of the motions orthogonal to the reaction path. These effects are also clearly observed in the same Figure 4 (lower panel), where the transmission factor is plotted against temperature. The LCT-3 approach overestimates the tunnelling contribution, while the LCT4 and LAT approaches, that deal in a better way with the anharmonicity of the modes orthogonal to the tunnelling path, give more reasonable values. In sum, we suggest that both CUS/LAT and RPMD overestimate the tunnelling contribution, although the first one in larger extension in the deep-tunnelling regime (200 K). Therefore we note that further accurate quantum mechanical calculations based on the PES-2014 are required to assess rigorously the tunnelling factor for the present system.

In Figure 2 we also include the results obtained with the PES-2000 surface for comparison. At low temperatures ($T < 300$K) it presents less tunnelling than the PES-2014 surface, which is the expected behaviour because the barrier is broader; while at higher temperatures ($T >300$ K) it reproduces the experimental data, which is not surprising because, as was noted in Introduction, the PES-2000 surface is semiempirical, i.e. the experimental rate coefficients were used in the fitting process.

Table 4 lists the phenomenological activation energies computed as local slopes of Arrhenius plots, in order to provide the most appropriate comparison with experiment. Both theoretical results, RPMD and CUS/LAT, increase with temperature, reproducing the experimental evidence. However, the behaviour at low and high temperatures is very different. At low temperatures, the theoretical values underestimate the experiment but at high temperatures the agreement is better. We
stress on the fact that both the intrinsic experimental difficulties and the limitations of semiclassical methods in the deep tunnelling regime can be responsible for the observed discrepancy between theory and experiment.

The activation energies at 298 K (from slopes of Arrhenius plots from 293 to 303 K) are 5.98 and 3.17 kcal mol\(^{-1}\) for the forward and reverse reactions from the CUS/LAT method, respectively. Using these theoretical activation energies, a value of 2.81 kcal mol\(^{-1}\) is obtained for the enthalpy of reaction at 298 K, in agreement with the experimental value from enthalpies of formation, \(\Delta H_r(298 \text{ K}) = 2.85\) kcal mol\(^{-1}\) (ref. 65). This result confirms some overall characteristics of the analytical surface, and the balance of the forward and reverse reactions.

We have also analysed the deuterated analogue, O\(^{(3P)}\) + CD\(_4\) (Figure 5). In contrast with the O\(^{(3P)}\) + CH\(_4\) reaction, note firstly that the free energy curve for the deuterated analogue presents a maximum located very close to the saddle point. Therefore, the “variational” and recrossing effects (important at high temperatures) are practically negligible in this case. Secondly, that due to transfer of the heavier deuterium atom, less tunnelling (important at low temperatures) is present. As it was expected, now the agreement RPMD and CUS/LAT is better, with an average difference smaller than 10\% in the common temperature range (see Arrhenius plot in Figure 5). However, while at high temperatures (2500 K) the agreement is reasonable at low temperatures (200 K) the agreement is very poor, with a RPMD/CUS-LAT factor of 0.31 demonstrating the important discrepancy in treating the tunnelling effect. As indicated above, further rigorous quantum mechanical calculations for the title reaction are necessary for assessing accuracy of these two approaches at low temperatures.

In order to complete the kinetic study we calculated the kinetics isotope effects (KIEs) (defined as the ratio of the rate coefficients of the unsubstituted reaction to the deuterated reaction, CH\(_4\)/CD\(_4\)). As a ratio of rate coefficients, the KIEs are less sensitive to the accuracy of the PES, representing an excellent opportunity to test the dynamics approaches. As far as we know, no experimentally determined isotope data are available for comparison and we hope that the present study will stimulate experiment.

The calculated KIEs from this work and the previous ones\(^{24}\) using the PES-2000 surface are listed in Table 5. In the whole temperature range (200-2500 K) the ratio between RPMD and CUS/LAT KIEs vary by factors between 2.1 and 1.0, decreasing with temperature, i.e., the reactive system approaches to the classical behaviour, where the recrossing and quantum effects are negligible. At the highest temperatures (T ≥
1500 K) both methods agree with each other. Since RPMD is exact in the high temperature regime, these results suggest that the harmonic approximation implemented in the CUS/LAT calculations leads to underestimation of the rate coefficients but cancels out in the KIE. The systematic and consistent performance of RPMD rate theory usually contrasts with the performance of TST-based methods which can both overestimate and underestimate KIEs\(^{34,55}\). At low temperatures, where tunnelling is important, in similar hydrogen abstraction reactions with heavy-light-heavy mass combination, OH + CH\(_4\), Suleimanov and co-workers\(^{55}\) found that the RPMD approach overestimates the KIEs, and concluded that since the deuterated reaction presents less tunnelling, the RPMD rate coefficients will be more accurate than the corresponding non-deuterated reaction. As a result, the overestimation of the KIE at low temperatures is due to the overprediction of the rate coefficients for the non-deuterated asymmetric reaction. At the same time, we also note that RPMD provided quantitative agreement for the title (non-deuterated) reaction even at very low temperatures on PES-2000.\(^{51} \) Therefore, for the present reaction, these results suggest that experimental KIEs should be close or no more than 2-3 times lower that the RPMD counterparts at very low temperatures.

In sum, when one compares theory and experiment many factors must be taken into account. Experimentally, the quality and accuracy of the measures, and theoretically the dynamics methods used and the accuracy of the PES. So, in a comparison theory/experiment, both the dynamics method and the PES are tested, while in a theoretical comparison using the same PES, the approaches and limitations in the dynamics methods are being tested.

5. Conclusions

In this work we have investigated the kinetics of the gas-phase reaction O\(^{(3P)}\) + CH\(_4\) → OH + CH\(_3\) and its deuterated analogue, using two dynamical approaches – ring polymer molecular dynamics (RPMD) and variational transition state theory with canonical unified statistical model and transmission coefficients calculated by the least-action tunnelling approximation (CUS/LAT), based on new analytical full-dimensional potential energy surface (PES), PES-2014.\(^{30} \) We found that, when the oxygen atom approaches to CH\(_4\), the reaction evolves with a high and thin barrier, which favours the tunnelling, with stabilized intermediate complexes in the entrance and exit channels.
In the low temperature regime, both approaches present noticeable curvature in the Arrhenius plots, indicating the important role played by tunnelling. This curvature is more pronounced than experimentally reported. Since the experimental measurements in this regime are problematic, we believe that these results can encourage to experimentalists to study more deeply the low-temperature kinetics.

In the classical high-temperature limit regime, where the RPMD rate theory is exact, this approach agrees with experiment perfectly, indicating the VTST/MT underestimates the rate coefficients by almost a factor of 2. Although further work would be required to explain this behaviour, we believe that the reason is related to the harmonic separable-modes approximation used for computing the partition functions in the CUS/LAT method. One would expect that anharmonicity plays a more significant role at high temperatures, where higher vibrational levels are more populated. On the other hand, very good agreement RPMD with experiment at high temperatures, indirectly lends confidence in the accuracy of the PES-2014.

The kinetic isotope effect (KIE) is another very sensitive test of dynamical approaches. The CUS/LAT and RPMD exhibit reasonable agreement in the high-temperature range but at low temperatures the RPMD KIEs are larger. Since no experimental results are available to shed any light in this comparison, we think that experimentalists may be encouraged to study more deeply the kinetics of this reaction, although we are aware of the very great experimental difficulty involved, especially at low temperatures.

Acknowledgments
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References
17. Wagner, A.F.; Dunning, T.H.; Walch, S.P.; Schatz, G.S. reported in ref. 21.


58. Li, Y.; Suleimanov, Y. V.; Green, W. H.; Guo, H. Quantum Rate Coefficients and Kinetic Isotope Effect for the Reaction Cl + CH₄ → HCl + CH₃ from Ring Polymer Molecular Dynamics, *J. Phys. Chem. A*, DOI: 10.1021/jp501043z.


63. Isaacson, A.D. Including anharmonicity in the calculation of rate constants. 1. the


**Table 1.** Input parameters for the RPMDrate calculations on the O + CH$_4$ reaction. The format of the input file is explained in detail in the RPMDrate code manual (see Ref. 50 and http://www.mit.edu/~ysuleyma/rpmdrate).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reaction</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>O + CH$_4$</td>
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<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Temp</td>
<td>200-2500</td>
<td>Temperature (K)</td>
</tr>
<tr>
<td>Nbeads</td>
<td>128-4</td>
<td>Number of beads (RPMD calculations)</td>
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<tr>
<td></td>
<td>1</td>
<td>Number of beads (classical calculations at 2500 K)</td>
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<table>
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<td>Dividing surface s$_1$ parameter (a$_0$)</td>
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<td>Number of forming and breaking bonds</td>
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<th>‘Andersen’</th>
<th>Thermostat option</th>
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| Biased sampling parameters |          |                                                |

<table>
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<tr>
<th>Parameter</th>
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<th>Explanation</th>
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<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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</thead>
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<tr>
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<td>( \delta )</td>
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</tr>
<tr>
<td>( d )</td>
<td>0.01</td>
<td>Window spacing step</td>
</tr>
<tr>
<td>( N )</td>
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<td>Center of the last window</td>
</tr>
<tr>
<td>( dt )</td>
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<td>Time step (ps)</td>
</tr>
<tr>
<td>( k_i )</td>
<td>2.72</td>
<td>Umbrella force constant ((T/\text{K}) \text{ eV})</td>
</tr>
<tr>
<td>( N_{\text{trajectory}} )</td>
<td>200</td>
<td>Number of trajectories</td>
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<tr>
<td>( t_{\text{equilibration}} )</td>
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<td>Equilibration period (ps)</td>
</tr>
<tr>
<td>( t_{\text{sampling}} )</td>
<td>100</td>
<td>Sampling period in each trajectory (ps)</td>
</tr>
<tr>
<td>( N_i )</td>
<td>( 2 \times 10^8 )</td>
<td>Total number of sampling points</td>
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**Potential of mean force calculation**

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<tr>
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<th>Value</th>
<th>Description</th>
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<td>( \theta )</td>
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<td>Start of umbrella integration</td>
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<tr>
<td>#</td>
<td>1.05</td>
<td>End of umbrella integration*</td>
</tr>
<tr>
<td>( N_{\text{bins}} )</td>
<td>5000</td>
<td>Number of bins</td>
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</table>

**Recrossing factor calculation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dt )</td>
<td>0.0001</td>
<td>Time step (ps)</td>
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<tr>
<td>( t_{\text{equilibration}} )</td>
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<td>Equilibration period (ps) in the constrained (parent) trajectory</td>
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<tr>
<td>( N_{\text{totalchild}} )</td>
<td>100000</td>
<td>Total number of unconstrained (child) trajectories</td>
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<tr>
<td>( t_{\text{childsampling}} )</td>
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<td>Sampling increment along the parent trajectory (ps)</td>
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<td>( N_{\text{child}} )</td>
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<td>Number of child trajectories per one initially constrained configuration</td>
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<tr>
<td>( t_{\text{child}} )</td>
<td>0.1</td>
<td>Length of child trajectories (ps)</td>
</tr>
</tbody>
</table>

* Maximum value of the free energy is detected automatically by RPMDrate.
Table 2. Thermal rate coefficients for the O(3P) + CH₄ reaction.⁴

<table>
<thead>
<tr>
<th>T(K)</th>
<th>CUS/LAT</th>
<th>RPMD</th>
<th>Ref. 18</th>
<th>Ref. 21</th>
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<tbody>
<tr>
<td>200</td>
<td>1.94E-19</td>
<td>1.29E-19</td>
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<td></td>
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<td>300</td>
<td>1.32E-17</td>
<td>1.50E-17</td>
<td>9.00E-18</td>
<td>5.50E-18</td>
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<td>400</td>
<td>2.28E-16</td>
<td>3.17E-16</td>
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<td>3.00E-16</td>
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<td>500</td>
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<tr>
<td>1000</td>
<td>3.60E-13</td>
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<td>6.00E-13</td>
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<td>1500</td>
<td>2.95E-12</td>
<td>4.58E-12</td>
<td>5.00E-12</td>
<td>6.00E-12</td>
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<tr>
<td>2000</td>
<td>1.03E-11</td>
<td>1.90E-11</td>
<td>1.80E-11</td>
<td>1.90E-11</td>
</tr>
<tr>
<td>2500</td>
<td>2.37E-11</td>
<td>4.05E-11</td>
<td>4.20E-11</td>
<td>4.20E-11</td>
</tr>
</tbody>
</table>

a) In cm³ molecule⁻¹ s⁻¹.
Table 3. Thermal rate coefficients for the $\text{O}(^3\text{P}) + \text{CH}_4$ reaction at 2500 K using different dynamical methods (see text for description).

<table>
<thead>
<tr>
<th>Method</th>
<th>$k(T = 2500 \text{ K})$, in cm$^3$ molecule$^{-1}$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>POLYRATE</td>
<td></td>
</tr>
<tr>
<td>CUS/LAT</td>
<td>2.37E-11</td>
</tr>
<tr>
<td>CUS</td>
<td>2.29E-11</td>
</tr>
<tr>
<td>CVT/LAT</td>
<td>2.55E-11</td>
</tr>
<tr>
<td>CVT</td>
<td>2.46E-11</td>
</tr>
<tr>
<td>Classical TST with quantum vibrational</td>
<td></td>
</tr>
<tr>
<td>partition functions (harmonic approximation)</td>
<td>7.05E-11</td>
</tr>
<tr>
<td>Classical TST with classical vibrational</td>
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</tr>
<tr>
<td>partition functions (harmonic approximation)</td>
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<tr>
<td>RPMDrate</td>
<td></td>
</tr>
<tr>
<td>RPMD</td>
<td>4.05E-11</td>
</tr>
<tr>
<td>Classical</td>
<td>3.54E-11</td>
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<tr>
<td>Classical TST$^a$</td>
<td>7.94E-11</td>
</tr>
</tbody>
</table>

$^a$ Obtained by placing the dividing surface at the saddle point
Table 4. Theoretical and experimental activation energies (kcal mol⁻¹)

<table>
<thead>
<tr>
<th>Method</th>
<th>300-400 K</th>
<th>400-500 K</th>
<th>500-600 K</th>
<th>900-1500 K</th>
<th>1500-2500 K</th>
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</thead>
<tbody>
<tr>
<td>CUS/LAT</td>
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<td>8.3</td>
<td>9.4</td>
<td>12.5</td>
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<tr>
<td>RPMD</td>
<td>7.3</td>
<td>8.0</td>
<td>10.2</td>
<td>11.4</td>
<td>16.2</td>
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<tr>
<td>Exp.ᵃ</td>
<td>8.7</td>
<td>9.1</td>
<td>10.1</td>
<td>12.4</td>
<td>15.8</td>
</tr>
<tr>
<td>Exp.ᵇ</td>
<td>9.6</td>
<td>9.9</td>
<td>10.5</td>
<td>12.0</td>
<td>14.4</td>
</tr>
</tbody>
</table>

a) Ref. 18  
b) Ref. 13 and 21
Table 5. H/D kinetic isotope effects for the O(\(^{18}\)P) + CH\(_4\) / CD\(_4\) reactions.

<table>
<thead>
<tr>
<th>T(K)</th>
<th>PES-2014</th>
<th>PES-2000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RPMD</td>
<td>VTST(^a)</td>
</tr>
<tr>
<td>200</td>
<td>290.4</td>
<td>136.6</td>
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<td>30.8</td>
<td>18.6</td>
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<tr>
<td>400</td>
<td>10.7</td>
<td>6.2</td>
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<tr>
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<td>5.1</td>
<td>3.4</td>
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<tr>
<td>600</td>
<td>3.8</td>
<td>2.4</td>
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<td>2.0</td>
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<tr>
<td>2500</td>
<td>1.1</td>
<td>1.10</td>
</tr>
</tbody>
</table>

\(^{a}\) Calculated using the CUS/LAT level. This work.

\(^{b}\) Calculated using the CUS/LCT-3 level, from Ref. 24.
FIGURE CAPTIONS

**Figure 1**: Schematic profile of the potential energy surface along the reaction path. CCSD(T): CCSD(T)=FULL/aug-cc-pVQZ//CCSD(T)=FC/cc-pVTZ single point level from Ref. 30; PES-2014: using the analytical surface developed Ref. 30; Benchmark: Accurate relative energies obtained at the all-electron CCSDT(Q)/complete-basis-set quality from Ref. 33.

**Figure 2.** Arrhenius plots of the O(3P) + CH₄ rate coefficients computed using the PES-2014 surface. CUS/LAT : solid blue line; RPMD: solid red line; Exp. From Ref. 18: solid squares; Exp. From Ref. 21: Solid triangles. CUS/LCT-3 using PES-2000 from Ref. 24: dashed blue line.

**Figure 3.** Vibrationally adiabatic ground-state potential energy, VₐG(s). The blue line is based on harmonic zero-point energies, and the red line is based on anharmonic zero-point energies.

**Figure 4.** Upper panel: Imaginary action integral computed using SCT method (green line), LCT version 4 (red line), LAT method (blue line), and LCT version 3 (dashed line) methods as a function of the energy. The zero of energy is set to reactants’ potential energy. Lower panel: Transmission coefficients computed using LCT version 3 (dashed line), LCT version 4 (red line) and LAT (blue line).

**Figure 5.** Arrhenius plots of the O(3P) + CD₄ rate coefficients computed using the PES-2014 surface. CUS/LAT : solid blue line; RPMD: solid red line.
Figure 2
Figure 3
Figure 4
Figure 5

![Graph showing the relationship between 1000/T (K) and k(T) (cm$^3$ molecule$^{-1}$ s$^{-1}$). The graph compares two curves labeled CUS/LAT and RPMD.](image-url)