SLOAN LOW-MASS PAIRS OF KINEMATICALLY EQUIVALENT STARS (SLoWPoKES): A CATALOG OF VERY WIDE, LOW-MASS PAIRS

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ABSTRACT

We present the Sloan Low-mass Wide Pairs of Kinematically Equivalent Stars (SLoWPoKES), a catalog of 1342 very-wide (projected separation $\gtrsim$500 AU), low-mass (at least one mid-K to mid-M dwarf component) common proper motion pairs identified from astrometry, photometry, and proper motions in the Sloan Digital Sky Survey. A Monte Carlo-based Galactic model is constructed to assess the probability of chance alignment for each pair; only pairs with a probability of chance alignment $\leq 0.05$ are included in the catalog. The overall fidelity of the catalog is expected to be 98.35%. The selection algorithm is purposely exclusive to ensure that the resulting catalog is efficient for follow-up studies of low-mass pairs. The SLoWPoKES catalog is the largest sample of wide, low-mass pairs to date and is intended as an ongoing community resource for detailed study of bona fide systems. Here, we summarize the general characteristics of the SLoWPoKES sample and present preliminary results describing the properties of wide, low-mass pairs. While the majority of the identified pairs are disk dwarfs, there are 70 halo subdwarf (SD) pairs and 21 white dwarf–disk dwarf pairs, as well as four triples. Most SLoWPoKES pairs violate the previously defined empirical limits for maximum angular separation or binding energies. However, they are well within the theoretical limits and should prove very useful in putting firm constraints on the maximum size of binary systems and on different formation scenarios. We find a lower limit to the wide binary frequency (WBF) for the mid-K to mid-M spectral types that constitute our sample to be 1.1%. This frequency decreases as a function of Galactic height, indicating a time evolution of the WBF. In addition, the semi-major axes of the SLoWPoKES systems exhibit a distinctly bimodal distribution, with a break at separations around 0.1 pc that is also manifested in the system binding energy. Compared with theoretical predictions for the disruption of binary systems with time, we conclude that the SLoWPoKES sample comprises two populations of wide binaries: an “old” population of tightly bound systems, and a “young” population of weakly bound systems that will not survive more than a few Gyr. The SLoWPoKES catalog and future ancillary data are publicly available on the Internet for utilization by the astronomy community.

Key words: binaries: general – binaries: visual – brown dwarfs – stars: late-type – stars: low-mass – stars: statistics – subdwarfs – white dwarfs

Online-only material: color figures, machine-readable and VO tables

1. INTRODUCTION

The formation and evolution of binary stars remains one of the key unanswered questions in stellar astronomy. As most stars are thought to form in multiple systems, and with the possibility that binaries may host exoplanet systems, these questions are of even more importance. While accurate measurements of the fundamental properties of binary systems provide constraints on evolutionary models (e.g., Stassun et al. 2007), knowing the binary frequency, as well as the distribution of the periods, separations, mass ratios, and eccentricities of a large ensemble of binary systems is critical to understanding binary formation (Goodwin et al. 2007, and references therein). To date, multiplicity has been most extensively studied for the relatively bright high- and solar-mass local field populations (e.g., Duquennoy & Mayor 1991, hereafter DM91). Similar studies of low-mass M and L dwarfs have been limited by the lack of statistically significant samples due to their intrinsic faintness. However, M dwarfs constitute $\sim 70\%$ of the Milky Way’s stellar population (Miller & Scalo 1979; Henry et al. 1999; Reid et al. 2002; Bochanski et al. 2010, hereafter B10) and significantly influence its properties.

Since the pioneering study of Heintz (1969), binarity has been observed to decrease as a function of mass: the fraction of primaries with companions drops from 75% for OB stars in clusters (Gies 1987; Mason et al. 1998, 2000) to $\sim 60\%$ for solar-mass stars (Abt & Levy 1976, DM91, Halbwachs et al. 2003) to $\sim 30\%$–$40\%$ for M dwarfs (Fischer & Marcy 1992, hereafter FM92; Henry & McCarthy 1993; Reid & Gizis 1997; Delos et al. 2004) to $\sim 15\%$ for brown dwarfs (BDs; Bouy et al. 2003; Close et al. 2003; Gizis et al. 2003; Martín et al. 2003). This decrease in binarity with mass is probably a result of preferential destruction of lower binding energy (BE) systems over time by dynamical interactions with other stars and molecular clouds, rather than a true representation of the multiplicity at birth (Goodwin & Kroupa 2005). In addition to having a smaller total mass, lower-mass stars have longer main-sequence (MS) lifetimes (Laughlin et al. 1997) and, as an ensemble, have lived longer and been more affected by dynamical interactions. Hence, they are more susceptible to disruption over their lifetime. Studies of young stellar populations (e.g., in Taurus, Ophiucus, Chameleon) appear to support this argument, as their multiplicity is twice as high as that in the field (Leinert et al. 1993; Ghez et al. 1997; Kohler & Leinert 1998). However, in denser star-forming regions in the Orion Nebula Cluster and IC 348, where more dynamical interactions are expected, the multiplicity is comparable to the field (Simon et al. 1999; Petr et al. 1998; Duchêne et al. 1999). Hence, preferential destruction
is likely to play an important role in the evolution of binary systems. DM91 found that the physical separation of the binaries could be described by a log-normal distribution, with the peak at $a \sim 30$ AU and $\sigma_{\log a} \sim 1.5$ for F and G dwarfs in the local neighborhood. The M dwarfs in the local 20 pc sample of FM92 seem to follow a similar distribution with a peak at $a \sim 3-30$ AU, a result severely limited by the small number of binaries in the sample.

Importantly, both of these results suggest the existence of very wide systems, separated in some cases by more than a parsec. Among the nearby ($d < 100$ pc) solar-type stars in the Hipparcos catalog, Lépine & Bongiorno (2007) found that 9.5% have companions with projected orbital separations $s > 1000$ AU. However, we do not have a firm handle on the widest binary that can be formed or on how they are affected by localized Galactic potentials as they traverse the Galaxy. Hence, a sample of wide binaries, especially one that spans a large range of heliocentric distances, would help in (1) putting empirical constraints on the widest binary systems in the field (e.g., Reid et al. 2001a; Burgasser et al. 2003, 2007; Close et al. 2003, 2007), (2) understanding the evolution of wide binaries over time (e.g., Weinberg et al. 1987; Jiang & Tremaine 2009), and (3) tracing the inhomogeneities in the Galactic potential (e.g., Bahcall et al. 1985 Weinberg et al. 1987; Yoo et al. 2004; Quinn et al. 2009).

Recent large-scale surveys, such as the Sloan Digital Sky Survey (SDSS; York et al. 2000), the Two Micron All Sky Survey (2MASS; Cutri et al. 2003), and the UKIRT Infrared Deep Sky Survey (UKIDSS; Lawrence et al. 2007), have yielded samples of unprecedented numbers of low-mass stars. SDSS alone has a photometric catalog of more than 30 million low-mass dwarfs (B10), defined as mid-K to late-M dwarfs for the rest of the paper, and a spectroscopic catalog of more than 44,000 M dwarfs (West et al. 2008). The large astrometric and photometric catalogs of low-mass stars afford us the opportunity to explore anew the binary properties of the most numerous constituents of the Milky Way, particularly at the very widest binary separations.

The orbital periods of very wide binaries (orbital separation $a > 100$ AU) are much longer than the human timescale ($P = 1000$ yr for $M_\star = 1 M_\odot$ and $a = 100$ AU). Thus, these systems can only be identified astrometrically, accompanied by proper motion or radial velocity matching. These also remain some of the most underexplored low-mass systems. Without the benefit of retracing the binary orbit, two methods have been historically used to identify very wide pairs:

1. Bahcall & Soneira (1981) used the two-point correlation method to argue that the excess of pairs found at small separations is a signature of physically associated pairs; binarity of some of these systems was later confirmed by radial velocity observations (Latham et al. 1984). See Garnavich (1988), Wasserman & Weinberg (1991), and Longhitano & Binggeli (2010) for other studies that use this method.

2. To reduce the number of false positives inherent in the above, one can use additional information such as proper motions. Orbital motions for wide systems are small; hence, the space velocities of a gravitationally bound pair should be the same, within some uncertainty. In the absence of radial velocities, which are very hard to obtain for a very large number of field stars, proper motion alone can be used to identify binary systems; the resulting pairs are known as common proper-motion (CPM) doubles. Luyten (1979, 1988) pioneered this technique in his surveys of Schmidt telescope plates using a blink microscope and detected more than 6000 wide CPM doubles with $\mu > 100$ mas yr$^{-1}$ over almost 50 yr. This method has since been used to find CPM doubles in the AGK 3 stars by Halbwachs (1986), in the revised New Luyten Two-tenths (rNLTT; Salim & Gould 2003) catalog by Chanamé & Gould (2004), and among the Hipparcos stars in the Lepine-Shara Proper Motion-North (LSPM-N; Lépine & Shara 2005) catalog by Lépine & Bongiorno (2007). All of these studies use magnitude-limited high proper-motion catalogs and, thus, select mostly nearby stars.

More recently, Sesar et al. (2008, hereafter SIJ08) searched the SDSS Data Release Six (DR6; Adelman-McCarthy et al. 2008) for CPM binaries with angular separations up to 30$''$ using a novel statistical technique that minimizes the difference between the distance moduli obtained from photometric parallax relations for candidate pairs. They matched proper motion components to within 5 mas yr$^{-1}$ and identified ~22,000 total candidates with excellent completeness, but with one-third of them expected to be false positives. They searched the SDSS DR6 catalog for pairs at all mass ranges and find pairs separated by 2000–47,000 AU, at distances up to 4 kpc. Similarly, Longhitano & Binggeli (2010) used the angular two-point correlation function to do a purely statistical study of wide binaries in the $\sim 675$ deg$^2$ centered at the North Galactic Pole using the DR6 stellar catalog and predicted that there are more than 800 binaries with physical separations larger than 0.1 pc but smaller than 0.8 pc. As evidenced by the large false positive rate in SIJ08, such large-scale searches for wide binaries generally involve a trade-off between completeness on the one hand and fidelity on the other, as they depend on statistical arguments for identification.

Complementing this type of ensemble approach, a high-fidelity approach may suffer from incompleteness and/or biases; however, there are a number of advantages to a “pure” sample of bona fide wide binaries such as that presented in this paper. For example, Faherty et al. (2010) searched for CPM companions around the BDs in the BDKP catalog (Faherty et al. 2009) and found nine nearby pairs; all of their pairs were followed up spectroscopically and, hence, have a much higher probability of being real. As mass, age, and metallicity can all cause variations in the observed physical properties, e.g., in radius or in magnetic activity, their effects can be very hard to disentangle in a study of single stars. Components of multiple systems are expected to have been formed of the same material at the same time, within a few hundred thousand years of each other (e.g., White & Ghez 2001; Goodwin et al. 2004; Stassun et al. 2008). Hence, binaries are perfect tools for separating the effects of mass, age, and metallicity from each other as well as for constraining theoretical models of stellar evolution. Some examples include benchmarking stellar evolutionary tracks (e.g., White et al. 1999; Stassun et al. 2007, Stassun et al. 2008), investigating the age–activity relations of M dwarfs (e.g., Silvestri et al. 2005), defining the dwarf–subdwarf boundary for spectral classification (e.g., Lépine et al. 2007), and calibrating the metallicity indices (Woolf & Wallerstein 2005; Bonfils et al. 2005). Moreover, equal-mass multiples can be selected to provide identical twins with the same initial conditions (same mass, age, and metallicity) to explore the intrinsic variations of stellar properties. In addition, wide binaries ($a > 100$ AU) are expected to evolve independently of each other; even their
disks are unaffected by the distant companion (Clarke 1992). Components of such systems are effectively two single stars that share their formation and evolutionary history. In essence they can be looked at as coeval laboratories that can be used to effectively test and calibrate relations measured for field stars. Finally, as interest has grown in detecting exoplanets and in characterizing the variety of stellar environments in which they form and evolve, a large sample of bona fide wide binaries could provide a rich exoplanet hunting ground for future missions such as SIM.

In this paper, we present a new catalog of CPM doubles from SDSS, each with at least one low-mass component, identified by matching proper motions and photometric distances. In Section 2, we describe the origin of the input sample of low-mass stars; Section 3 details the binary selection algorithm and the construction of a Galactic model built to assess the fidelity of each binary in our sample. The resulting catalog and its characteristics are discussed in Section 4. We compare the result of our CPM double search with previous studies in Section 5 and summarize our conclusions in Section 6.

2. SDSS DATA

2.1. SDSS Sample of Low-mass Stars

SDSS is a comprehensive imaging and spectroscopic survey of the northern sky using five broad optical bands (ugriz) between \( \sim 3000 \) and 10,000 Å using a dedicated 2.5 m telescope (York et al. 2000; Fukugita et al. 1996; Gunn et al. 1998). Data Release Seven (DR7; Abazajian et al. 2009) contains photometry of 357 million unique objects over 11,663 deg\(^2\) of the sky and spectra of over 1.6 million objects over 9380 deg\(^2\) of the sky. The photometry has calibration errors of 2\% in \( u \) and \( \sim 1\% \) in griz, with completeness limits of 95\% down to magnitudes 22.0, 22.2, 22.2, 21.3, 20.5 and saturation at magnitudes 12.0, 14.1, 14.1, 13.8, 12.3, respectively (Gunn et al. 1998). We restricted our sample to \( r \lesssim 20 \) where the SDSS/USNO-B proper motions are more reliable (see Section 2.3); hence, photometric quality and completeness should be excellent for our sample.

Stellar sources with angular separations \( \gtrsim 0'5-0'7 \) are resolved in SDSS photometry; we determined this empirically from a search in the NEIGHBORS table. For sources brighter than \( r \sim 20 \), the astrometric accuracy is 45 mas r.m.s per coordinate while the relative astrometry between filters is accurate to 25–35 mas r.m.s (Pier et al. 2003).

The SDSS DR7 photometric database has more than 180 million stellar sources (Abazajian et al. 2009); to select a sample of low-mass stars, we followed the procedures outlined in B10 and required \( r-i \gtrsim 0.3 \) and \( i-z \gtrsim 0.2 \), which represents the locus of sample K5 or later dwarfs. The star table was used to ensure that all of the selected objects had morphology consistent with being point sources (\( \text{type} = 6 \)) and were not duplicate detections of the same source (\( \text{primary} \)).\(^5\) This yielded a sample of \( \gtrsim 109 \) million low-mass stars with \( r \sim 14-24 \), at distances of \( \sim 0.01-5 \) kpc from the Sun. Figure 1 shows a graphical flow chart of the selection process and criteria used; the steps needed in identifying kinematic companions, which are detailed in this paper, are shown in gray boxes. The white boxes show subsets where kinematic information is not available; however, it is still possible to identify binary companions based on their close proximity. We will discuss the selection of binaries, without available kinematic information, in a future paper.

2.2. Proper Motions

The SDSS/USNO-B matched catalog (Munn et al. 2004), which is integrated into the SDSS database in the PROPER MOTIONS table, was used to obtain proper motions for this study. We used the proper motions from the DR7 catalog; the earlier data releases had a systematic error in the calculation of proper motion in right ascension (see Munn et al. 2008, for details). This catalog uses SDSS galaxies to recalibrate the USNO-B positions and USNO-B stellar astrometry as an additional epoch for improved proper motion measurements. The PROPER MOTIONS catalog is resolution limited in the USNO-B observations to \( 7'' \) and is 90\% complete to \( g = 19.7, \)\(^6\) corresponding to the faintness limit of the POSS-II plates used in USNO-B. The completeness also drops with increasing proper motion; for the range of proper motions in our sample (\( \mu = 40-350 \) mas yr\(^{-1} \); see Section 2.3), the completeness is \( \sim 85\% \). The typical 1\( \sigma \) error is 2.5–5 mas yr\(^{-1} \) for each component.

2.3. Quality Cuts

To ensure that the resultant sample of binaries is not contaminated due to bad or suspect data, we made a series of cuts on the stellar photometry and proper motions. With \( \gtrsim 109 \) million low-mass stars, we could afford to be very conservative in our quality cuts and still have a reasonable number of stars in our input sample. We restricted our sample, as shown in Figure 1, to stars brighter than \( r = 20 \) and made a cut on the standard quality flags on the riz magnitudes (PEAKCENTER, NOTCHECKED, PSF_FLUX_INTERP, INTERP_CENTER, BAD_COUNTS_ERROR, SATURATED—all of which are required to be 0),\(^7\) which are the only bands pertinent to low-mass stars and the only bands used in our analysis.

On the proper motions, Munn et al. (2004) recommended a minimum total proper motion of 20 mas yr\(^{-1} \) and cuts based on different flags for a “clean” and reliable sample of stellar sources. Therefore, we required that each star (1) matched a unique USNO-B source within \( 1'' \) (MATCH > 0), (2) had no other SDSS source brighter than \( g = 22 \) within \( 7'' \) (DIST22 > 7), (3) was detected on at least 4 of the 5 USNO-B plates and in SDSS (NFIT = 6 or (NFIT = 5 and \( O < 2 \) or \( J < 2 \))), (4) had a good least-squares fit to its proper motion (SIGRA < 1000 and SIGDEC < 1000), and (5) had 1\( \sigma \) error for both components less than 10 mas yr\(^{-1} \). A challenge inherent in using a deep survey like SDSS to identify CPM binaries, is that most of the stars are far away and, therefore, have small proper motions. To avoid confusing real binaries with chance alignments of stars at large distances, where proper motions are similar but small, a minimum proper motion cut needs to be applied. Figure 2 shows the distribution of candidate binaries (selected as detailed in Section 3.1) with minimum proper motion cuts of 20, 30, 40, and 50 mas yr\(^{-1} \); the histograms have been normalized by the area of the histogram with the largest area to allow for relative comparisons. All four distributions have a peak at small separations of mostly \( \lesssim 0.01 \); the proportion of chance alignments at wider separations becomes larger and more dominant at smaller \( \mu \) cutoffs.

\(^{5}\) We note that binaries separated by less than \( \sim 1'' \) might appear elongated in SDSS photometry and might not be listed in the STAR table.

\(^{6}\) This corresponds to \( r = 18.75 \) for K5 and \( r = 18.13 \) for M6 dwarfs.

\(^{7}\) A PRIMARY object is already selected to not be BRIGHT and NODEREND or DEBLEND, NOPEAK.
If our aim were to identify a complete sample of binaries, we might have chosen a low \( \mu \) cutoff and accepted a relatively high contamination of false pairs. Such was the approach in the recent study of SDSS binaries by SIJ08 who used \( \mu \geq 15 \) mas yr\(^{-1}\). Our aim, in this paper, was to produce a “pure” sample with a high yield of bona fide binaries. Thus, in our search for CPM pairs we adopted a minimum proper motion of \( \mu = 40 \) mas yr\(^{-1}\) for our low-mass stellar sample since the number of matched pairs clearly declines with increasing \( \Delta \theta \) with this cutoff (Figure 2). While this still allows for a number of (most likely) chance alignments, they do not dominate the sample and can be sifted more effectively as discussed below.

Thus, on the \( \sim 15.7 \) million low-mass stars that satisfied the quality cuts, we further imposed a \( \mu \geq 40 \) mas yr\(^{-1}\) cut on the total proper motion that limits the input sample to 577,459 stars with excellent photometry and proper motions. As Figure 1 shows, this input sample constitutes all the stars around which we searched for companions. Figure 3 shows the distributions of photometric distances, total proper motions, \( r-i \) versus \( i-z \) color–color diagram, and the \( r \) versus \( r-z \) Hess diagram for the input sample. As seen from their \( r-z \) colors in the Hess diagram, the sample consists of K5–L0 dwarfs. Metal-poor halo SDs are also clearly segregated from the disk dwarf population; however, due to a combination of the magnitude and color limits that were used, they are also mostly K subdwarfs (SDs) and are limited to only the earliest spectral types we probe.

2.4. Derived Properties

2.4.1. Photometric Distances

**Disk dwarfs (DDs).** We determined the distances to the DDs in our sample by using photometric parallax relations, measured empirically with the SDSS stars. For M and L dwarfs (\( \sim M0–L0; 0.94 < r-z < 4.34 \)), we used the relation derived by B10 based on data from D. Golimowski et al. (2010, in preparation).

For higher mass dwarfs (\( \sim O5–K9; -0.72 < r-z < 0.94 \)), we fit a third-order polynomial to the data reported in Covey

**Figure 1.** Graphical representation of the selection process used in the identification of SLoWPoKES pairs. The gray boxes show the steps involved in SLoWPoKES, with the results presented in this paper. The pairs without proper motions (shown in white boxes) will be presented in a follow-up paper.
We adopted a cutoff of \( \mu \geq 40 \, \text{mas yr}^{-1} \) as it allows for identification of CPM pairs with a reasonable number of false positives that are later sifted with our Galactic model.

In both of the above relations, we used extinction-corrected magnitudes. Ideally, \( M_r \) would have been a function of both color and metallicity, but the effect of metallicity is not quantitatively known for low-mass stars. This effect, along with unresolved binaries and the intrinsic width of the MS, causes a non-Gaussian scatter of \( \sim 0.3 – 0.4 \, \text{mag} \) in the photometric parallax relations (West et al. 2005; SJ08; B10). Since we are matching the photometric distances, using the smaller error bars ensures fewer false matches. Hence, we adopted 0.3 mag as our error, implying a 1σ error of \( \sim 14\% \) in the calculated photometric distances.

Subdwarfs (SDs). Reliable photometric distance relations are not available for SDs, so instead we used the relations for DDs above. As a result of appearing underluminous at a given color, their absolute distances will be overestimated. However, the relative distance between two stars in a physical binary should have a small uncertainty. Because we are interested in determining if two candidate stars occupy the same volume in space, the relative distances will suffice. While photometric parallax relations based on a few stars of a range of metallicities (Reid 1998; Reid et al. 2001b) or calibrated for solar-type stars (Ivezić et al. 2008) do exist and can provide approximate distances for low-mass SDs, we refrained from using them due to the large uncertainties involved.

White dwarfs (WDs). We calculated the photometric distances to WDs using the algorithm used by Harris et al. (2006): ugriz magnitudes and the \( u-g, g-r, r-i, i-z \) colors, corrected for extinction, were fitted to the WD cooling models of Bergeron et al. (1995) to get the bolometric luminosities and, hence, the distances. The bolometric luminosity of WDs is a function of gravity as well as the composition of its atmosphere (hydrogen/helium), neither of which can be determined from the photometry. Therefore, we assumed pure hydrogen atmospheres with \( \log g = 8.0 \) to calculate the distances to the WDs. As a result, distances derived for WDs with unusually low mass and gravity (\( \sim 10\% \) of all WDs) or with unusually high mass and gravity (\( \sim 15\% \) of all WDs) will have larger uncertainties. Helium WDs redder than \( g-i > 0.3 \) will also have discrepant distances (Harris et al. 2006).

2.4.2. Spectral Type and Mass

The spectral type of all (O5–M9; \(-0.6 \leq r-z \leq 4.5\)) disk dwarfs and SDs were inferred from their \( r-z \) colors using the following two-part fourth-order polynomial:

\[
\text{SpType} = \begin{cases} 
50.4 + 9.04(r-z) - 2.97(r-z)^2 + 0.516(r-z)^3 - 0.028(r-z)^4; & \text{for O5–K5} \\
33.6 + 47.8(r-z) - 20.1(r-z)^2 - 33.7(r-z)^3 - 58.2(r-z)^4; & \text{for K5–M9}
\end{cases}
\]

(2)

where SpType ranges from 0 to 67 (O0 = 0 and M9 = 67 with all spectral types, except K for which K6, K8, and K9 subtypes are not defined, having ten subtypes) and is based on the data reported in Covey et al. (2007) and West et al. (2008). The spectral types should be correct to \( \pm 1 \) subtype.

Similarly, the mass of B8–M9 disk dwarfs and SDs was determined from their \( r-z \) colors using a two-part fourth-order polynomial:

\[
M(M_\odot) = \begin{cases} 
1.21 - 1.12(r-z) + 1.06(r-z)^2 - 2.73(r-z)^3 + 2.97(r-z)^4; & \text{for B8–K5} \\
0.640 + 0.362(r-z) - 0.539(r-z)^2 + 0.170(r-z)^3 - 0.016(r-z)^4; & \text{for K5–M9}
\end{cases}
\]

(3)

based on the data reported in Kraus & Hillenbrand (2007), who used theoretical models, supplemented with observational constraints when needed, to get mass as a function of spectral type. The scatter of the fit, as defined by the median absolute deviation, is \( \sim 2\% \).

As the polynomials for both the spectral type and mass are monotonic functions of \( r-z \) over the entire range, the component with the bluer \( r-z \) color was classified as the primary star of each binary found.

3. METHOD

3.1. Binary Selection

Components of a gravitationally bound system are expected to occupy the same spatial volume, described by their semi-axes,
and to move with a common space velocity. To identify physical binaries in our low-mass sample, we implemented a statistical matching of positional astrometry (R.A., $\alpha$, and decl., $\delta$), proper motion components ($\mu_\alpha$ and $\mu_\delta$), and photometric distances ($d$). The matching of distances is an improvement to the methods of previous searches for CPM doubles (Lépine & Bongiorno 2007; Chanamé & Gould 2004; Halbwachs 1986) and serves to provide further confidence in the binarity of identified systems.

The angular separation, $\Delta \theta$, between two nearby point sources, A and B, on the sky can be calculated using the small angle approximation:

$$\Delta \theta \approx \sqrt{(\alpha_A - \alpha_B)^2 \cos \delta_A \cos \delta_B + (\delta_A - \delta_B)^2}. \quad (4)$$

We searched around each star in the input sample for all stellar neighbors, brighter than $r = 20$ and with good photometry and proper motions, within $\Delta \theta \lesssim 180^\circ$ in the SDSS photometric database using the Catalog Archive Server query tool. Although CPM binaries have been found at much larger angular separations (up to 900$^\circ$ in Chanamé & Gould 2004; 1500$^\circ$ in Lépine & Bongiorno 2007; 570$^\circ$ in Faherty et al. 2010), the contamination rate of chance alignments at such large angular separations was unacceptably high in the deep SDSS imaging (see Figure 2). In addition, searching the large number of matches at larger separations required large computational resources. However, since the SDSS low-mass star sample spans considerably larger distances than in previous studies (see below), our cutoff of 180$^\circ$ angular separation probes similar physical separations of up to ~0.5 pc, which is comparable to the typical size of prestellar cores (0.35 pc; Benson & Myers 1989; Clemens et al. 1991; Jessop & Ward-Thompson 2000).

For all pairs that were found with angular separations of $7^\circ \leq \Delta \theta \leq 180^\circ$, we required the photometric distances to be within

$$\Delta d \leq (1\sigma_{\Delta d}, 100 \text{ pc}) \quad (5)$$

and the proper motions to be within

$$\left(\frac{\Delta \mu_\alpha}{\sigma_{\Delta \mu_\alpha}}\right)^2 + \left(\frac{\Delta \mu_\delta}{\sigma_{\Delta \mu_\delta}}\right)^2 \leq 2, \quad (6)$$

where $\Delta d$, $\Delta \mu_\alpha$, and $\Delta \mu_\delta$ are the scalar differences between the two components with their uncertainties calculated by adding the individual uncertainties in the quadrature. An absolute upper limit on $\Delta d$ of 100 pc was imposed to avoid $\Delta d$ being arbitrarily large at the very large distances probed by SDSS; hence, at $d \gtrapprox 720$ pc, distances are matched to be within 100 pc and results in a significantly lower number of candidate pairs. The proper motions are matched in two-dimensional vector space, instead of just matching the total (scalar) proper motion as has been frequently done in the past. The latter approach allows for a significant number of false positives as stars with proper motions with the same magnitude but different directions can be misidentified as CPM pairs.

For our sample, the uncertainties in proper motions are almost always larger than the largest possible Keplerian orbital motions of the identified pairs. For example, a binary with a separation of 5000 AU and $M_{\text{tot}} = 1 M_\odot$ at 200 pc, which is a typical pair in the resultant SLoWPoKES sample, will have a maximum orbital motion of ~0.27 mas yr$^{-1}$, much smaller than error in our proper motions (typically 2.5–5 mas yr$^{-1}$). However, a pair with a separation of 500 AU pair and $M_{\text{tot}} = 1 M_\odot$ at 50 pc has a maximum orbital motion of ~5.62 mas yr$^{-1}$, comparable to the largest errors in component proper motions. Hence, our algorithm will reject such nearby, relatively tight binaries.

From the mass estimates and the angular separations from the resulting sample, we calculated the maximum Keplerian orbital velocities, which are typically less than 1 mas yr$^{-1}$; only 104, 34, and 3 out of a total of 1342 systems exceeded 1, 2, and 5 mas yr$^{-1}$, respectively. More importantly, only seven pairs had maximum orbital velocities greater than 1σ error in the proper motion; so apart from the nearest and/or tightest pairs, our search should not have been affected by our restrictions on the proper motion matching in Equation (6).

Applying these selection criteria to the stellar sample described in Section 2.1, we found a total of 3774 wide CPM binary candidates, where each has at least one low-mass component. Among these, we found 118 pairs shown in red pluses and dashed histograms in Figure 4, concentrated in a $20^\circ \times 150^\circ$ stripe, in the direction of open cluster IC 5146. As there is significant nebulousity that is not reflected in the extinction values, the photometry and, hence, the calculated photometric distances are not reliable. In addition, while the kinematics are not characteristic of IC 5146, they are more likely to be part of a moving group rather than individual CPM pairs. Thus, we rejected all of these candidates. No other distinct structures, in space and kinematics, were found. Then, we made the quality cuts described in Section 2.3 on the companions; 906 and 1085 companions did not meet our threshold for the photometry and proper motions, respectively, and were rejected. The majority of these rejections were near the Galactic Plane, which was expected due to higher stellar density. Thus, at the end, the resulting sample had 1598 CPM double candidates from the statistical matching (Figure 1). False positives are inherent in statistical samples, arising from chance alignments within the uncertainties of the selection criteria. For any star, the probability of chance alignment grows with the separation making the wider companions much more likely to be chance alignments despite the selection criteria we just implemented. Hence, it is necessary to complete a detailed analysis of the fidelity of the sample.

### 3.2. Galactic Model: Assessing False Positives in the Binary Sample

To quantitatively assess the fidelity of each binary in our sample, we created a Monte Carlo based Galactic model that

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9 http://casjobs.sdss.org/CasJobs/

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Figure 4. Spatial distribution of SLoWPoKES binaries, shown in equatorial coordinates, closely follows the SDSS footprint. The upper and right panels show the histogram of R.A. and decl. in $5^\prime$ bins. We rejected the 118 CPM candidates found in the direction of open cluster IC 5146 (shown in red pluses and dashed histograms) due to the highly anomalous concentration of pairs and possibly contaminated photometry due to nebulosity (see Section 3.1).

(A color version of this figure is available in the online journal.)
mimics the spatial and kinematic stellar distributions of the Milky Way and calculates the likelihood that a given binary could arise by chance from a random alignment of stars. Clearly, for this to work the underlying distribution needs to be carefully constructed such that, as an ensemble, it reflects the statistical properties of the true distribution. The model needs to take into account the changes in the Galactic stellar density and space velocity over the large range of galactocentric distances and heights above/below the Galactic disk plane probed by our sample. Previous studies, focused on nearby binaries, have been able to treat the underlying Galactic stellar distribution as a simple random distribution in two-dimensional space. For example, Lépine & Bongiorno (2007) assigned a random shift of $1^{\circ}$–$5^{\circ}$ in R.A. for the secondary of their candidate pairs and compared the resultant distribution with the real one. As they noted, the shift cannot be arbitrarily large and needs to be within regions of the sky with similar densities and proper motion systematics. With stars at much larger heliocentric distances in our sample, even a $1^{\circ}$ shift would correspond to a large shift in Galactic position ($1^{\circ}$ in the sky at 1000 pc represents $\sim$17.5 pc). Therefore, we could not use a similar approach to assess the false positives in our catalog.

The Galactic model is built on the canonical view that the Galaxy comprises three distinct components—the thin disk, the thick disk, and the halo—that can be cleanly segregated by their age, metallicity, and kinematics (Bahcall & Soneira 1980; Gilmore et al. 1989; Majewski 1993). We note that some recent work has argued for the disk to be a continuum instead of two distinct components (Ivezić et al. 2008) or for the halo to be composed of two distinct components (Carlberg et al. 2008, 2009). However, for our purposes, the canonical three-component model was sufficient. We also did not try to model the over-densities or under-densities, in positional or kinematic space, caused by co-moving groups, open clusters, star-forming regions, or Galactic streams. If such substructures were found in the SDSS data, they were removed from our sample (see Section 3.1). In essence, this model strictly describes stars in the field and produces the three-dimensional position and two-dimensional proper motion, analogous to what is available for the SDSS photometric catalog.

3.2.1. Galactic Stellar Density Profile

In the canonical Galactic model, the stellar densities ($\rho$) of the thin and the thick disks, in standard Galactic coordinates $R$ (Galactic radius) and $z$ (Galactic height), are given by

$$\rho_{\text{thin}}(R, Z) = \rho(R, 0) \exp \left(-\frac{|Z|}{H_{\text{thin}}}ight) \times \exp \left(-\frac{|R - R_\odot|}{L_{\text{thin}}}ight) \quad (7)$$

$$\rho_{\text{thick}}(R, Z) = \rho(R, 0) \exp \left(-\frac{|Z|}{H_{\text{thick}}}ight) \times \exp \left(-\frac{|R - R_\odot|}{L_{\text{thick}}}ight) \quad (8)$$

where $H$ and $L$ represent the scale height above (and below) the plane and the scale length within the plane, respectively. The halo is described by a bi-axial power-law ellipsoid

$$\rho_{\text{halo}}(R, Z) = \rho(R, 0) \left(\frac{R_\odot}{\sqrt{R^2 + (Z/q)^2}}\right)^{-r_{\text{halo}}} \quad (9)$$

with a halo flattening parameter $q$ and a halo density gradient $r_{\text{halo}}$. The three profiles are added together, with the appropriate scaling factors, $f$, to give the stellar density profile of the Galaxy:

$$\rho(R, Z) = f_{\text{thin}}\rho_{\text{thin}} + f_{\text{thick}}\rho_{\text{thick}} + f_{\text{halo}}\rho_{\text{halo}}. \quad (10)$$

The scaling factors are normalized such that $f_{\text{thin}} + f_{\text{thick}} + f_{\text{halo}} = 1$. With the large number of stars imaged in the SDSS, robust stellar density functions have been measured for the thin and thick disks using the low-mass stars (Jurič et al. 2008; B10) and for the halo using the MS turn-off stars (Jurič et al. 2008). The values measured for the disk in the two studies are in rough agreement. We adopted the disk parameters from B10 and the halo parameters from Jurič et al. (2008); Table 1 summarizes the adopted values.

### Table 1

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter Name</th>
<th>Parameter Description</th>
<th>Adopted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin disk</td>
<td>$f_{\text{thin}}$</td>
<td>Fraction$^a$</td>
<td>$1 - f_{\text{thick}} - f_{\text{halo}}$</td>
</tr>
<tr>
<td></td>
<td>$H_{\text{thin}}$</td>
<td>Scale height</td>
<td>260 pc</td>
</tr>
<tr>
<td></td>
<td>$L_{\text{thin}}$</td>
<td>Scale length</td>
<td>2500 pc</td>
</tr>
<tr>
<td>Thick disk</td>
<td>$f_{\text{thick}}$</td>
<td>Fraction$^a$</td>
<td>9%</td>
</tr>
<tr>
<td></td>
<td>$H_{\text{thick}}$</td>
<td>Scale height</td>
<td>900 pc</td>
</tr>
<tr>
<td></td>
<td>$L_{\text{thick}}$</td>
<td>Scale length</td>
<td>3500 pc</td>
</tr>
<tr>
<td>Halo</td>
<td>$r_{\text{halo}}$</td>
<td>Density gradient</td>
<td>2.77</td>
</tr>
<tr>
<td></td>
<td>$q = (c/a)^b$</td>
<td>Flattening parameter</td>
<td>0.64</td>
</tr>
</tbody>
</table>

*Notes.* The parameters were measured using M dwarfs for the disk (Bochanski et al. 2010) and MS turn-off stars for the halo (Jurič et al. 2008) in the SDSS footprint.

$^a$ Evaluated in the solar neighborhood.

$^b$ Assuming a bi-axial ellipsoid with axes $a$ and $c$.

3.2.2. Galactic Kinematics

Compared to the positions, the kinematics of the stellar components are not as well characterized; in fact, apart from their large velocity dispersions, little is known about the halo kinematics. We seek to compare the proper motions of a candidate pair with the expected proper motions for that pair given its Galactic position. Thus, we found it prudent to (1) ignore the halo component, with its unconstrained kinematics, at distances where its contributions are expected to be minimal and (2) limit our model to a distance of 2500 pc, which corresponds to the Galactic height where the number of halo stars begins to outnumber disk stars (Jurič et al. 2008). In practice, all the SLoWPoKES CPM pairs, with the exceptions of SDs for which distances were known to be overestimated, were within $\sim$1200 pc (see Figure 9 later); so we did not introduce any significant systematics with these restrictions.

An ensemble of stars in the Galactic Plane can be characterized as having a purely circular motion with a velocity, $V_{\text{circ}}$. The orbits become more elliptic and eccentric over time due to kinematic heating causing the azimuthal velocity, $V_\phi$, to decrease with the Galactic height, $z$. However, the mean radial ($V_r$) and perpendicular ($V_\perp$) velocities for the ensemble at any $z$ remains zero, with a given dispersion, as there is no net flow of stars in either direction. In addition, this randomization of orbits also causes the asymmetric drift, $V_a$, which increases with the age of stellar population and is equivalent to $\sim$10 km s$^{-1}$ for M dwarfs. Hence, the velocities of stars in the Galactic disk can be
This would tend to underestimate the true number of chance alignments for SDSS low-mass dwarfs (Bochanski et al. 2007). All the velocity dispersions were well described by the power law radial velocity; Johnson & Soderblom 1987), analogous to our can be converted to a two-dimensional proper motion (and velocities, when complemented with the dispersions, velocities, when complemented with the dispersions, UVW can be converted to a two-dimensional proper motion (and velocities, when complemented with the dispersions, UVW...)

summarized, in Galactic cylindrical coordinates, as

\[
\begin{align*}
\langle V_r(Z) \rangle &= 0 \\
\langle V_\phi(Z) \rangle &= V_{\text{circ}} - V_a - f(Z) \\
\langle V_z(Z) \rangle &= 0,
\end{align*}
\]

where \( V_{\text{circ}} = 220 \text{ km s}^{-1}, V_a = 10 \text{ km s}^{-1}, f(Z) = 0.013 |Z| - 1.56 \times 10^{-5} |Z|^2 \) was derived by fitting a polynomial to the data in West et al. (2008), and \( z \) is in parsecs. This formulation of \( V_\phi(Z) \) is consistent with a stellar population composed of a faster thin disk (\( (V_\phi) = 210 \text{ km s}^{-1} \)) and a slower thick disk (\( (V_\phi) = 180 \text{ km s}^{-1} \)). Then, we converted these galactocentric polar velocities to the heliocentric, Cartesian UVW velocities. The UVW velocities, when complemented with the dispersions, can be converted to a two-dimensional proper motion (and radial velocity; Johnson & Soderblom 1987), analogous to our input catalog. We used the UVW velocity dispersions measured for SDSS low-mass dwarfs (Bochanski et al. 2007). All the dispersions were well described by the power law

\[
\sigma(Z) = k|Z|^\eta,
\]

where the values of constants \( k \) and \( n \) are summarized in Table 2. As the velocity dispersions in Bochanski et al. (2007) extend only up to \( \sim 1200 \text{ pc} \), we extrapolated the above equation for larger distances. While the velocity ellipsoids of F and G dwarfs have been measured to larger distances (e.g., Bond et al. 2009), we preferred to use the values measured for M dwarfs for our low-mass sample.

### 3.2.3. The Model

By definition, a chance alignment occurs because two physically unassociated stars randomly happen to be close together in our line of sight (LOS), within the errors of our measurements. Due to the random nature of these chance alignments, it is not sufficient to estimate the probability of chance alignment along a given LOS simply by integrating Equation (10). This would tend to underestimate the true number of chance alignments because the density profiles in Equations (7)–(9) are smooth functions that, in themselves, do not include the random scatter about the mean relation that real stars in the real Galaxy have. Thus, the stars need to be randomly redistributed spatially about the average given by Equation (10) in order to properly account for small, random fluctuations in position and velocity that could give rise to false binaries in our data.

In principle, one could simulate the whole Galaxy in this fashion in order to determine the probabilities of chance alignments as a function of LOS. In practice, this requires exorbitantly amounts of computational time and memory. Since our aim was to calculate the likelihood of a false positive along specific LOSs, we, instead, generated stars in much smaller regions of space, corresponding to the specific binaries in our sample. For example, a \( 30' \times 30' \) cone integrated out to a distance of 2500 pc from the Sun will contain at least a few thousand stars within any of the specific LOSs in our sample. The number of stars is large enough to allow for density variations similar to that of the Milky Way while small enough to be simulated with ease. With a sufficient number of Monte Carlo realizations, the random density fluctuations along each LOS can be simulated. We found that \( 10^5 \) realizations allowed for the results to converge, within \( \sim 0.5\% \).

We implemented the following recipe to assess the false-positive likelihood for each candidate pair using our Galactic model:

1. The total number of stars in the LOS volume defined by a \( 30' \times 30' \) area, centered on the \( \alpha \) and \( \delta \) of a given binary, over heliocentric distances of 0–2500 pc was calculated by integrating Equation (10) in 5 pc deep, discrete cylindrical “cells.”

Integrating Equation (10) for \( \Delta \theta = 30' \) and \( d = 0–2500 \text{ pc} \) resulted in \( \sim 3300–1580 \) stars, with the higher numbers more likely to be the LOSs along the Galactic plane. This number of stars, when randomly redistributed in the entire volume, was more than enough to recreate over-densities and under-densities.

2. The stars were then distributed in three-dimensional space defined by the LOS using the rejection method (Press et al. 1992), generating \( \alpha, \delta, \) and \( d \) for each star. The rejection method ensured that the stars were randomly distributed while following the overlying distribution function, which, in this case, was the stellar density profile given by Equation (10).

The model did an excellent job of replicating the actual distribution of stars in the three-dimensional space. The red histograms in Figure 5 show the number of stars from the center of the LOS as a function of angular separation for the model (dashed lines) and the data (solid lines), averaged...
over all LOSs where candidate binaries were identified in Section 3.1. There is an excess of pairs at close separations, a signature of genuine, physically associated pairs, while the two distributions follow the same functional form at larger separations, where chance alignments dominate. The increasing number of pairs with angular separation is an evidence of larger volume that is being searched. Integrating the model predicts \( \sim 8.0 \) stars within a search radius of 60\(^\circ\) and \( \sim 71 \) within 180\(^\circ\). In other words, for a typical LOS with candidate binaries, we would, on average, expect to find eight chance alignments within 60\(^\circ\) and \( \sim 71 \) within 180\(^\circ\) when considering only angular separation.

For the 180\(^\circ\) radius around each LOS, the average number of stars in the model and the data were within a few \( \sigma \) of each other, with the largest deviations found along the LOSs at low Galactic latitudes or large distances. As our model only integrated to \( d = 2500 \) pc, deviations for LOSs at large distances were expected. Deviations for LOSs at low Galactic latitudes are reasonable as the parameters in Equations (7) and (8) as not as well constrained along the Galactic disk. Hence, we concluded that the rejection method used in redistributing the stars in the 30\(^\circ\) \times 30\(^\circ\) LOS is correctly implemented and that the model successfully replicates the three-dimensional spatial distribution of the stars in the Galaxy.

When the distances were matched, the number of pairs decreased as chance alignments were rejected (blue histograms in Figure 5); there are, on average, only \( \sim 0.41 \) and \( \sim 3.6 \) chance alignments within 60\(^\circ\) and 180\(^\circ\), respectively. Note that matching distances, in addition to the angular separation, significantly enhanced the peak at the small separations.

3. Based on the Galactic position of each randomly generated star, mean UVW space velocities and their dispersions were generated based on Equations (11) and (12), respectively. Proper motions were then calculated by using the inverse of the algorithm outlined in Johnson & Soderblom (1987). These generated proper motions represent the expected kinematics of stars at the given Galactic position.

Figure 6 shows the comparison between the proper motions in the SDSS/USNO-B catalog and our model; component proper motions of stars within 60\(^\circ\) of all LOSs, where candidate pairs were found. For the purposes of this plot, we restricted the stars to be within 1200 pc and of spectral type K5 or later so we could compare kinematics with a sample similar to our resultant catalog. We also did not compare the distributions at \( \mu_\alpha \) or \( \mu_\delta \) < 10 mas yr\(^{-1}\) where the proper motions are comparable to the 1\( \sigma \) errors and, hence, not reliable. Since our initial sample was has \( \mu \geq 40 \) mas yr\(^{-1}\), stars with the small proper motion component are either rejected or their motion is dominated by the other component in our analysis. As evidenced in the figure, our model reproduced the overall kinematic structure of the thin and thick disks.

When the proper motions were matched (purple histograms in Figure 5), in addition to the angular separation and distances, the number of chance alignments fell drastically, especially at the smaller separations. In fact, at \( \Delta \theta \lesssim 15\(^\circ\) \), there were \( \sim 10^{-3} \) chance alignments, on average; even at \( \Delta \theta = 180\(^\circ\) \), the real pairs outnumbered the chance alignments by a factor of 4–5. Cumulatively, for a typical LOS, there were, on average, only \( \sim 0.0097 \) and \( \sim 0.086 \) chance alignments within 60\(^\circ\) and 180\(^\circ\), respectively. As a result of matching distance and proper motions components, the number of chance alignments within 180\(^\circ\) were reduced by a factor of \( \sim 800 \).

4. In the model galaxy, we repeated the selection process used to find CPM pairs in the SDSS photometric catalog, as described in Section 3.1. To avoid double-counting, the input coordinates of the LOS were considered to be the primary star. Note that we did not intend to model and reproduce both stars of a given pair but wanted to see if a random chance alignment could produce a companion for a given primary. Hence, each additional star that was found to satisfy Equations (5) and (6) was counted as an optical companion. The average number of companions found in the 10\(^5\) Monte Carlo realizations is the probability of chance alignment or the probability that the candidate pair is a false positive, \( P_f \), for that candidate pair. The number of realizations sets the resolution of \( P_f \) at \( 10^{-5} \).

5. Finally, this was repeated for all candidate pairs that were found in Section 3.1. Figure 7 shows the distribution of \( P_f \) all of the candidate pairs; we further discuss this distribution is Section 3.3.

To conclude, the result of our Galactic model was a 30\(^\circ\) \times 30\(^\circ\) area of the sky, centered around the given CPM pair, with the surrounding stars following the Galactic spatial and kinematic distributions of the Milky Way. Each star in this model galaxy was described by its position \((\alpha, \delta, \mu_\alpha, \mu_\delta)\), the same as is available for the SDSS photometric catalog. Based on the above results, we concluded that the Galactic model sufficiently reproduced the five-dimensional (three spatial and two kinematic\(^{10} \)) distribution of stars along typical LOSs in the Milky Way and, thus, allowed for the calculation of probability that a given CPM double is a chance alignment.

### 3.3. Fidelity

As described in the previous section, we have implemented a very stringent selection algorithm in identifying the CPM pairs. In addition to only including objects with the most robust photometry and proper motions, we also used a relatively high

\(^{10}\)The model also predicts radial velocities. We have an observational program underway to obtain radial velocities of the binaries for further refinement of the sample.
proportional to the number of low-mass stars. As described above, an algorithm optimized to reject the most false positives, even at the expense of real pairs, was used in the statistical matching of angular separation, photometric distance, and proper motion components. Finally, we used the Galactic model to quantify the probability of chance alignment, \( P_f \), for each of the 1558 candidate pairs.

Normally all candidate pairs with \( P_f \leq 0.5 \), i.e., a higher chance of being a real rather than a fake pair, could be used to identify the binaries. However, to minimize the number of spurious pairs, we required

\[
P_f \leq 0.05
\]

for a pair to be classified as real. Here we note that \( P_f \) represents the false-alarm probability that a candidate pair identified by matching angular separation, photometric distance, and proper motion components, as described by Equations (5) and (6), is a real pair; it is not the probability that a random low-mass star is part of a wide binary.

Making the above cut on \( P_f \) resulted in a catalog of 1342 pairs, with a maximum of 5\% or 67 of the pairs expected to be false positives. However, as a large number of the pairs have extremely small \( P_f \) (see Figure 7), the number of false positives is likely to be much smaller. Adding up the \( P_f \) for the pairs included in the catalog gives an estimated 22 (1.65\%) false positives. In other words, the overall fidelity of SLoWPoKES is 98.35\%. This is a remarkably small proportion for a sample of very wide pairs, especially since they span a large range in heliocentric distances and is a testament to our selection criteria. For example, if the proper motion components were matched to within 2\( \sigma \), Equation (13) would have rejected \( \geq 60\% \) of the candidates. Our choice of Equation (13) is a matter of preference; if a more efficient (or larger) sample is needed, it can be changed to suit the purpose.

To get a first-order approximation of how many real binaries we are missing or rejecting, we applied our selection algorithm to the rNLTT CPM catalog with 1147 pairs (Chanamé & Gould 2004). Note that Chanamé & Gould (2004) did not match distances of the components, as they did not have reliable distance estimates available and matched total proper motions instead of a two-dimensional vector matching in our approach. Out of the 307 rNLTT pairs, which have \( \Delta \theta \leq 180^\circ \) and are within the SDSS footprint, we recover both components of 194 systems (63\%) and one component of another 56 (18\%) systems, within 2\( \sigma \) of the rNLTT coordinates. Of the 194 pairs for which both components had SDSS counterparts, 59 (30\%) pairs satisfied our criteria for proper motions, Equation (6), while 19 (10\%) pairs satisfied our criteria for both proper motions and distances, Equations (5) and (6). In other words with our selection criteria, we recovered only 10\% of the rNLTT pairs, with SDSS counterparts, as real binaries. If we relax our selection criteria to match within 2\( \sigma \), the number of recovered pairs increases to 82 with matching proper motions and 37 with matching proper motions and distances. Of course, as the rNLTT is a nearby, high proper motion catalog, Chanamé & Gould (2004) had to allow for larger differences in proper motions due to the stars’ orbital motion, which is not applicable to the SLoWPoKES sample (see Section 3.1). In conclusion, compared to previous catalogs of CPM doubles, we find a small fraction of previously identified very wide binaries. The reasons for the low recovery rate are twofold: (1) the restrictive nature of our matching algorithm that rejects the most false positives, even at the expense of real pairs and (2) improvement in the identification method—e.g., matching proper motions in vector space and being able to use photometric distance as an additional criterion.

4. CHARACTERISTICS OF THE SLoWPoKES CATALOG

Using statistical matching of angular separation, photometric distance, and proper motion components, we have identified 1598 very wide, CPM double candidates from SDSS DR7. We have built a Galactic model, based on empirical measurements of the stellar density profile and kinematics, to quantitatively evaluate the probability that each of those candidate doubles is real (Figure 7). Using Equation (13), we classify 1342 pairs as real, associated pairs. In deference to their extremely slow movement around each other, we dub the resulting catalog SLoWPoKES for Sloan Low-mass Wide Pairs of Kinematically Equivalent Stars. Table 3 lists the properties of the identified pairs. The full catalog is publicly available on the Internet.

SLoWPoKES is intended to be a “live” catalog of wide, low-mass pairs, i.e., it will be updated as more pairs are identified and as follow-up photometric and spectroscopic data become available.

Figure 8 shows a collage of \( gri \) composite images, 50" on a side, of a selection of representative SLoWPoKES systems; the images are from the SDSS database. In the collage high-mass ratio pairs (top row; mass ratio \( m_2/m_1 < 0.5 \)), equal-mass pairs (middle row; having masses within \( \sim 5\% \) of each other), white dwarf–disk dwarf pairs (bottom row, left), and halo SD pairs (bottom row, right) are shown. Table 4 summarizes the different types of systems in the catalog.

In the following sections, we summarize various aspects of the systems that constitute the SLoWPoKES catalog and briefly examine some of the follow-up science that can be pursued with SLoWPoKES. We wish to emphasize that the SLoWPoKES catalog is intended principally as a high-fidelity sample of CPM doubles that can be used for a variety of follow-up investigations where the reliability of each object in the catalog is more important than sample completeness. Thus, we have not attempted to fully account for all sources of incompleteness or bias; and we intentionally have not applied any form of statistical “corrections” to the catalog.

We note that some of the most important sources of incompleteness in the present catalog could be at least partially over-
Proper motions either do not exist or are not reliable for stars. Motions for SDSS stars that were not detected in USNO-B can be ruled out most of the mid- to late-M dwarf spectrum, SDSS saturates at \( r \lesssim 20 \) imply that while we are sensitive to K5 dwarfs at \( \sim 7000–18,000 \) AU at 100 pc but \( \sim 14–180 \) pc. This means that (1) we are entirely insensitive to pairs with the most extreme mass ratios (i.e., a K5 paired with an M6 or M7) and (2) we cannot directly compare the properties of K5 and M5 spectral subtypes in identical distance ranges. In addition, as illustrated in Figure 9, with our \( 7'' \) of a brighter star. The first criterion currently rules out most of the mid- to late-M dwarf companions, while the latter rejects close binaries and most hierarchical higher-order systems. However, as the SDSS photometric and astrometric data are available for these systems, their multiplicity could be verified with more rigorous analysis or through cross-matching with other catalogs. For example, by cross-matching SDSS with 2MASS, an M4.5–L5 binary (Radigan et al. 2008) and an M6–M7 binary (Radigan et al. 2009) have already been identified. At the other end of the spectrum, SDSS saturates at \( r \approx 14 \), resulting in saturated or unreliable photometry for the brighter stars. We found that more than 1000 candidate pairs were rejected for this reason; with reliable follow-up photometry these could be added as additional genuine SLoWPoKES binaries.

Even so, a fully volume-complete sample is likely to be impossible to compile over the full ranges of spectral types and distances spanned by our catalog. For example, our magnitude limits of \( 14 \lesssim r \lesssim 20 \) imply that while we are sensitive to K5 dwarfs at \( \sim 250–3900 \) pc, we are sensitive to M5 dwarfs at \( \sim 14–180 \) pc. This means that (1) we are entirely insensitive to pairs with the most extreme mass ratios (i.e., a K5 paired with an M6 or M7) and (2) we cannot directly compare the properties of K5 and M5 spectral subtypes in identical distance ranges. In addition, as illustrated in Figure 9, with our \( 7'' \) of a brighter star. The first criterion currently rules out most of the mid- to late-M dwarf companions, while the latter rejects close binaries and most hierarchical higher-order systems. However, as the SDSS photometric and astrometric data are available for these systems, their multiplicity could be verified with more rigorous analysis or through cross-matching with other catalogs. For example, by cross-matching SDSS with 2MASS, an M4.5–L5 binary (Radigan et al. 2008) and an M6–M7 binary (Radigan et al. 2009) have already been identified. At the other end of the spectrum, SDSS saturates at \( r \approx 14 \), resulting in saturated or unreliable photometry for the brighter stars. We found that more than 1000 candidate pairs were rejected for this reason; with reliable follow-up photometry these could be added as additional genuine SLoWPoKES binaries.
Figure 8. 50″ × 50″ gri composite images of CPM pairs found in the SLoWPoKES survey. Pictured are high-mass ratio pairs (top row), identical twins (middle row), white dwarf–disk dwarf pairs (bottom row, left), and halo SD pairs (bottom row, right). Spectral types based on their $r - z$ colors are shown. Overall, 1342 wide, low-mass binaries were identified.

1000 pc. Thus, it is important with the current catalog that statistical determinations of ensemble system properties be performed within narrowly defined slices of separation and distance. We do so in the following subsections as appropriate, but we emphasize again that our intent here is primarily to characterize the SLoWPoKES sample and will proffer any interpretive conclusions only tentatively.

Finally, as is evident from Table 4, the current SLoWPoKES catalog is clearly not well suited for study of higher-order multiples (i.e., triples, quadruples, etc.). Identifying CPM higher-order multiple systems in SDSS is very challenging due to the lack of reliable proper motions in the SDSS/USNO-B matched catalog at $\Delta \theta < 7^\prime\prime$ and $r > 20$. Unless all components are widely separated and are all bright, they will be rejected in our search. In addition, we are currently rejecting hierarchical triples consisting of a close pair that is unresolved in SDSS and a wide, CPM tertiary. If the mass ratio of the close, unresolved pair is near unity, it will appear as an overluminous single star that will then be misinterpreted by our algorithm as having a discrepant photometric distance from its wide tertiary companion. The available SDSS photometry and astrometry shows evidence of a substantial number of such multiple systems, and we plan to make these the subject of a future study. Already, four CPM triples are identified in our search (Table 4). Moreover, the current SLoWPoKES catalog is likely to contain quadruple systems in which the two components of the identified wide binary are themselves in fact spatially unresolved binaries with near-equal mass components. We have initiated an adaptive optics program to identify such higher-order systems in the SLoWPoKES catalog.

4.1. Kinematic Populations

Luyten (1922) devised the reduced proper motion (RPM) diagram to be used instead of the H-R diagram when distances to the objects are not available, as is the case in large imaging surveys. The RPM of an object is defined as

$$H \equiv m + 5 \log \mu + 5 = M + 5 \log v_t - 3.25,$$

Figure 9. Projected linear separation of SLoWPoKES pairs is plotted as a function of angular separation, with the iso-distance lines plotted across the grid. DD (black), SD (red), and WD–DD pairs (blue) are shown. Systematically larger distances for the SDs are clearly seen, which are expected as photometric DD parallax relations were used to calculate the SD distances as well. Hence, SDs are excluded in all analysis involving their distances. (A color version of this figure is available in the online journal.)
where \( v_t \) is the heliocentric tangential velocity in \( \text{km s}^{-1} \) given by \( v_t = 4.74 \mu d \) and \( \mu \) is the proper motion in arcsec yr\(^{-1}\). Just as in an H-R diagram, the RPM diagram effectively segregates the various luminosity or kinematic classes from each other (e.g., Chanamé & Gould 2004; Harris et al. 2006; Lépine & Bongiorno 2007; SJ08).

WDs, in addition to being relatively very blue, are less luminous than either the DDs or the SDs; hence, the observed WDs tend to be nearby disk WDs with high tangential velocities. Specifically, spectroscopic follow-up has shown that the disk WDs with \( v_t = 25–150 \text{ km s}^{-1} \) (Figure 10, black dashed lines) can be effectively identified from the \( g \)-band RPM diagram, when complemented by photometric parallax relations (Kilic et al. 2006). However, we note that spectroscopic follow-up is needed to confirm that the identified objects are actually WDs. The available SDSS spectra confirm that 9 of the 21 WD primaries, identified in SLoWPoKES, are indeed WDs. As the WDs were identified from the \( g \) band, they scatter toward the SD and DD locus in the \( r \)-band RPM diagram.

SDs are low-metallicity halo counterparts of the MS dwarfs found in the Galactic disk. Hence, they have bluer colors at a given absolute magnitude (however, the \( g-r \) colors for M SDs are redder; West et al. 2004; Lépine & Scholz 2008) and have higher velocity dispersions. As a result, the SDs lie below the disk dwarfs in the RPM diagram.\(^{12}\) To segregate the SDs from the DDs, we used the DD photometric parallax relations complemented by the mean tangential velocity of halo stars, \( v_t = 180 \text{ km s}^{-1} \). This relation is shown in blue dashed lines in the \( r \)-band RPM diagram in Figure 10. As the mean halo velocity was used, SDs can scatter above the line, but DDs would not be expected to be below the blue line. For comparison, \( v_t = 25 \text{ km s}^{-1} \), the mean tangential velocity of disk stars, is also shown. Note that DDs in SLoWPoKES have tangential velocities larger than the mean velocity of the disk, which is expected as we rejected all stars with \( \mu < 40 \text{ mas yr}^{-1} \).

Figure 10 shows the \( H_g \) versus \( g-i \) and \( H_r \) versus \( r-z \) RPM diagrams with both the primary (triangle) and the secondary component (circle) for the 21 WD–DD (blue), 70 SD–SD

\(^{12}\) SJ08 showed that the RPM diagram becomes degenerate at \( Z > 2–3 \text{ kpc} \) due to the decrease in rotational velocity. This does not affect our sample.
biomodal structure in between. The cutoff at small separations is observational, however, that these $a$ values are valid only for ensemble comparisons and should not be taken as an accurate measure of $a$ for any individual system. In addition, the above equation implies that, at the extrema of angular separations probed, we are biased toward systems that are favorably oriented, either because of $\sin i$ projection effects or eccentricity effects that lead to a changing physical separation as a function of the orbital position. For example, at $\Delta \theta \approx 7^\circ$, pairs at their maximal apparent separation are more likely to be identified while we are biased toward pairs with smaller apparent separations at $\Delta \theta \approx 180^\circ$.

The distribution of $a$ is plotted for the SLoWPoKES sample in Figure 13 (right). The semi-major axes for the identified pairs range from $\sim 10^2$ to $10^5$ AU ($\sim 0.005$–$0.5$ pc), with sharp cutoffs at both ends of the distribution, and with a clear bimodal structure in between. The cutoff at small separations is observational, resulting from our bias against angular separations $\Delta \theta < 7^\circ$. Indeed, the range of physical separations probed by SLoWPoKES is at the tail-end of the log-normal distribution ($\langle a \rangle = 30$ AU, $\sigma_{\log a} = 1.5$) proposed by DM91 and FM92.

Figure 14 (left panel) shows the $r - z$ color distributions of the primaries and secondaries of SLoWPoKES pairs, where the primary is defined as the component with the bluer $r - z$ color (and, thus, presumably more massive). As the SLoWPoKES sample is dominated by dwarfs, the $r - z$ color distribution should correspond directly to the mass distribution. Both the primary and secondary distributions show a peak at the early to mid-M spectral types (as inferred from their colors; Section 2.4.2), probably due to, at least in part, the input sample being mostly M0–M4 dwarfs. However, apart from the saturation at $r \lesssim 14$, there is no bias against finding highermass companions. Hence, it is notable that more than half of the primaries have inferred spectral types of M0 or later, with mass companions. Hence, it is notable that more than half of the primaries have inferred spectral types of M0 or later, with mass companions.

4.3. Mass Distribution

As discussed above, selection biases in SLoWPoKES are in general a function of distance because of the large range of
Table 5
A Subsample of Previously Known Wide Binaries with the Projected Separation, \( s \gtrsim 10^4 \) AU (\( \sim 0.05 \) pc)

<table>
<thead>
<tr>
<th>ID</th>
<th>Secondary</th>
<th>( \Delta \theta )</th>
<th>( s )</th>
<th>Spectral Type(^a)</th>
<th>BE(^c)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIP 38602</td>
<td>LSPM J0753+5845</td>
<td>109.90</td>
<td>10175</td>
<td>G8.1</td>
<td>M5.1</td>
<td>132.18</td>
</tr>
<tr>
<td>HIP 25278</td>
<td>HIP 25220</td>
<td>707.10</td>
<td>10249</td>
<td>F1.8</td>
<td>K5.8</td>
<td>307.44</td>
</tr>
<tr>
<td>HIP 52469</td>
<td>HIP 52498</td>
<td>288.00</td>
<td>10285</td>
<td>A8.9</td>
<td>F9.3</td>
<td>379.74</td>
</tr>
<tr>
<td>HIP 86036</td>
<td>HIP 86087</td>
<td>737.90</td>
<td>10400</td>
<td>F8.4</td>
<td>M2.6</td>
<td>184.33</td>
</tr>
<tr>
<td>HIP 58939</td>
<td>LSPM J1205+1931</td>
<td>117.20</td>
<td>10558</td>
<td>K0.5</td>
<td>M4.7</td>
<td>107.44</td>
</tr>
<tr>
<td>HIP 51669</td>
<td>LSPM J1033+4722</td>
<td>164.30</td>
<td>10668</td>
<td>K7.3</td>
<td>M5.7</td>
<td>52.10</td>
</tr>
<tr>
<td>HIP 81023</td>
<td>LSPM J1633+0311N</td>
<td>252.00</td>
<td>10723</td>
<td>K3.2</td>
<td>M3.8</td>
<td>81.51</td>
</tr>
<tr>
<td>HIP 50802</td>
<td>LSPM J1022+1204</td>
<td>311.90</td>
<td>11139</td>
<td>K5.9</td>
<td>M3.1</td>
<td>56.44</td>
</tr>
<tr>
<td>HIP 78128</td>
<td>LSPM J1557+0745</td>
<td>144.60</td>
<td>11385</td>
<td>G7.1</td>
<td>M5.0</td>
<td>123.01</td>
</tr>
<tr>
<td>HIP 116106</td>
<td>2MASS J2331−04AB</td>
<td>451.00</td>
<td>11900</td>
<td>F8</td>
<td>( \cdots )</td>
<td>161.10</td>
</tr>
<tr>
<td>( \alpha ) Cen AB</td>
<td>NLTT 37460</td>
<td>9302.00</td>
<td>12000</td>
<td>G2+K1</td>
<td>M6</td>
<td>430.21</td>
</tr>
</tbody>
</table>

Notes.

\(^a\) We have tried to use HIP and NLTT identifiers, whenever they exist, for consistency.

\(^b\) Spectral types are from the referenced papers, SIMBAD, or inferred from their \( V−J \) colors using Kenyon & Hartmann (1995).

\(^c\) Binding energies are calculated using estimated masses as a function of spectral type (Kraus & Hillenbrand 2007). When spectral type for the secondary was not available, it was assumed to be an equal-mass binary.

References.


(This table is available in its entirety in machine-readable and Virtual Observatory (VO) forms in the online journal. A portion is shown here for guidance regarding its form and content.)

Figure 15. Color distribution for the primary (blue) and secondary (red) components of SLoWPoKES pairs as a function of distance, in 100 pc bins shown in a box-and-whisker plot; the number of pairs in each bin is printed along the top. The bar inside the boxes shows the median of the distribution; the boxes show the inter-quartile range; the whiskers extend to either the 1.5 times the inter-quartile range or the maximum or the minimum value of the data, whichever is larger; and the open circles show the outliers of the distribution. The black dashed lines show the bright and faint limits of our sample (14 \( \lesssim r \lesssim 20 \)). As the figure indicates, the primary and secondary distributions are bluer at increasing distances, as would be expected due to the bright and faint limits; but the secondary distribution does not change as compared to the primary distribution as a function of distance. However, this is likely to be a strict function of the faintness limit of our catalog: the stellar mass function peaks around M4 spectral type (B10) while we cannot see M4 or later stars beyond \( \sim 400 \) pc. Hence, with our current sample, we cannot discern whether the tendency for the secondary distribution to follow the primary distribution is a tendency toward \( q \sim 1 \) or a tendency for the secondary distribution to be drawn from the field mass function.

We can also examine the distribution of the secondary masses relative to their primaries, i.e., the mass ratio distribution, which is an important parameter in the study of binary star formation and evolution. As shown in Figure 14 (right panel, solid histogram), the distribution of mass ratios, \( q \equiv M_2/M_1 \), is strongly skewed toward equal-mass pairs: 20.9%, 58.5%, and 85.5% of pairs have masses within 10%, 30%, and 50%, respectively, of each other. To determine whether this is strictly due to the magnitude limits in SLoWPoKES, we calculated the mass ratios for hypothetical pairs with the observed primary and the faintest observable secondary. The resulting distribution (dashed histogram) is considerably different from what is observed and shows that we could have identified pairs with much lower mass ratios, within the \( r \lesssim 20 \) faintness limit. The same result is obtained when we pair the observed secondaries with the brightest possible primary. Hence, we conclude that the observed distribution peaked toward equal-masses among wide, low-mass stars is real and not a result of observational biases.

4.4. Wide Binary Frequency

To measure the frequency of wide binaries among low-mass stars, we defined the wide binary frequency (WBF) as

\[
WBF = \frac{\text{number of CPM pairs found}}{\text{number of stars searched around}}. \tag{16}
\]
With 1342 CPM doubles among the 577,459 stars that we searched around, the raw WBF is 0.23%. However, given the observational biases and our restrictive selection criteria, 0.23% is a lower limit. Figure 16 shows the WBF distribution as a function of \( r - z \) color with the primary (solid histogram), secondary (dashed histogram), and total (solid dots with binomial error bars) WBF plotted; the total number of pairs found in each bin are also shown along the top. The WBF rises from \( -0.23\% \) at the bluest \( r - z \) color (\( \sim K7 \)) to \( \sim 0.57\% \) at \( r - z = 1.6 \) (\( \sim M2 \)), where it plateaus. This trend is probably due at least in part to our better sensitivity to companions around nearby early to mid-M dwarfs as compared to the more distant mid-K dwarfs or to the much fainter mid- to late-M dwarfs. To get a first-order measure of the effects of the observational biases, we can look at the WBF in specific distance ranges where all stars of the given \( r - z \) colors can be seen and the biases are similar for all colors, assuming the range of observable magnitude are \( r = 14-20 \). In particular, it would be useful to look at \( r - z = 0.7-1.5 \) where the WBF increases and \( r - z = 1.5-2.5 \) where the WBF plateaus in Figure 16. Figure 17 shows the WBF in the two regimes for \( d = 247-1209 \) pc and 76–283 pc, respectively, where all companions of the given color range are expected to be detected at those distances. In the restricted ranges, the WBF is generally higher for a given color than in Figure 16, ranging between 0.44% and 1.1%. More importantly, neither panel reproduces the trend in WBF seen for the same color range in Figure 16, indicating that observational biases and incompleteness play a significant role in the observed WBF. As these two samples are pulled from two different distance ranges, some of the differences in the value of WBF as well as the observed trend in WBF as a function of \( r - z \) might be due to Galactic structure or age of the sampled pairs. Hence, even the maximum observed WBF of 1.1% in SLoWPoKES is likely a lower limit on the true WBF.

SLoWPoKES should be useful for studying the WBF as a function of Galactic height (Z), which can be taken as a proxy for age. Figure 18 shows the WBF versus Z, in \( r - z \) color bins. Again, to keep the observational biases similar across color bins, we selected distances ranges such that CPM companions with \( r - z \) colors within ±0.3 could be seen in SDSS; for example, in the first bin (\( r - z = 0.7-1.0 \)) all companions with \( r - z = 0.4-1.3 \) are expected to be seen. Note that this approach is biased toward equal-mass pairs, with high mass-ratio pairs never counted. Consequently, the WBF is lower than in Figures 16 and 17, with maximum at \( \sim 0.35\% \) for \( r - z = 1.3-1.9 \); the WBF peaks at around the same color range in all three figures. However, our method ensures that we are sensitive to all similar mass pairs across all of the distance bins. Figure 18 suggests that the WBF decreases with increasing Z. As this trend appears for both primary and secondary components for almost all spectral types (colors) probed, it is likely not an artifact of observational biases and is strong evidence for the time evolution of the WBF. Wide binaries are expected to be perturbed by inhomogeneities in the Galactic potential, giant molecular clouds, and other stars and, as a result, to dissipate over time. As pairs at larger Galactic heights are older as an ensemble, it is expected that a larger fraction of pairs at larger Z, which are older, have dissipated.

5. DISCUSSION

As discussed above, SIJ08 recently conducted a search of CPM pairs in the SDSS DR6 and found ~22,000 wide pairs, of all spectral types, with the probability of being a real binary of
\(\sim 67\%\) at angular separations of 5–30″. This search was done for all stars in the DR6 photometry with \(\mu \geq 15 \text{ mas yr}^{-1}\) and matched both components of proper motions within 5 mas yr\(^{-1}\). As SJ08 note, the sample is not very efficient for follow-up studies due to the likely large number of chance alignments (i.e., false positives). Our analysis in this paper indicates that their low proper-motion cutoff is chief culprit for the contamination; with a higher proper-motion cutoff similar to ours, the SJ08 sample could in principle be sifted of many false positives. SJ08 used a statistical subtraction to correct the ensemble for the chance alignments, which suffices for the purpose of their work. While SLoWPoKES is intended to serve a very different purpose—namely, to provide a “pure” catalog of high-fidelity systems suitable for targeted follow-up studies—it is useful to compare some of our tentative findings and interpretations with those of SJ08 based on their more complete sample.

For example, the upper limit of the WBF in SLoWPoKES (\(\sim 1.1\%\); e.g., Figure 17) is consistent with the 0.9% at \(Z = 500\) pc from SJ08. This suggests that, for specific color and distance ranges, the selection biases in SLoWPoKES may not significantly affect the ability of this sample to characterize the WBF among late-type stars at the widest separations. Kraus & Hillenbrand (2009) similarly find a WBF of at most a few percent for masses 0.012–2.5 \(M_{\odot}\) at separations of 500–5000 AU in their study of the Taurus and Upper Sco star-forming regions. Moreover, they find that the distribution of binary separations in their sample remains flat out to the limits of their survey (\(\sim 5000\) AU) for all but the lowest mass systems with \(M_{\text{tot}} < 0.3\) \(M_{\odot}\). At the same time, all of these studies fall well below the WBF (for \(s > 1000\) AU) of \(\sim 9.5\%\) determined for solar-type Hipparcos stars (Lépine & Bongiorno 2007) and the 10% suggested by Longhitano & Binggeli (2010) for G5 or later stars in the solar neighborhood from SDSS DR6. Similarly, integrating the Gaussian distribution of DM91 suggests a WBF of \(\sim 9.2\%\) for 1000 AU \(\leq a \leq 0.5\) pc and \(\sim 0.64\%\) for \(a > 0.5\) pc for solar-type stars. Even accounting for the lower multiplicity seen in M dwarfs by FM92, this is much higher than our result. Similarly, the decrease of WBF with increasing \(Z\) in SLoWPoKES (Figure 18) was noted by SJ08 in their sample as well.

SJ08 had noted that the companions of red stars, unlike the bluer stars, seemed to be drawn randomly from the field stellar mass function. We do not find evidence to support that finding, but our results are probably limited by the faintness limit. With respect to the distribution of mass ratios (Figure 15), Reid & Gizis (1997) observed, as we do, a strong skew toward equal-mass pairs with a peak at \(q \geq 0.8\) in their 8 pc sample. However, FM92 found a flat distribution in their sample of dMs within 20 pc. More recently, Kraus & Hillenbrand (2009) also found that a bias toward equal-masses in their study of 0.012–2.5 \(M_{\odot}\) wide binaries (500–5000 AU separations) in the Taurus and Upper-Sco star-forming regions. In contrast, solar-type field stars exhibit a definitive peak at \(q \sim 0.3\) (DM91; Halbwachs et al. 2003). With the field mass function peaking around \(\sim M4\) spectral type (B10), a peak at \(q \sim 0.3\) for solar-type stars and at \(q \sim 1\) for M dwarfs are both consistent with the secondary mass function being a subset of the field mass function.

With some systems wider than \(\sim 1\) pc, SLoWPoKES provides the largest sample to date of the widest CPM doubles. However, these are not the only systems known at this separation regime; Table 5 lists 84 systems with projected physical separations greater 0.05 pc (\(\sim 10^4\) AU). This sample is biased toward pairs with at least one Hipparcos star, with about half of them from the CPM catalog of Lépine & Bongiorno (2007). This is not a comprehensive list of all known wide binaries; for example, the existing CPM catalogs of Luyten (1979, 1988), Bahcall & Soneira (1981), Halbwachs (1986), and Chanamé & Gould (2004) likely contain at least a few hundred pairs in this separation range. Even among the BPs, where most known pairs have separations smaller than 15 AU, two very-wide systems—the 5100 AU 2MASS J0126555−502239 (Artigau et al. 2007, 2009) and the 6700 AU 2MASS J12583501+40130 (Radigan et al. 2009)—have already been identified. All of these wide pairs and other low-Be VLM systems from the VLM binary archive\(^\text{13}\) are shown, when relevant, in Figures 13 and 20 for comparison with the SLoWPoKES distribution. Most of the CPM doubles in Table 5 were found in nearby, high proper motion catalogs. Due to the depth of SDSS imaging and the larger distances probed, SLoWPoKES significantly increases CPM pairs with large physical separations. For the same reason, the existence of pairs as wide as \(\geq 1\) pc in SLoWPoKES (Figure 13) should perhaps not be surprising.

At the same time, the question of how such wide, weakly bound systems survive over time and how they form in the first place is a very interesting issue that the SLoWPoKES sample will be well suited to address. The initial distribution of separations of wide binaries is not static over time but is modified by interactions with other stars, molecular clouds, and variations in the Galactic potential. Over the lifetime of a binary, the small and dissipative, but numerous, encounters that it undergoes with other stars are much more efficient at disrupting the system than single catastrophic encounters, which are very rare (Weinberg et al. 1987). Using the Fokker–Planck coefficients to describe the advection and diffusion of the orbital BE due to such small encounters over time, Weinberg et al. (1987) calculated that the average lifetime of a binary is given by

\[
t_s(a) \approx 18 \text{ Gyr} \left(\frac{n_s}{0.05 \text{ pc}^{-3}}\right)^{-1} \left(\frac{M_{\bullet}}{M_{\odot}}\right)^{-2} \left(\frac{M_{\text{tot}}}{M_{\odot}}\right)^{-1} \left(\frac{V_{\text{rel}}}{20 \text{ km s}^{-1}}\right) \left(\frac{a}{0.1 \text{pc}}\right)^{-1} (\ln \Lambda)^{-1},
\]

where \(n_s\) and \(M_{\bullet}\) are the number density and average mass of the perturbers, \(V_{\text{rel}}\) is the relative velocity between the binary system and the perturber, \(M_{\text{tot}}\) and \(a\) are the total mass and semi-major axis of the binary, and \(\Lambda\) is the Coulomb logarithm. Using the average Galactic disk mass density of 0.11 M\(\odot\) pc\(^{-3}\), an average perturber mass of 0.7 M\(\odot\), \(V_{\text{rel}} = 20\) km s\(^{-1}\), and \(\Lambda = 1\) (see also Close et al. 2007), we can rewrite the above equation as

\[
a \approx 1.212 \frac{M_{\text{tot}}}{t_s}\text{ pc}
\]

for \(M\) in M\(\odot\) and \(t_s\) in Gyr. This describes, statistically, the widest binary that is surviving at a given age. For example, at 1 Gyr 1 M\(\odot\) binaries as wide as 1.2 pc are likely to be bound, but by 10 Gyr, all systems wider than 0.12 pc will likely have been disrupted. Hence, the combination of Equation (18) and the distribution of binary separation at birth may describe the current binary population of the Galaxy. Figure 19 shows the physical separation versus total mass for SLoWPoKES pairs and the other known wide CPM pairs from the literature (see above); characteristic disruption timescales of 1, 2, and 10 Gyr for Equation (18) are over-plotted. From this, most SLoWPoKES

\(^{13}\) http://vlmbinaries.org
pairs can be expected not to dissipate before 1–2 Gyr, with approximately half of the population stable enough to last longer than 10 Gyr.

Interestingly, the widest SLoWPoKES systems appear to violate previously proposed empirical limits on maximum separations (dashed and dotted green lines in Figure 19; Reid et al. 2001a; Burgasser et al. 2003). Similarly, previously proposed empirical limits based on binding energies, $10^{41}$ erg for stellar and $10^{42.5}$ erg very low-mass regimes (Close et al. 2003, 2007; Burgasser et al. 2007), are also violated by SLoWPoKES pairs as well as other known CPM doubles. This is clearly evident in Figure 20, where we see that the most weakly bound SLoWPoKES systems have binding energies of only $\sim 10^{39}$ erg, comparable to the BE of Neptune about the Sun.14 Faherty et al. (2010) have also noted such transgressions in their sample of wide, very low-mass CPM double.

It seems that previously proposed empirical limits were too restrictive as the widest and the VLM pairs that clearly violate them were not available at the time. We can follow the approach of Reid et al. (2001a) and Burgasser et al. (2003) to fit a log normal that forms an empirical envelope around the SLoWPoKES pairs (Figure 19, green solid lines). Although this new fit is not likely to define the absolute stability limit for wide pairs, the fit is notable for how well it describes the envelope. We note that Equation (18) as well as most previously proposed empirical limits have used the total mass of the system and do not take into account the mass ratio of the components. This creates a degeneracy as the BE is dependent on the product of masses. Instead, it may be more physical to consider the wide, low-mass pairs as a function of the product of the masses of their components—i.e., the BE of the system. Figure 20 (left panels) present the $a$ and BE versus $M_1 \times M_2$ for the SLoWPoKES pairs. That the distribution of $a$ as a function of either sum or product of the component masses appears qualitatively similar is a manifestation of the fact that SLoWPoKES pairs tend to be equal mass (see Figure 14); for high-$q$ pairs there would be a more noticeable difference in these distributions.

Perhaps, of particular interest to the issue of wide binary stability, formation, and evolution is our observation of a distinctly bimodal distribution of binary separations, seen in Figure 20, which equates to a bimodal distribution of system binding energies as well. The break in the bimodal distribution at BE $\sim 10^{41.3}$ erg is remarkable for how well it agrees with the previously proposed empirical limit in BE for stellar binaries to be dynamically stable (Close et al. 2003, 2007; Burgasser et al. 2007). In addition, as can be seen in Figure 19, the 10 Gyr disruption “isochrone” of Weinberg et al. (1987) nicely traces the observed break in the distribution over nearly a full decade of total system mass. The trend of the break in the distribution with mass is also important because it makes it very unlikely that the bimodality is the result of observational biases in the SLoWPoKES sample. Similarly, there is no effect of the heliocentric distances or the Galactic heights (Figure 20, right panels) of the pairs on the physical separation or BE. As discussed earlier, the principal observational biases in SLoWPoKES affect only (1) the smallest separations most severely because of the strict cutoff at 7″ and (2) to a lesser extent the largest separations as we limit our search to 180″.

Thus, the bimodal distribution of binary separations suggests two populations of binaries, perhaps representing (1) systems

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14 SLoWPoKES systems are much more likely than the Sun–Neptune system to be disrupted by passing stars due to the binaries’ larger cross-section of interaction.
of stars that formed with sufficient BE to survive for the age of the Galaxy and (2) relatively young systems that formed within the past 1–2 Gyr but that will likely not survive much longer.

Figure 21 shows that the dispersion in the tangential velocity is indeed smaller for the “young” population, relative to the “old” population that has had more time to get kinematically heated.\(^\text{15}\) Quantitatively, the median absolute deviations for the two populations are 46.1 km s\(^{-1}\) and 32.7 km s\(^{-1}\). Curiously, the separations of the binaries in the second group (up to 1 pc; Figure 13, right panel) is larger than the typical sizes of prestellar cores (∼0.35 pc), suggesting that these systems may not have formed via the “standard” collapse and fragmentation of individual cloud cores.

Recent N-body simulations predict bimodal distributions of wide binary separations and may provide important theoretical insight for understanding the formation and evolution of wide pairs. Although the specific simulations predict bimodality on different scales than seen in our observations, some of the resulting predicted physical properties of pairs are similar to those found in SLOWPOKES. For example, Kouwenhoven et al. \((2010)\) find that the distribution of binary semi-major axes will in general be bimodal on the size scale of star-forming clusters (i.e., ∼0.1 pc), with a group of tight systems formed primarily via dynamical interactions near the cluster center and a second group of loosely bound systems formed through random pairing during the slow dissolution of the cluster. In their simulations, up to ∼30% of the resulting binaries will be in the latter “dissolution peak,” with separations of ∼0.1 pc. They predict that the mass ratios of these systems will reflect random pairing. Alternatively, Jiang & Tremaine \((2009)\) have found that a bimodal distribution arises due to the slow (as opposed to instantaneous), diffusive disruption of systems on timescales of several Gyr, and that the details of the resulting bimodality are mostly independent of the initial distribution of separations arising from the formation process. Their simulations predict that the bimodal break in the distribution occurs at a few Jacobi radii, which corresponds to a separation of ∼2 pc for a total system of 1 M\(_{\odot}\). Perhaps the dissipative dissipation mechanism is more efficient than predicted by Jiang & Tremaine \((2009)\), or perhaps the “tidal-tail peak” of dissolving systems has not yet been observed. In the future, the SLOWPOKES catalog can be extended to even larger physical separations to probe this possibility.

6. CONCLUSIONS

We have created the SLOWPOKES catalog, comprising 1342 CPM binary pairs, identified through statistical matching of angular separation, photometric distances, and proper motion components. We have sifted the sample of chance alignments using a Galactic model based on empirical observations of stellar spatial and kinematic distributions. With the objective that each pair can be confidently used to investigate various science questions regarding low-mass stars, we have adopted a very restrictive set of selection criteria. This approach clearly underestimates the number of binary systems. Moreover, the sample includes several biases, the most important of which are a lack of systems with physical separations smaller than ∼1000 AU (arising from a strict bias against angular separations smaller than 7′′), and the exclusion of certain types of higher-order multiples (e.g., triples) due to the strict photometric distance matching. However, as a consequence the catalog should contain very few false positives, making follow-up studies efficient.

We built a Monte Carlo-based six-dimensional Galactic model that is able to replicate the positional and kinematic properties of the stars in the Milky Way. In its current incarnation, we used it to calculate the number of stars within a certain spatial volume in the Galaxy and the likelihood that those stars have common kinematics (proper motions) by chance. One of the things this model underscores is how difficult it is to find two physically unassociated stars close together in space: along a typical SDSS LOS, there are expected to be only 0.52 chance alignments within 15′ and a minuscule 0.03 chance alignments if the volume is considered. The additional matching of proper motions gives each of the accepted SLOWPOKES binaries a very low probability of being a false positive.

Due to their intrinsic faintness and the resulting small numbers, binarity studies of low-mass stars have been limited in scope. However, with the advent of large-scale deep surveys, detailed and statistically significant studies of M (and L) dwarfs are being done. SLOWPOKES is now the largest sample to date of very wide, low-mass binaries. In particular, SLOWPOKES provides a large sample of systems with physical separations up to ∼1 pc that will be useful for putting firm constraints on the maximum size of physically associated systems. How the widest of these systems form, and how long they survive, is in particular an interesting question that SLOWPOKES is well suited to address. While numerical calculations suggest that approximately half of SLOWPOKES systems can remain bound for at least 10 Gyr \((\text{Weinberg et al. } 1987)\), previously proposed empirical limits are violated by many SLOWPOKES systems.

Indeed, the distribution of SLOWPOKES binary separations is distinctly bimodal, suggesting the presence of (1) a population of tightly bound systems formed with sufficient BE to remain intact for the age of the Galaxy and (2) a population of weakly bound systems that recently formed and that are unlikely to survive past 1–2 Gyr. Recent N-body simulations \((e.g., \text{Kouwenhoven et al. } 2010; \text{Jiang & Tremaine } 2009)\) in fact predict a bimodal distribution of binary separations on the scales probed by the widest SLOWPOKES systems.

We observed a WBF of ∼1.1%, which is likely to be a minimum given the nature of our sample. While this is consistent with the results from SJ08 who found 0.9% of stars at Z = 500 pc had

\(^{15}\) One might expect lower mass stars to move with higher velocities relative to higher mass stars if equipartition of kinetic energy holds. However, observations have shown that the equipartition of kinetic energy does not hold and that kinematic heating is purely a function of age for field stars. Unlike in clusters, individual stars can be thought of as small, point masses when compared to the much larger Galactic potential or the giant molecular clouds that cause dynamical heating \((\text{Bochanski et al. } 2007; \text{Schmidt et al. } 2010)\).
wide companions, it is significantly lower than the solar-type *Hipparcos* stars having wide companions (Lépine & Bongiorno 2007). While the incompleteness involved in the searching for companions at large distances probably causes some of this, both this study and SJ108 saw a decrease in binary fraction as a function of Galactic height, evidence of dynamical destruction of older systems. Hence, the wide binary fraction around our initial sample of low-mass stars might actually be significantly lower than the *Hipparcos* stars.

Besides the importance of SLoWPoKES for constraining models of formation and evolution of binary stars, SLoWPoKES systems are coeval laboratories—sharing an identical formation and evolutionary history without affecting each other—making them ideal for measuring and calibrating empirical relationships between rotation, activity, metallicity, age, etc. We have started programs to test and calibrate the age–activity relationship measured by West et al. (2008) and to explore whether gyrochronology (Barnes 2003, 2007) can be applied in the fully convective regime. As coeval laboratories allow for the removal of one or more of the three fundamental parameters (mass, age, and metallicity), much more science can be done with a large sample of such systems.

Future astrometric missions, such as the Space Interferometry Mission (SIM), should provide exquisite astrometry, perhaps enabling us to trace the orbits of some of the SLoWPoKES systems. While tracing orbits with periods $\geq 10^{4–6}$ yr sounds ambitious, with SIM’s microarcsecond level (or better) astrometry (Unwin et al. 2008) combined with SDSS, DSS, and/or other epochs, it is not unrealistic. Similarly, the “identical” twins in SLoWPoKES would be ideal sites to probe for the presence and differences in the formation mechanism of planets. As each identical twin in a CPM double provides similar environment for the formation and evolution of planets, these systems can be ideal sites to study planetary statistics. Due to their large separations the stars are not expected to influence each others evolution but have similar mass, age, and metallicity, as we noted earlier in Section 1.

The SLoWPoKES catalog, as the name suggests, only contains systems for which kinematic information is available. We can, however, use the results from the Galactic model to identify pairs at the small separations ($\Delta \theta < 7\arcsec$), albeit with a larger uncertainty, for which no kinematic information is available. Similarly companions which are fainter than $r = 20$ can also be identified as the SDSS photometry is complete to $r = 22.5$. The latter systems are likely to be skewed toward late-type dMs and unequal-mass pairs. A follow-up paper will study such systems and will add a large proportion of wide systems.

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