A Methodology for Evaluating Liquefaction Susceptibility in Shallow Sandy Slopes

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ABSTRACT: This paper illustrates a modeling approach for evaluating the liquefaction susceptibility of shallow sandy slopes. The methodology is based on a theoretical framework for capturing undrained bifurcation in saturated granular media. In order to provide predictive capabilities, the theory is combined with the MIT-S1 constitutive model. The role of a non-homogeneous density profile is investigated, distinguishing among the different forms of undrained response that can be induced by rapid shearing. The first part of the paper describes the general methodology and illustrates the use of a stability index for static liquefaction. In the second part, the practical significance of the approach is discussed by back-analyzing the well-known series of flow failures in an underwater berm at the Nerlerk site. The analyses predict that prior to failure the Nerlerk slopes were not yet beyond the limits of stability for incipient liquefaction. Model simulations, however, also indicate that very small shear perturbations could have activated an undrained instability. These results suggest that static liquefaction was a mechanically plausible failure mechanism and provide the first interpretation of the classical Nerlerk collapse based on the combined use of the theory of material stability and an advanced constitutive model for sands.

INTRODUCTION

Landslides and slope failures are widely recognized as one of the major natural hazards affecting both the development of densely populated areas in rugged terrain and the design of artificial earthworks. The wide distribution of these phenomena has always attracted the attention of the entire geotechnical research community. Within the general class of slope failures, runaway instabilities or flow slides represent impressive phenomena that still raise several open questions. The massive features of the collapse of natural soil masses of liquefiable cohesionless materials is well known in the literature, and is confirmed by several case histories of subaqueous collapses (Terzaghi 1957). Flow slides of submerged artificial sandy slopes can also represent a major threat during construction in harbor and offshore environments, where deposition, dredging and excavation can produce rapid shear perturbations in the soil mass (Sladen et al 1985-b; Hight et al., 1999).
One of the first interpretations of these slope failures was due to Terzaghi (1957). In order to justify the observed phenomenon, Terzaghi used for the first time the term *spontaneous liquefaction*, relating these slope instabilities to the metastable state of the involved sand deposit. Even though several studies have been carried out on the subject, there is a need for advanced tools of analysis that can explain catastrophic failures, evaluate hazard levels in landslide prone areas and define geotechnical design criteria. The purpose of this work is to suggest an advanced modeling methodology to study flow slide phenomena. The proposed approach is aimed at evaluating the shear perturbations that can trigger a flow slide, the spatial distribution of soil masses prone to a liquefaction and the post-failure response of the slope.

**MATHEMATICAL CAPTURE OF STATIC LIQUEFACTION**

It is well known that the undrained behavior of sands is affected by stress and density conditions (Been et al., 1991), as well as by other state variables such as the evolution of anisotropic properties during consolidation and loading (Kramer and Seed, 1988). Even minor changes in the initial state (e.g., due to deposition processes or changes in drainage conditions) can alter the expected undrained response (Fig. 1) and are crucial in developing predictive frameworks (Andrade, 2009).

![Image](image_url)

**FIG. 1.** Effect of changes of the initial state: experimental evidence against model predictions (data after Verdugo, 1992; simulations after Pestana et al., 2002).

Although modern elastoplastic constitutive models have the capabilities of capturing such dependencies, there are some key theoretical questions that are still open, such as: (i) How is it possible to identify instabilities for general in situ conditions? And (ii) what are the analytical criteria that should be combined with elastoplastic soil models in determining the inception of liquefaction? Within an elastoplastic framework, a convenient mathematical strategy for expressing the onset of an instability is based on the use of the concept of critical hardening modulus (Maier and Hueckel, 1979). An intuitive example is given by the particular case of stress-controlled loading, for which the usual notion of failure is expressed through the achievement of a vanishing value for the plastic hardening modulus, \( H \) and the consequent violation of the plastic admissibility requirements. The concept of a critical hardening modulus generalizes this idea to very general static-kinematic conditions, that for geomechanical problems
are usually characterized by mixed stress-strain loading paths. In fact, it can be proved that, once control conditions are given, the critical hardening modulus is unequivocally defined and allows evaluating a specific bifurcation index for the deformation mechanism at stake (Buscarnera et al., 2011). According to this formalism, undrained loading is a particular case of mixed stress-strain control, in which volumetric strains are held constant while independently imposing the deviatoric stresses. In the latter case it can be demonstrated that the critical hardening modulus is:

\[ H_{LIQ} = -\frac{\partial f}{\partial p'} K \frac{\partial g}{\partial p'} \]  

(1)

where \( f \) and \( g \) are the yield surface and plastic potential, respectively, \( K \) the elastic bulk modulus and \( p' \) the mean effective stress. This allows defining a particular stability index for liquefaction, here given by:

\[ \Lambda_{LIQ} = H - H_{LIQ} \]  

(2)

in which \( H \) encapsulates the shear strength properties of the sand, while \( H_{LIQ} \) represents the effect of control conditions (Buscarnera and Whittle, 2012-a).

The sign of the stability index \( \Lambda_{LIQ} \) is related with the undrained incremental response predicted by an elastoplastic model [Buscarnera and Whittle, 2012-a]. In particular, positive values of \( \Lambda_{LIQ} \) reflect the capability of the material of sustaining further incremental undrained loading, while vanishing or negative values reflect the lack of undrained resistance capabilities. Figure 2 illustrates the mechanical significance of this stability criterion, showing that vanishing values of \( \Lambda_{LIQ} \) reflect the occurrence of a peak in the stress deviator upon undrained shearing. In loose sands this condition marks the entrance within an unstable domain, in which negative values for \( \Lambda_{LIQ} \) corresponds to a pseudo-softening response upon undrained shearing [Buscarnera and Whittle, 2012-a].

**MODELING APPROACH FOR FLOW SLIDE TRIGGERING**

Application of stability criteria to field conditions requires accurate consideration of the realistic static-kinematic characteristics of the problem at stake. An important example is represented by shallow slopes, in which initial stress conditions and kinematics are highly anisotropic and cannot be appropriately represented through classical triaxial testing. One of the first approaches to consider the role of soil behavior through a comprehensive constitutive model was suggested by di Prisco, Matiotti and Nova (1995). In order to study the onset of a flow slide, these authors considered the geometry of an infinite slope and modeled sand behavior through simple shear simulations (Figure 3). This idea allowed the response of the slope to be described simply by performing material point analyses.

This paper discusses a methodology which is inspired by the original idea suggested by di Prisco et al. (1995), but tries to link it directly to the framework of critical state for sands and in situ observations. The key contribution is the incorporation of predictive capabilities for describing transitions from contractive to dilative volumetric behavior upon shearing. As a result, the approach is able to distinguish among
different types of sand response induced by an undrained perturbation (complete liquefaction, partial liquefaction, etc.), which is an essential aspect to define the expected post-failure behavior of a sliding mass.

FIG. 2. MIT-S1 simulations for very loose Toyoura sand: stress-strain response and evolution of the stability index $\Lambda_{LIQ}$ as a function of the axial strain.

FIG. 3. Reference system for a submerged infinite slope and initial stress conditions.

In order to maintain the same conceptual link to the mathematical theory of bifurcation, it is possible to adapt the stability index (2) to the case of undrained simple shear kinematics. This is possible by redefining the critical hardening modulus for undrained simple shear conditions (Buscarrera and Whittle, 2012-b), as follows:

$$H_{LSS} = H_c + \frac{\partial f}{\partial \tau_{\xi\eta}} G \frac{\partial g}{\partial \tau_{\xi\eta}}$$  

The stability index that governs undrained failure in a shallow infinite slope is therefore given by:
The simple shear response predicted by a constitutive model can be then interpreted by means of (4), identifying the stress states at the initiation of a flow failure and the residual margin of safety. For example, Figure 4 illustrates two MIT-S1 simulations of undrained simple shear response at the same level of initial vertical effective stress but with different magnitudes of initial shear stresses (representing different slope angles).

\[ \Lambda_{LSS} = H - H_{LSS} \]  

(4)

FIG. 4. Example of simple shear simulations (loose Toyoura Sand simulated with the MIT-S1 model): a) stress path in the \( \sigma' \)-\( \tau \) plane; b) stress strain behavior.

The simulations have been performed using model parameters calibrated for Toyoura sand (Pestana et al., 2002), with vertical stress \( \sigma'_v = 150 \text{ kPa} \) and \( e = 0.93 \). The results illustrate that the initial state of stress affects significantly the magnitude of the shear perturbation required to induce instability (\( \Delta \tau_1 \) vs \( \Delta \tau_2 \)). The onset of a mechanical instability coincides with the peak in the shear stress, and can be readily interpreted through the stability index (4) (Buscarrnera and Whittle, 2012-b).

As is well known, the undrained behavior of sands is also influenced by changes in the effective stress and density. For example, even very loose sands can exhibit a tendency to dilate at low effective stress levels, but will collapse for undrained shearing at high levels of effective stress. Hence, the prediction of liquefaction potential requires a constitutive framework that can simulate realistically the stress-strain properties as functions of stress level and density. To illustrate this aspect, Figure 5 shows MIT-S1 simulations for a pre-shear void ratio ranging from 0.87 to 0.94, with the model predicting a sharp transition from a stable behavior (\( e_0 = 0.87 \)) to complete collapse (\( e_0 = 0.94 \)).

FIG. 5. MIT-S1 predictions: effect of void ratio on undrained simple shear response of Toyoura Sand. a) stress path; b) stress-strain behavior.
The effect of confining pressure and density on the undrained response of sands is that the perturbation shear stress ratio, \( \Delta \tau(\alpha, z) / \tau'_{Vc} \), associated with the initiation of liquefaction is not only a function of the slope angle, but must be evaluated at the depth of interest. Figure 6 gives an example of this notion by using model simulations for Toyoura sand. In the first series of simulations (Fig. 6-a) the effect of initial density has been considered, keeping constant the initial mean effective stress. By contrast, the second series (Fig. 6-b) considers the effect of a different stress level for a constant value of void ratio.

**FIG. 6.** Effect of void ratio (a) and effective stress (b) on the stability charts.

Once the stability charts expressing the shear resistance potential \( \Delta \tau(\alpha, z) / \tau'_{Vc} \) have been obtained, it is possible to define the variation of the triggering perturbation at any depth. These capabilities are illustrated in the next section by applying the theory to a case study of flow failures in a sandy deposit.

**EXAMPLE OF APPLICATION: THE NERLERK CASE HISTORY**

The Nerlerk berm case history refers to an impressive series of slope failures that took place in 1983 during construction of an artificial island in the Canadian Beaufort Sea (Sladen et al., 1985-b; Hicks and Boughrarou, 1998). Based on detailed studies of the slide morphology, Sladen et al. (1985-b) classified these slope instabilities as flow slides. The present paper uses the MIT-S1 model to investigate potential static liquefaction mechanisms in the Nerlerk berm. In order to use the theory to the Nerlerk case, it is assumed that the local behavior of the sides of the berm can be studied through the scheme of infinite slope. Although this choice represents an important simplification of the real geometry, this assumption allows an immediate mechanical evaluation of possible incipient instability within the fill and provides an insight on the type of expected undrained phenomena.

The application of the methodology is based on the calibration of the MIT-S1 model parameters for the site-specific properties of the Nerlerk sands. Given the lack of data (especially on their compression response), the calibration procedure required a
number of approximations. Hereafter only some key aspects of the calibration process are described, while more details are available in Buscarnera and Whittle (2012-b). First, the parameters governing the critical state of Nerlerk sands have been evaluated on the basis of the available literature data (Sladen et al., 1985-a). The critical state properties of the Nerlerk sands have been compared with those of similar Arctic sands (i.e., Erksak sand, Fig. 7; Been et al., 1991), for which one-dimensional compression data were available (Jefferyes and Been, 1991).

![Critical State Lines and Limit Compression Curves](image)

FIG. 7. Comparison of Critical State Lines (CSL) and Limit Compression Curves (LCC; dotted lines) for Erksak and Nerlerk sands (while fines content affects the CSL of Nerlerk sand, no influence on the LCC is assumed given the lack of data).

Such comparisons, together with empirical considerations compiled for a broad set of sands (Pestana and Whittle, 1995), allowed the definition of a set of parameters for the compression response of Nerlerk sands. The remaining model constants were calibrated using data on drained and undrained response. Figure 8 shows a comparison between data and model predictions for the undrained behavior of Nerlerk sand with 12% of fines. The model captures first order features of the behavior but underestimates the peak shear strength mobilized in the tests. The accuracy of model predictions at low confinements, however, is considered to be satisfactory for the range of effective stresses relevant for this application.

In order to use the calibrated MIT-S1 model for the Nerlerk berms it is finally necessary to define the in situ initial void ratios along the slope profile and evaluate the stability charts of the Nerlerk berm for several depths within the slope. The first step is largely dependent on a reliable interpretation of the available in situ tests. Several CPT tests were performed on the hydraulic fills at Nerlerk, with the aim of estimating the in situ density. For consistency with prior studies (Sladen et al., 1985-b; Lade, 1993), the current analyses assume that relative density (D_r) can be estimated using the CPT correlation proposed by Baldi et al. (1982). It is clear that the choice of a specific interpretation method for CPT test results will affect the estimation of relative density (and, in turn, the model predictions). This uncertainty, however, is probably unavoidable in any method of interpretation.
FIG. 8. Comparison of computed and measured undrained shear behavior for Nerlerk Sand 12% fines.

Figure 9-a shows that the estimated values for $D_r$ range from 30 to 55 %, while Figure 9-b illustrates the distribution of these initial states relative to the CSL of Nerlerk sands with 12% fines content.

FIG. 9. In situ relative density from CPT tests (Baldi et al. 1982); b) Location of in situ state on the CSL plane.

Figure 10 shows the computed stability charts, $\Delta \tau(\alpha, z)/\gamma'z$, at selected depths for infinite slopes in Nerlerk sand with 12% fines content. The results show that the magnitude of the shear perturbation needed to cause instability can be significantly affected by the specific depth within the slope profile. In addition, the analyses define the initial state of stability within the Nerlerk berm slopes in a proper mechanical sense, allowing a prediction of the critical inclination for incipient instability. Since the Nerlerk berm was constructed at slope angles in the range $\alpha = 10^\circ-13^\circ$, these results suggest that Nerlerk slopes were likely not in an incipient state of instability and additional shear stress were required to trigger flow failures. In other locations where steeper slopes were recorded, however, only very small perturbations in shear stress could have triggered failure, i.e., an undrained collapse triggered by rapid deposition can be considered as a mechanically feasible failure mechanism for the berm.
CONCLUSIONS

This paper has presented a framework for evaluating the triggering of flow slides in shallow sandy slopes. The undrained shear behavior has been modeled by using the anisotropic MIT-S1 model and stability charts have been derived from simulations of undrained simple shear behavior. The current approach follows the same kinematic assumptions previously used by di Prisco et al. (1995), but introduces predictive capabilities for simulating instability as a function of the in situ stress and density within the slope. The selected soil model, in fact, is able to simulate realistic transitions in the contractive/dilative response of sands. Such an enhancement enables predictions of the critical shear perturbations at different depths and the location of potentially unstable zones within the soil mass. In practice the model needs to be calibrated for the site specific properties of the soil, and requires reliable data on in situ density.

To illustrate its capabilities, the proposed methodology has been applied to the well-known case of slope failures in the Nerlerk berm. A general picture of the distribution of liquefaction susceptibility on the Nerlerk slope profile has been obtained. The analysis are based on the calibration of model input parameters based on published laboratory tests results and empirical correlations for $D_r$ based on CPT data. The results show that there were two zones within the slope that were vulnerable to flow failure (complete liquefaction). Although some sections of the berm slope were oversteepened, most were deposited with a slope angle $\alpha=10^\circ$-$13^\circ$. For these slope angles, the current analyses show that instability could have been triggered by the undrained perturbations possibly induced by the rapid deposition of hydraulic fill. Thus, static liquefaction is likely to have contributed to the observed failures, confirming earlier hypothesis by Sladen et al. (1985-b). The current analysis offers a simple, consistent and complete mechanical framework for interpreting and predicting the triggering of flow slides in sands that can be easily applied to other similar engineering cases.
REFERENCES


