UNDERSTANDING TRENDS ASSOCIATED WITH CLOUDS IN IRRADIATED EXOPLANETS

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<th>Citation</th>
<th>Heng, Kevin, and Brice-Olivier Demory. “UNDERSTANDING TRENDS ASSOCIATED WITH CLOUDS IN IRRADIATED EXOPLANETS.” The Astrophysical Journal 777, no. 2 (October 18, 2013): 100. © 2013 American Astronomical Society.</th>
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</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1088/0004-637x/777/2/100">http://dx.doi.org/10.1088/0004-637x/777/2/100</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>Institute of Physics/American Astronomical Society</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Accessed</td>
<td>Thu Dec 13 08:30:49 EST 2018</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/93752">http://hdl.handle.net/1721.1/93752</a></td>
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UNDERSTANDING TRENDS ASSOCIATED WITH CLOUDS IN IRRADIATED EXOPLANETS

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Received 2013 May 13; accepted 2013 August 29; published 2013 October 18

ABSTRACT

Unlike previously explored relationships between the properties of hot Jovian atmospheres, the geometric albedo and the incident stellar flux do not exhibit a clear correlation, as revealed by our re-analysis of Q0–Q14 Kepler data. If the albedo is primarily associated with the presence of clouds in these irradiated atmospheres, a holistic modeling approach needs to relate the following properties: the strength of stellar irradiation (and hence the strength and depth of atmospheric circulation), the geometric albedo (which controls both the fraction of starlight absorbed and the pressure level at which it is predominantly absorbed), and the properties of the embedded cloud particles (which determine the albedo). The anticipated diversity in cloud properties renders any correlation between the geometric albedo and the stellar flux weak and characterized by considerable scatter. In the limit of vertically uniform distributions of scatterers and absorbers, we use an analytical model and scaling relations to relate the temperature–pressure profile of an irradiated atmosphere and the photon deposition layer and to estimate whether a cloud particle will be lofted by atmospheric circulation. We derive an analytical formula for computing the albedo spectrum in terms of the cloud properties, which we compare to the measured albedo spectrum of HD 189733b by Evans et al. Furthermore, we show that whether an optical phase curve is flat or sinusoidal depends on whether the particles are small or large as defined by the Knudsen number. This may be an explanation for why Kepler-7b exhibits evidence for the longitudinal variation in abundance of condensates, while Kepler-12b shows no evidence for the presence of condensates despite the incident stellar flux being similar for both exoplanets. We include an “observer’s cookbook” for deciphering various scenarios associated with the optical phase curve, the peak offset of the infrared phase curve, and the geometric albedo.

Key word: planets and satellites: atmospheres

Online-only material: color figures

1. INTRODUCTION

1.1. Observational Motivation

Built primarily to study the occurrence of Earth-like exoplanets in our local cosmic neighborhood, the actual scientific scope of the Kepler Space Telescope has expanded to include the study of hot Jupiters and their atmospheres. The detection of transits and eclipses in the broad optical channel of Kepler has enabled the geometric albedos of about a dozen hot Jupiters to be measured. In Figure 1, we show the geometric albedo ($A_{g}$) versus the incident stellar flux at the substellar point ($F_{0}$) for a sample of hot Jupiters, for which we have performed an improved analysis of Q0–Q14 data (see Section 2.1 for details). Beginning our discussion requires defining a few temperatures (Cowan & Agol 2011b),

\begin{align*}
T_{0} & \equiv T_{*} \left( \frac{R_{*}}{a} \right)^{1/2}, \\
T_{eq} & \equiv T_{*} \left( \frac{R_{*} f_{\text{dist}}}{a} \right)^{1/2} \left( 1 - A_{B} \right)^{1/4}, \\
T_{eq,0} & \equiv T_{*} \left( \frac{R_{*}}{2a} \right)^{1/2},
\end{align*}

(1)

with $T_{*}$ denoting the stellar effective temperature, $R_{*}$ the stellar radius, and $a$ the orbital semi-major axis of the exoplanet. The quantity $T_{0}$ may be regarded as the “irradiation temperature” at the substellar point, such that $F_{0} = \sigma_{SB} T_{0}^{4} = 4 \sigma_{SB} T_{eq,0}^{4}$ is the incident stellar flux (or “stellar constant”), with $\sigma_{SB}$ denoting the Stefan–Boltzmann constant. The quantity $T_{eq}$ is our conventional definition of the “equilibrium temperature” and includes a dimensionless factor $f_{\text{dist}}$ that describes the efficiency of heat redistribution from the dayside to the nightside hemisphere: we have $f_{\text{dist}} = 2/3$ and 1/2 for no and full redistribution, respectively (Hansen 2008). The quantity $T_{eq,0}$ is the equilibrium temperature assuming full redistribution and a vanishing albedo. Strictly speaking, the geometric albedo is defined at a specific wavelength $\lambda$ and at zero viewing angle. The spherical albedo ($A_{s}$) is the geometric albedo considered over all viewing angles (Russell 1916; Marley et al. 1999; Sudarsky et al. 2000; Seager 2010; Madhusudhan & Burrows 2012). For a Lambertian sphere (isotropic scattering), we have $A_{s} = 2 A_{g}/3$. When integrated over all wavelengths, we obtain the Bond albedo ($A_{B}$). Since we are dealing with observations integrated over a broad optical bandpass, we assume $A_{B} = 3 A_{s}/2$; the consideration of more sophisticated scattering behavior will introduce an order-of-unity correction factor to this relation.

For the range of $T_{eq,0}$ values listed in Table 1, we estimate that the hot Jupiters examined in Figure 1 radiate mostly at wavelengths of about 1–2 $\mu$m (using Wien’s law). Nevertheless, since the Kepler bandpass extends from $\lambda_{1} \approx 0.4 \mu$m to $\lambda_{2} \approx 0.9 \mu$m (Koch et al. 2010), it is instructive to estimate the fraction of thermal flux from the exoplanet radiated in this bandpass,

\begin{equation}
\frac{f_{\text{thermal}}}{\sigma_{SB} T_{eq}^{4}} = \frac{\pi \int_{\lambda_{1}}^{\lambda_{2}} B_{\lambda}(T_{eq}) d\lambda}{\sigma_{SB} T_{eq}^{4}},
\end{equation}

(2)

by approximating the spectral energy distribution of a hot Jupiter to be a blackbody function. In Figure 1, we plot $f_{\text{thermal}}$ as a function of $F_{0}$ for both $f_{\text{dist}} = 2/3$ and 1/2. It is apparent that $f_{\text{thermal}}$ may not be small and generally increases with $F_{0}$,
implying that the measured geometric albedo \( A_g \) (Seager 2010),

\[
A_g = \frac{F_{p,\oplus}}{F_{\ast,\oplus}} \left( \frac{a}{R_p} \right)^2,
\]

may be contaminated by thermal emission “leaking” into the Kepler bandpass, causing \( A_g \) to be overestimated. The quantities \( F_{p,\oplus} \) and \( F_{\ast,\oplus} \) are the fluxes from the star and the exoplanet, respectively, received at Earth, while the radius of the exoplanet is given by \( R_p \). One may approximately correct for the contamination by thermal emission by considering the following equation:

\[
A_g = \left[ \frac{F_{p,\oplus}}{F_{\ast,\oplus}} - \frac{\pi f_{\text{thermal}}^{\text{obs}} B_{\lambda}(T_{\text{eq}}) d\lambda}{F_0} \left( \frac{R_p}{R_\ast} \right)^2 \left( \frac{a}{R_p} \right)^2 \right] = A_{g,\text{obs}} - \frac{\pi f_{\text{thermal}}^{\text{obs}} B_{\lambda}(T_{\text{eq}}) d\lambda}{F_0} \left( \frac{a}{R_\ast} \right)^2,
\]

where \( A_{g,\text{obs}} \) is the measured value of the geometric albedo obtained by applying Equation (3). Since \( T_{\text{eq}} \) depends on \( A_g \), the preceding expression is an implicit equation for the geometric albedo, which may be solved to obtain the “de-contaminated” \( A_g \), also shown in Figure 1. The small and large open circles represent the corrected geometric albedos for contamination by thermal emission (see the text) assuming no and full redistribution, respectively.

(A color version of this figure is available in the online journal.)

Table 1

<table>
<thead>
<tr>
<th>Object Name</th>
<th>( A_g )</th>
<th>( T_{\text{eq,0}} )</th>
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<tbody>
<tr>
<td>TrES-2b</td>
<td>0.015 ± 0.003</td>
<td>1444 ± 13</td>
</tr>
<tr>
<td>HAT-P-7b</td>
<td>0.225 ± 0.004</td>
<td>2139 ± 27</td>
</tr>
<tr>
<td>Kepler-5b</td>
<td>0.134 ± 0.021</td>
<td>1572 ± 17</td>
</tr>
<tr>
<td>Kepler-6b</td>
<td>0.091 ± 0.021</td>
<td>1451 ± 16</td>
</tr>
<tr>
<td>Kepler-7b</td>
<td>0.352 ± 0.023</td>
<td>1586 ± 13</td>
</tr>
<tr>
<td>Kepler-8b</td>
<td>0.051 ± 0.029</td>
<td>1638 ± 40</td>
</tr>
<tr>
<td>Kepler-12b</td>
<td>0.078 ± 0.019</td>
<td>1477 ± 26</td>
</tr>
<tr>
<td>Kepler-14b</td>
<td>0.012 ± 0.023</td>
<td>1573 ± 26</td>
</tr>
<tr>
<td>Kepler-15b</td>
<td>0.078 ± 0.044</td>
<td>1225 ± 31</td>
</tr>
<tr>
<td>Kepler-17b</td>
<td>0.106 ± 0.011</td>
<td>1655 ± 40</td>
</tr>
<tr>
<td>Kepler-41b</td>
<td>0.135 ± 0.014</td>
<td>1745 ± 43</td>
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</table>

contamination by thermal emission assuming full redistribution \( (f_{\text{dist}} = 1/2) \), the \( A_g \) values do not change much (and the Spearman rank coefficient remains, to the first significant figure, unchanged).

2. When the correction is performed assuming no redistribution \( (f_{\text{dist}} = 2/3) \), the trend flattens as expected (with a Spearman rank coefficient of \(-0.1\)). Three of the data points are consistent with being zero. The key point is that the correlation between \( A_g \) and \( F_0 \) can only weaken, and not strengthen, when heat redistribution is taken into account.

Our conclusion is that there exists no clear correlation between \( A_g \) and \( F_0 \). Values of \( A_g \approx 0.1 \) may be consistent with Rayleigh scattering caused by hydrogen molecules alone (Sudarsky et al. 2000), without the need for the presence of clouds or condensates. The high geometric albedo associated with Kepler-7b \( (A_g \approx 0.35) \) may require an explanation that includes the effects of clouds or condensates (Demory et al. 2011, 2013).

1.2. Theoretical Motivation

Unlike other previously examined relationships between various properties of hot Jupiters \( (\text{e.g.}, \text{radius and heat redistribution versus } T_{\text{eq}}; \text{e.g., } \text{Cowan} \& \text{Agol} 2011 \text{b}; \text{Demory} \& \text{Seager} 2011; \text{Laughlin} \text{et al.} 2011; \text{Perna} \text{et al.} 2012) \), there is no clear trend of \( A_g \) with the incident stellar flux. One of the goals of the present study is to suggest that the absence of a clear trend is caused by a combination of opacity effects, possibly due to the presence of condensates or clouds, and atmospheric circulation, the latter of which is often ignored in spectral analyses of hot Jupiters. The study of clouds or hazes is emerging as a major theme in the observations of hot Jupiters \( (\text{e.g., } \text{Lecavelier des Etangs et al.} 2008; \text{Pont et al.} 2008; \text{Sing et al.} 2011; \text{Gibson et al.} 2012) \) and directly imaged exoplanets \( (\text{e.g., } \text{Barman et al.} 2011; \text{Madhusudhan et al.} 2011; \text{Marley et al.} 2012; \text{Lee et al.} 2013) \) and has long been an obstacle plaguing advances in the understanding of brown dwarfs \( (\text{e.g., } \text{Saumon} \& \text{Marley} 2008; \text{Artigau et al.} 2009; \text{Burrows et al.} 2011; \text{Helling et al.} 2011; \text{Buenzli et al.} 2012) \).

On the theoretical front, several trends are now understood.

1. The strength and depth of atmospheric circulation are intimately tied to the intensity of stellar irradiation \( \text{(Perna et al.} 2012) \). An “eddy diffusion coefficient” \( (K_{zz}) \) is often used to mimic this behavior, but ultimately the relationship between atmospheric circulation and stellar flux can—and should—be calculated from first principles using global, three-dimensional (3D) simulations.
employ a Markov Chain Monte Carlo (MCMC) framework to characterize the posterior distribution of \( A_p \). MCMC is a Bayesian inference method based on stochastic simulations that samples the posterior probability distributions of adjusted parameters for a given model. Our MCMC implementation (described in, e.g., Gillon et al. 2012) uses the Gibbs sampler and the Metropolis–Hastings algorithm to estimate the posterior distribution function of all jump parameters. Our nominal model is based on a star and a transiting planet on a Keplerian orbit about their center of mass.

Input data provided to the MCMC for each system consist of the Q0–Q14 Kepler photometry and the spectroscopic stellar parameters (effective temperature \( T_e \), metallicity \([\text{Fe/H}]\), and \( \log g_\star \)) published in the references mentioned above. We correct for the photometric dilution induced by neighbor stellar sources using a quarter-dependent dilution factor based on the dilution values presented in the literature and on the contamination values reported in the FITS files headers (Bryson et al. 2010).

We divide the total light curve into segments of duration of about 24–48 hr and fit for each of them the smooth photometric variations due to stellar variability or instrumental systematic effects with a time-dependent quadratic polynomial. Baseline model coefficients are determined at each step of the MCMC for each light curve with the singular value decomposition method (Press et al. 1992). The resulting coefficients are then used to correct the raw photometric light curves.

We assumed a quadratic law for the stellar limb darkening (LD) and used \( c_1 = 2u_1 + u_2 \) and \( c_2 = u_1 - 2u_2 \) as jump parameters, where \( u_1 \) and \( u_2 \) are the quadratic coefficients. \( u_1 \) and \( u_2 \) were drawn from the theoretical tables of Claret & Bloemen (2011) for the stellar parameters obtained from the references above.

The MCMC has the following set of jump parameters: the planet/star flux ratio, the impact parameter \( b \), the transit duration from first to fourth contact, the time of minimum light \( t_0 \), the orbital period, the occultation depth, the two LD combinations \( c_1 \) and \( c_2 \), and the two parameters \( \sqrt{e} \cos \omega \) and \( \sqrt{e} \sin \omega \). The latter two parameters allow us to fit for the occultation phase and width. A uniform prior distribution is assumed for all jump parameters but \( c_1 \) and \( c_2 \), for which a normal prior distribution is used, based on theoretical tables and the stellar parameters used as input data to the MCMC fit.

We run two Markov chains of 100,000 steps each. The good mixing and convergence of the chains are assessed using the Gelman–Rubin statistic criterion (Gelman & Rubin 1992). We use the posterior distribution functions for the jump parameters \( F_p/\sigma_p \), \( a/R_\star \), and \( (R_p/R_\star)^2 \) obtained from the MCMC to derive the geometric albedo posteriors for each planet. We show the median of the posterior distribution function and its associated 1\(\sigma\) probability interval for \( A_p \) and \( T_{eq,0} \) in Table 2.

As already mentioned in Section 1, the non-parametric Spearman rank coefficient is 0.6 for the \( A_p \) versus \( F_0 \) data shown in Figure 1. Values of \( \pm 1 \) indicate a perfectly monotonically increasing or decreasing trend, while those close to zero indicate no correlation between the two quantities.

2.2. Models

Any model constructed to describe clouds in exoplanetary atmospheres has to include both their radiative and dynamical effects. Even when clouds contribute negligible mass to the atmosphere, they introduce strong radiative forcing to it in the form of the reflection of incident starlight (cooling) and the retention of reprocessed starlight in the infrared (heating).
On Earth, these two significant effects almost cancel, but not quite—it is this imperfect cancellation that is important for determining details of terrestrial weather and the climate system. Getting these details correct to high precision remains a formidable challenge (see, e.g., Pierrehumbert 2010). Clouds also interact with the atmospheric flow. To first order, we may neglect the dynamical effects of clouds on the flow (unless the dust-to-gas ratio is close to unity), but we may not neglect the dynamical effects of the flow on the clouds. Atmospheric circulation sets the background state of velocity, temperature, pressure, and density—and all of its dependent quantities—that allows us to determine if a cloud particle will remain at a certain location in the atmosphere.

To place the present study in context, we note that the 3D simulations of Parmentier et al. (2013) examine the effects of the atmospheric flow on embedded tracers that mimic the presence of cloud particles, but these tracers do not feed back on the flow in any way, either dynamically or radiatively. By contrast, the 3D simulations of Dobbs-Dixon & Agol (2012) include an extra opacity source to describe Rayleigh scattering, but do not consider the interaction between the flow and embedded particles.

It is practically impossible to include all of these effects in an analytical model in any rigorous manner. Instead, we subsume the absorptivity of the clouds into a general, infrared, absorption opacity that also describes the atmospheric gas. The geometric albedo is a consequence of the scattering and absorption properties of the atmosphere, as we will discuss in Section 2.2.3. While being mindful of this fact, we prescribe the geometric albedo as a free parameter when computing the analytical temperature–pressure profile. To describe the action of the atmospheric flow on the cloud particles, we use an analytical expression for the local terminal velocity of a given cloud particle and approximate the vertical velocity of the flow to be a fixed fraction of the local sound speed. Finally, we discuss the relationship between the properties of the cloud particles and the albedo.

2.2.1. Radiative Forcing

We use analytical models of the temperature–pressure profiles of hot Jupiters in the present study from Heng et al. (2012), where the cloud-free models of Guillot (2010) were generalized to include a non-zero albedo. Whether the albedo is due to Rayleigh scattering associated with molecules or scattering associated with condensates or dust grains is unspecified. These models solve the one-dimensional (1D) radiative transfer equation in the plane-parallel, two-stream approximation using the method of moments, while the incoming stellar radiation is approximated as a collimated beam. The free parameters involved are the irradiation temperature \(T_0\), the Bond albedo \(A_B\), the absorption opacity in the infrared \(\kappa_{IR}\), and the absorption opacity in the optical \(\kappa_O\). We set collision-induced absorption, associated with hydrogen molecules, to be the dominant opacity source at pressures of 0.1 bar and greater \((\epsilon = 51\) for a bottom pressure of 10 bar). There is an option to specify a purely absorbing cloud deck of intermediate width, which we ignore for the purpose of simplicity and clarity.

Starlight incident upon an atmosphere is predominantly absorbed at the following pressure level, known as the “photon deposition layer” (Heng et al. 2012),

\[
P_D = \frac{0.63 g}{\kappa_O} \left( \frac{1 - A_B}{1 + A_B} \right),
\]

where \(g\) denotes the surface gravity of the exoplanet. In a purely reflecting atmosphere \(A_B = 1\), there is no penetration of starlight \(P_D = 0\). Note that this unique relationship between the photon deposition layer and the Bond albedo only exists in the limit of uniform, vertical populations of scatterers and absorbers in the 1D model atmosphere. When the assumption of spatial uniformity is relaxed, this unique relationship is broken and it is now possible to specify cloud decks of varying spatial and optical thicknesses, located at different altitudes, that will produce the same Bond albedo. While being aware of this possibility, we do not explore this complexity for several reasons: we are interested in elucidating trends rather than making detailed predictions; to develop intuition, we confine ourselves to analytical models as much as possible and it has been previously shown that such a generalization breaks the analytical nature of the model for the temperature–pressure profile (Heng et al. 2012); and the atmospheric data associated with hot Jupiters are not (yet) of a high enough quality to warrant such sophisticated investigations (unlike, for example, in the case of brown dwarfs).

2.2.2. Effects of Atmospheric Flow on Condensates

In this sub-section, we use simple scaling relations to elucidate the relationship between vertical, atmospheric flow and its ability to loft condensates. The mean free path of molecules within an atmosphere is \(L = m/\rho \sigma_m\), where \(m\) is the molecular mass, \(\rho\) is the mass density, and \(\sigma_m \sim 10^{-15} \text{ cm}^2\) is the cross section for inter-molecular interactions. To describe the influence of the atmospheric flow on a particle embedded within it, we use the analytical formulae previously described in Li & Wang (2003) and Spiegel et al. (2009) (and references therein). The terminal velocity of the (spherical) particle is

\[
v_t = \frac{2C r_c^2 \rho g}{\rho v},
\]

where \(v\) is the local sound speed. Its internal mass density is \(\rho_i\), while its radius is \(r_c\). The kinematic viscosity of the atmospheric gas is \(v \approx L c_s\), with \(c_s\) being the local sound speed. The quantity \(C\) is a correction factor accounting for the enhancement of the terminal velocity in rarefied media,

\[
C = 1 + \frac{N_k}{1.256 + 0.4 \exp \left( - \frac{1.1}{N_k} \right)},
\]

and depends on the Knudsen number,

\[
N_k = \frac{L}{r_c} = \frac{k_B T}{P \sigma_m r_c} \sim 10 \left( \frac{T}{1500 \text{ K}} \right) \left( \frac{P}{0.1 \text{ bar}} \right) \left( \frac{r_c}{1 \mu\text{m}} \right)^{-1},
\]

which in turn depends on the pressure level \(P\) and local temperature \(T\) of the atmosphere considered. The Boltzmann constant is given by \(k_B\).

We define a quantity \(S \equiv v_c / v_t\), which describes whether a particle is likely to be lofted by atmospheric circulation. We approximate the vertical component of the velocity to be

\[
v_c = M_z c_s = M_z \left( \gamma k_B T / m \right)^{-1/2},
\]

where \(\gamma \approx 7/5\) is the adiabatic gas index. The vertical Mach number \(M_z\) is approximated to be constant, although
we fully anticipate that it generally has the functional form \( M_c = M_c(P, T, \gamma) \). It follows that

\[
S \approx \frac{9 M_c \gamma k_B T}{2 \mathcal{C} \rho c r_c^2 \sigma g}.
\]

(10)

When \( S \gg S_0 \), atmospheric circulation keeps the particles lofted. When \( S < S_0 \), the particles sink under the action of gravity. We expect \( S_0 \approx 1 \). When \( N_k \ll 1 \), we obtain

\[
S \approx 2.7 M_c \gamma P \rho c r_c g^{-1}
\]

\[
\sim 1 \left( \frac{M_c}{10^{-6} \, \text{g}} \right) \left( \frac{\gamma}{7/5} \right) \left( \frac{P}{0.1 \, \text{bar}} \right) \left( \frac{\rho_c}{1 \, \text{g cm}^{-3}} \right) \left( \frac{r_c}{1 \, \mu\text{m}} \right) \left( \frac{g}{10^3 \, \text{cm} \, \text{s}^{-2}} \right)^{-1}.
\]

(11)

We thus expect micron-sized particles to be lofted by atmospheric circulation.

To obtain the correct order-of-magnitude value for \( M_c \), we use the 3D simulations of atmospheric circulation of hot Jupiters by Heng et al. (2012). Further details are described in Heng et al. (2011a, 2011b). Specifically, we employ their Models C (“cold”), W (“warm”), and H (“hot”), which have \( T_{eq,0} \approx 545, 969, \) and 1723 K, respectively. For each model, we also examine atmospheres with \( (T_0 = 2.0) \) and without \( (T_0 = 0.5) \) temperature inversions. In Figure 2, we show the rms vertical velocity \( \langle v_{z,\text{rms}} \rangle \) and vertical Mach number \( \langle M_{z,\text{rms}} \rangle \) as functions of the vertical pressure \( P \). We see that at \( P \approx 0.1 \) bar, we have \( \langle M_{z,\text{rms}} \rangle \sim 10^{-6} \) with a somewhat weak dependence on the equilibrium temperature and the absolute pressure or pressure of a temperature inversion. A caveat is that the simulations of Perna et al. (2012) essentially assume \( A_s = 0 \) and do not include a treatment of scattering. Secondly, like most other published general circulation models of hot Jupiters, they solve the primitive equations of meteorology, which assumes hydrostatic balance. Hydrostatic balance does not preclude \( v_z \neq 0 \), since the vertical velocity is assumed to be sub-dominant only in the vertical component of the momentum equation, but it does imply that any simulated value is probably a lower limit to the one obtained by a fully non-hydrostatic simulation. It is with these caveats that we adopt \( M_c = 10^{-6} \) at \( P \approx 0.1 \) bar.

Since we are interpreting \( v_z \) to be the rms vertical velocity in Equations (9)–(11), \( \min \{1, S\} \) may be interpreted as the relative abundance of embedded cloud particles summed horizontally over an entire atmospheric layer.

### 2.2.3. Relationship between Condensates and Albedo

The scattering and absorption properties of cloud particles are mainly described by two quantities. The first quantity is the “single scattering albedo” \( \omega_0 = \sigma_{\text{scat}} / (\sigma_{\text{scat}} + \sigma_{\text{abs}}) \), where \( \sigma_{\text{scat}} \) and \( \sigma_{\text{abs}} \) are the scattering and absorption cross sections, respectively. The second quantity is the “asymmetry parameter” \( (g_0) \), which is the mean cosine of the relative angle between the initial and scattered directions of a photon incident upon the particle. It is the sole parameter in the Henyey–Greenstein scattering phase function, which determines the spatial distribution of scattered light (Henyey & Greenstein 1941). Isotropic scattering occurs for \( g_0 = 0 \). Particles described by \( g_0 = 1 \) produce predominantly forward scattering, while those described by \( g_0 = -1 \) produce mostly backward scattering. Generally, \( \omega_0 \) and \( g_0 \) depend on both the particle radius \( (r_c) \) and the wavelength \( (\lambda) \). (See also Madhusudhan & Burrows 2012.)

Asymptotically, we expect that (e.g., Section 5.4 of Pierrehumbert 2010)

\[
\begin{align*}
g_0 & \to 0, \quad r_c / \lambda \to 0, \\
g_0 & \to G_0, \quad r_c / \lambda \to \infty
\end{align*}
\]

(12)

where \( G_0 > 0 \) is a constant. In other words, small particles scatter isotropically, while large ones tend to produce more forward scattering. Whether a particle is “small” or “large” depends on the wavelength \( \lambda \) considered—the relevant quantity is \( 2 \pi r_c / \lambda \), rather than \( r_c \). To compute the precise relationship between \( r_c \), \( g_0 \), and \( \lambda \) requires the full machinery of Mie scattering (Draine & Lee 1984; Laor & Draine 1993; Draine 2003). Generally, the scattering properties of a particle are determined mainly by its size and to a lesser extent by its composition, implying that a diagnosis of the cloud composition is a challenging and degenerate task (Pierrehumbert 2010).

In an atmosphere populated by cloud particles, the geometric albedo \( A_g \) is determined by both \( \omega_0 \) and \( g_0 \). The simplest model one can construct of the function \( A_g(\omega_0, g_0) \) involves solving the two-stream Schwarzschild equations with scattering, as described in Section 5 of Pierrehumbert (2010). Here, we generalize the formula to allow for \( g_0 \neq 0 \) and a finite blackbody efficiency of the cloud particle. Denoting the incoming and outgoing fluxes by \( F_\uparrow \) and \( F_\downarrow \), respectively, we define

\[
F_\pm \equiv F_\uparrow \pm F_\downarrow.
\]

(13)

Subtracting and adding the pair of Schwarzschild equations for \( F_\uparrow \) and \( F_\downarrow \) yields

\[
\frac{dF_\downarrow}{d\tau} = -2 \varepsilon_1 (1 - \omega_0) F_\downarrow + 4 \varepsilon_2 (1 - \omega_0) \pi B + \omega_0 \xi_0 \tan \tau \left( \frac{\tau - \tau_0}{\cos \xi_0} \right) \exp \left( \frac{\tau - \tau_0}{\cos \xi_0} \right),
\]

\[
\frac{dF_\uparrow}{d\tau} = -2 \varepsilon_1 (1 - g_0 \omega_0) F_\uparrow - 2 \varepsilon_1 \omega_0 g_0 \cos \xi_0 \xi_0 F_\downarrow \exp \left( \frac{\tau - \tau_0}{\cos \xi_0} \right).
\]

(14)

Here, \( \tau \) denotes the optical depth measured from some reference depth in the model atmosphere, while \( \tau_0 \) is the optical depth...
associated with the distance from this reference depth to the top of the model atmosphere. The blackbody flux is given by $\pi B = \frac{\sigma_\text{SB} T^4}{\cos \xi_0}$. The zenith angle $\xi_0$ is the angle between the incident stellar flux and the vertical axis; we allow for $\cos \xi_0 \neq 1$ in our derivation but later set it to be unity in our calculations. The dimensionless quantities $\epsilon_1$ and $\epsilon_2$ are closure relations that depend on assumptions about the angular distribution of incoming versus outgoing radiation (Pierrehumbert 2010). Physically, the terms involving $F_0$ represent “direct beam” emission from the star, while those involving $F_{\pm}$ represent the diffuse emission.

Taking the derivative of the second equation in (14) and eliminating $F_-$ using the first equation yields a second-order ordinary differential equation for $F_+$,

$$\frac{d^2 F_+}{d\tau^2} = K^2 F_+ - 8\epsilon_1\epsilon_2(1 - g_0)(1 - g_0\omega_0)\pi B \cdot \left[ -2\epsilon_1\omega_0 F_0 \left[ 1 + g_0(1 - \omega_0) \right] \exp \left( \frac{\tau - \tau_0}{\cos \xi_0} \right) \right] + \frac{F_{\pm} \exp [\pm K (\tau - \tau_0)]}{\pm K (\tau - \tau_0)}.$$  

(15)

where $K^2 \equiv 4\epsilon_1\epsilon_2(1 - g_0\omega_0)(1 - \omega_0)$. The homogeneous solution for $F_+$ takes the form

$$F_{+,h} = f_{\pm} \exp \left[ \pm K (\tau - \tau_0) \right].$$  

(16)

For a deep atmosphere, we expect the homogeneous solution to not diverge when $\tau \rightarrow -\infty$, which compels us to set $f_{\pm} = 0$. The particular solution takes the form

$$F_{+,p} = f_{p,1} \exp \left( \frac{\tau - \tau_0}{\cos \xi_0} \right) + f_{p,2}. $$  

(17)

The constants $f_{p,1}$ and $f_{p,2}$ are determined by substituting Equation (17) into Equation (15) and matching the coefficients found on both sides of the equation. The full solution ($F_{+,h} + F_{+,p}$) then becomes

$$F_+ = f_{+,h} \exp \left[ K (\tau - \tau_0) \right] + f_{p,1} \exp \left( \frac{\tau - \tau_0}{\cos \xi_0} \right) + f_{p,2}. $$  

(18)

All that remains is to determine the constant $f_{+,h}$, which may be accomplished by substituting the full solution into the second equation in (14), which yields an expression for $F_-$ and hence $F_+ - F_- = 2F_1$. Applying the boundary condition of $F_1 = 0$ when $\tau = \tau_0$ produces an expression for $f_{+,h}$.

Returning to our expression for $F_+$, we again apply the condition $\tau = \tau_0$ to obtain $F_+ = 3/2A_y F_0\cos \xi_0$, which yields the somewhat unwieldy formula for the geometric albedo,

$$A_y = \frac{2(g_0\omega_0 + \omega_0(1 - g_0\omega_0))}{3(1 - g_0\omega_0)(1 - K^2 \cos^2 \xi_0)} + \frac{K}{2\epsilon_1(1 - g_0\omega_0) + K} \left[ \frac{16\epsilon_1\epsilon_2 f_B(1 - \omega_0)(1 - g_0\omega_0)}{3K^2} - \frac{4\epsilon_1(1 - g_0\omega_0)}{3[2\epsilon_1(1 - g_0\omega_0) + K]} \left( \frac{g_0\omega_0}{1 - g_0\omega_0} \right) \right] - \frac{3[2\epsilon_1(1 - g_0\omega_0) + K]}{2K} \times \frac{[1 + 2\epsilon_1(1 - g_0\omega_0)\cos \xi_0][g_0\omega_0 + \omega_0(1 - g_0\omega_0)]}{(1 - g_0\omega_0)(1 - K^2 \cos^2 \xi_0)}.$$  

(19)

The preceding equation constitutes a solution of the radiative transfer equation. The blackbody efficiency of the particle is defined as $f_B \equiv \frac{\pi B}{f_0 \cos \xi_0}$. In an isothermal atmosphere, we expect large particles to behave like blackbodies ($f_B = 1$) when they are observed at a wavelength $\lambda < 2\pi r_c$. By contrast, small particles are inefficient emitters of radiation ($f_B = 0$) at a wavelength $\lambda > 2\pi r_c$.

In the limit of $\omega_0 = 0$, we expect $3A_y/2 = f_B$, which occurs only if $\epsilon_1 = \epsilon_2$. We adopt the “hemi-isotropic closure” ($\epsilon_1 = \epsilon_2 = 1$), which asserts that the flux is isotropic in each of the upward and downward hemispheres (Pierrehumbert 2010). We checked that Equation (19) yields $3A_y/2 = 1$ when $\omega_0 = 1$, independent of $g_0$ and $f_B$. In the limit of $g_0 = f_B = 0$, we recover Equation (5.49) of Pierrehumbert (2010),

$$\frac{3A_y}{2} = \frac{\omega_0}{(1 + 2\sqrt{1 - \omega_0 \cos \xi_0})(1 + \sqrt{1 - \omega_0})}. $$  

(21)

3. RESULTS

3.1. Albedo Spectra: The Degeneracy between Condensate Size and Relative Abundance to Sodium

The top panel of Figure 3 shows $A_y$ as a function of $g_0$ for different values of $f_B$ and $f_\text{NS}$. As the particle becomes more forward scattering ($g_0 > 0$), the geometric albedo generally decreases (see also Sudarsky et al. 2000). When $\omega_0 = 0$ and 1, $3A_y/2 = f_B$ and 1, respectively, as expected. Generally, larger particles (higher $f_B$ values) correspond to higher geometric albedos. Next, we compute albedo spectra by considering the limiting case where Rayleigh scattering is entirely due to the presence of small condensates ($2\pi r_c/\lambda \ll 1$) and absorption is due to the sodium D doublet (Sudarsky et al. 2000). (The potassium doublet absorbs at somewhat longer wavelengths: about 0.77 $\mu$m.) Details of the scattering and absorption cross sections used are described in the Appendix. This simple model contains three parameters: the particle radius ($r_c$), the scattering refractive index of the condensates ($n_c$), and the relative abundance of sodium atoms to the condensates by number ($f_{\text{Na}}$). The single scattering albedo becomes

$$\omega_0 = \frac{\sigma_{\text{scat}}}{\sigma_{\text{scat}} + f_{\text{Na}} \sigma_{\text{Na}}},$$  

(22)

and is somewhat insensitive to $n_c$. We choose $n_c = 1.6$ to mimic the presence of silicates such as enstatite. We set $g_0 = 0$. In the middle and bottom panels of Figure 3, we show calculations of $A_y$ using Equations (19) and (22) for $r_c \sim 0.1 \mu$m and $\sim 10$ nm and compare them to the measurement of the albedo spectrum of the hot Jupiter HD 189733b by Evans et al. (2013). These assumed particle radii are consistent with those inferred by Lecavelier des Etangs et al. (2008). Generally, the computed geometric albedo is a degenerate function of $r_c$ and $f_{\text{NS}}$. However, since the Rayleigh cross section scales as $\sigma_{\text{scat}} \propto r_c^6$, a small change in the particle radius needs to be compensated by a large change in the relative abundance of sodium atoms to condensates. This property may prove to be useful for constraining $r_c$ in future observations of exoplanetary atmospheres. We do not use Equation (19) to compute $A_y$ for use in the analytical temperature–pressure profiles, but rather specify it as a free parameter while being mindful that the geometric albedo is an emergent property of the scattering and absorption properties of the atmosphere.
3.2. The Effects of Geometric Albedo on Thermal Structure

Figure 4 shows some examples of temperature–pressure profiles. We have adopted some of the parameters of Kepler-7b: \( T_{\text{eq}} = 1586 \) K and \( \log g = 2.62 \) (Demory et al. 2011). We pick \( \kappa_{\text{IR}} = 0.004 \) cm\(^2\) g\(^{-1}\) such that the infrared photosphere lies at \( \sim 0.1 \) bar. Since the opacity sources in the optical range of wavelengths remain poorly known for hot Jupiters in general, we adopt \( \kappa_{\text{O}} = 0.003 \) cm\(^2\) g\(^{-1}\) as an illustration. We consider \( A_g = 0, 0.17, \) and \( 0.34 \) to demonstrate the effects of varying the albedo, corresponding to \( P_D \approx 0.09, 0.04, \) and \( 0.03 \) bar.

In the limit of vertically uniform populations of scatterers and absorbers, starlight is absorbed and reflected mostly at the photon deposition layer \( (P_D) \). It is also the pressure level the Kepler optical bandpass is probing. Thus, the presence of a non-zero albedo has two effects. The first, obvious one is to diminish the amount of heat deposited in an atmosphere. The second, less obvious effect is to alter the location at which most of the starlight is being deposited. More reflecting atmospheres tend to have their starlight deposited higher in altitude. Both properties are reflected in Figure 4. Consequently, the effect of a non-zero albedo is to produce a temperature inversion if \( A_g \) is of a high enough value.

3.3. The Relationship between Thermal Structure, Condensate Size, and Condensate Abundance

Next, we wish to examine the lofting properties of the atmosphere at \( P = P_D \). We evaluate the temperature at the photon deposition layer using our analytical \( T-P \) profiles: \( T = T_D \) where \( T_D \equiv T(P_D) \). In Figure 5, we show \( S \) as a function of \( T_{\text{eq},0} \) for various values of \( A_g \). We show three sets of curves.
for $r_e = 1$, 10, and 100 μm. For $r_e = 1$ μm, we obtain $S \gtrsim 1$ for $T_{eq,0} = 1000–3000$ K, implying that micron-sized particles should be readily lofted in hot Jovian atmospheres, consistent with the results from the 3D simulations of Parmentier et al. (2013). Micron-sized particles have $N_e \gg 1$ and $S \propto P_D$.

However, since $P_D$ depends on $A_q$, the pressure level probed is lower (higher altitude) and the corresponding value of $S$ is lower for larger values of $A_q$. For $r_e = 100$ μm ($N_e \ll 1$), $S$ now has a dependence on $T_{eq,0}$ as we have $S \propto T_D$. However, since $S \ll 1$ for all values of the equilibrium temperature examined, we do not expect particles with $r_e = 100$ μm to be lofted by atmospheric circulation. Particles with $r_e = 10$ μm have $S \sim 0.1–1$ and are expected to be partially lofted as a population, again consistent with the results of Parmentier et al. (2013).

If small cloud particles or grains ($N_e \gg 1$) are robustly formed in hot Jovian atmospheres, then they should be omnipresent due to the ease at which atmospheric circulation will keep them aloft. The variations in their scattering and absorption properties (see Section 2.2.3), which are determined by their sizes and compositions, are expected to produce a scatter in the values of the measured geometric albedos.

4. DISCUSSION

4.1. Comparative Exoplanetology: Kepler-7b versus Kepler-12b

The bounty of exoplanets discovered by the Kepler mission has emphasized the importance of comparative exoplanetology. Within our sample of hot Jupiters examined in Figure 1, Kepler-7b (Latham et al. 2010; Demory et al. 2011) and Kepler-12b (Fortney et al. 2011) provide for an intriguing comparison. They receive similar degrees of stellar heating ($T_{eq,0} \approx 1500$ K), and thus we expect the strength of atmospheric circulation to be comparable in both atmospheres. They possess comparable surface gravities ($\log g \approx 2.6$) and radii ($R \approx 1.6–1.7 R_J$). They orbit somewhat quiescent, Sun-like stars of comparable metallicity ([Fe/H] $\approx 0.1$). Yet their measured geometric albedos are non-negligibly different: $A_g \approx 0.35$ (for Kepler-7b) versus 0.08 (for Kepler-12b). For Kepler-7b, there is evidence for the presence of condensates at the atmospheric layer probed by the Kepler bandpass (Demory et al. 2011, 2013). Furthermore, the optical phase curve of Kepler-7b exhibits a sinusoidal functional form with a peak that is offset from the secondary eclipse (Demory et al. 2013). The infrared secondary eclipses of Kepler-7b, as detected by the Spitzer Space Telescope at 3.6 and 4.5 μm, imply brightness temperatures that are markedly lower than that in the optical (Demory et al. 2013).

By contrast, the phase curve of Kepler-12b exhibits no sinusoidal variations (at the sensitivity level of Kepler) with a period similar to the orbital motion of the exoplanet, while registering brightness temperatures of about 1400–1600 K in the infrared (Fortney et al. 2011).

We apply the lessons learnt from the analytical models presented in this study. All else being equal, we expect the photon deposition layer to reside at a higher altitude (lower pressure) for Kepler-7b due to its higher albedo. If large cloud particles ($N_e \ll 1$) are embedded in the atmosphere of Kepler-7b, then we expect $S \propto T_D$ to describe the lofting property of its photon deposition layer. Large particles generally produce a higher geometric albedo if $r_e \gg \lambda/2\pi$. There are two possible configurations of $T_D$ that will produce a sinusoidal functional form for $S$ with a peak that is offset from the secondary eclipse. The first and simplest configuration is a shifted Heaviside function, i.e., $T_D$ has two values, one in each hemisphere, but it is translated in longitude by some amount. In this case, the corresponding brightness temperature in the Kepler bandpass as a function of orbital phase will be a sinusoidal function that has a peak offset (Cowan & Agol 2008). The second configuration is for $T_D \propto \sin(\phi \pm \phi_0)$, with $\phi$ being the longitude of the exoplanet and $\phi_0$ being a constant offset, which also produces a sinusoidal function for $S$. In the right circumstances (i.e., $r_e \sim 10$ μm), the variation in temperature and pressure may cause $S$ to possess values ranging from 0.1 to 1. Such a variation in $S$ produces a longitudinal variation in the abundance of lofted cloud particles, which in turn produces a longitudinal variation in the associated albedo and the flux of reflected starlight.

The observations of Kepler-7b (Demory et al. 2013) are consistent with such a scenario. By contrast, if small cloud particles ($N_e \gg 1$) are embedded in the atmosphere of Kepler-12b, we expect $S \propto P_D$, which produces a flat phase curve for a given albedo value if the opacity in the optical range of wavelengths is roughly constant with longitude. In other words, when small clouds are present, we expect their abundance to be zonally uniform; if $g$, $\kappa_0$, and $A_g$ are constant, then $P_D$ and hence $S$ are constant across longitude. Small particles are consistent with a lower albedo if $r_e \ll \lambda/2\pi$.

However, there is an important detail in the optical phase curve of Kepler-7b that is difficult to reconcile with our simple explanation. The peak of the optical phase curve peaks after secondary eclipse, implying that the corresponding brightness map peaks westward of the substellar point (Demory et al. 2013). Irradiated atmospheres in the hot Jupiter regime are expected to possess temperature maps that peak eastward of the substellar point (Showman & Polvani 2011), a theoretical expectation that is corroborated by 3D simulations of atmospheric circulation. That this property is independent of whether Newtonian cooling (e.g., Showman & Guillot 2002; Heng et al. 2011b), broadband radiative transfer (Heng et al. 2011a; Rauscher & Menou 2012; Perna et al. 2012), or multi-wavelength radiative transfer (Showman et al. 2009) is utilized suggests that it is a robust outcome of the hot Jupiter regime. If an infrared phase curve of Kepler-7b is measured with this property (eastward shift), then it is a “smoking gun” for the presence of condensates—reflected light and thermal emission are not tracing each other, implying that the spatial distribution of the condensates is being modified by atmospheric dynamics.

4.2. Caveats and Future Work

Several caveats and unexplored aspects provide opportunities for future work. We have not used an eddy diffusion coefficient ($K_{zz}$) to mimic atmospheric circulation and instead approximated the vertical velocity as a fixed fraction of the local sound speed. Although such an approach includes the effect of varying the stellar irradiation, since it is tied to the temperature–pressure profile, it fails to capture the dependence of the depth of atmospheric circulation on the incident stellar flux (Fortney et al. 2008; Dobbs-Dixon et al. 2012; Perna et al. 2012). The vigor of atmospheric circulation in irradiated exoplanetary atmospheres means that if clouds form, they will be well mixed from ~1 mbar all the way down to ~10 bar, depending on $F_0$ and $A_g$. Performing 3D simulations of atmospheric circulation with a diversity of cloud configurations will inform 1D models on how to “paint” clouds onto their T-P profiles.
Another important interplay we have not explored concerns atmospheric chemistry. Specifically, the strength and spectral distribution of the incident stellar flux modify the absorption ($\kappa_\nu$) and scattering ($A_\nu$) properties of both the condensates and the gas in the irradiated atmosphere. To understand the lifting behavior of small cloud particles as a function of the incident stellar flux requires this interplay to be elucidated.

In recent years, retrieval models, originally developed for the modestly irradiated planets/moons of the solar system, have been employed to infer the temperature–pressure profiles and atmospheric chemistry/composition of hot Jupiters (Madsudhan & Seager 2009; Lee et al. 2012; Line et al. 2012). While being an important development in the study of exoplanetary atmospheres, these published works have so far been “blue sky” and omitted the effects of clouds. Benneke & Seager (2012) specify the albedo as a free parameter in their analysis, but only include one of its effects as suggested by their use of the Guillot (2010) model: the diminution of the incident stellar flux, but not the modification of its vertical absorption profile. They also allow for the cloud-top pressure to be a fitting parameter. To illustrate the importance of including a non-zero albedo, consider the limiting case of $A_\nu = 1$, in which case we expect the photon deposition layer to reside at the top of the atmosphere and the infrared photosphere to be undefined (if interior heat from the irradiated exoplanet is negligible). When the geometric albedo is just below unity, we expect the infrared photosphere(s) to be located just below the top of the atmosphere. When $A_\nu = 0$, the contribution functions are correctly computed by the published retrieval models. Since we expect physical quantities to vary continuously, the contribution functions, at various infrared wavelengths, should shift to lower pressures as $A_\nu$ increases, an effect that needs to be included in the retrieval models. Future work that includes a simple model of clouds, with a small number of free parameters, in the retrieval technique will set some empirical constraints on the cloud properties, although some degeneracy is anticipated.

Although 3D simulations that treat only the dynamical or radiative interaction between the atmospheric flow and the cloud particles are insufficient for a full understanding of the effects of clouds in irradiated exoplanetary atmospheres, neither is a 1D analytical model that employs simple, approximate treatments for these effects, as presented in this study. Nevertheless, both approaches drive us toward constructing falsifiable models that are realistic yet simple enough to be confronted by observations and feasibly included in future 3D simulations.

4.3. An “Observer’s Cookbook”

To provide an executive summary of the observational relevance of our study, we highlight a few scenarios and suggest possible (and possibly non-unique) interpretations. When the peak amplitude of the phase curve is on the order of the occultation depth, then we term it to be “sinusoidal”; if not, we term it to be “flat.” When the angular offset of the peak of the phase curve from secondary eclipse is $\sim 1^\circ$ or close to zero, we term it to be “small.” Phase offsets of $\sim 10^\circ$ are “large.”

1. **High albedo, sinusoidal optical phase curve.** Large cloud particles or dust grains ($\sim 10\mu m$). As discussed, a possible example is Kepler-7b.
2. **Low albedo, flat optical phase curve.** Small cloud particles ($\ll 1\mu m$). As discussed, a possible example is Kepler-12b.
3. **High albedo, small infrared phase offset.** A high albedo implies that the photon deposition layer resides higher in the atmosphere. With most of the starlight being deposited at lower pressures, the infrared photosphere also lies at lower pressures. With the atmosphere being more radiative (or less advective) at higher altitudes, a small phase offset in the infrared is expected (Showman & Guillot 2002; Cowan & Agol 2011a; Perna et al. 2012; Heng 2012). Conversely, a larger infrared phase offset is expected for a low albedo (Fortney et al. 2008).

4. **Low albedo, small infrared phase offset.** For highly irradiated hot Jupiters, the intense stellar flux trumps any effect due to opacity (e.g., albedo), and small infrared phase offsets are generally expected (Perna et al. 2012). Conversely, hot Jupiters with lower levels of incident irradiation are expected to possess large infrared phase offsets.

As an example, we consider the prototypical case of the hot Jupiter HD 189733b, for which large peak offsets have been measured in the infrared phase curves (Knutson et al. 2007, 2009). This is consistent with the low intensity of stellar irradiation impinging upon its atmosphere ($T_{eq,0} \approx 1200$ K). Transit observations in the ultraviolet and optical range of wavelengths reveal a spectral slope, punctuated by sodium lines, consistent with Rayleigh scattering by condensates present at the day–night terminators (Lecavelier des Etangs et al. 2008; Pont et al. 2008, 2013; Sing et al. 2011; Gibson et al. 2012). That the infrared peak offsets are not small suggests that the albedo associated with the condensates is small, or even close to zero, at and near the peak of the stellar spectrum. Based on a tentative comparison of several brightness temperature points with an analytical temperature–pressure profile, Heng et al. (2012) estimate that the Bond albedo of HD 189733b is about 0.1. The albedo spectrum of HD 189733b suggests a low to vanishing geometric albedo in the *Kepler* bandpass (Evans et al. 2013). Optical phase curves of HD 189733b will further constrain the properties of the condensates, including if they are small or large (as already described by the preceding scenarios).

Case studies that contradict these scenarios (e.g., Crossfield et al. 2010) hint at the possibility of missing physics or chemistry and will inspire novel ways of thinking about irradiated exoplanetary atmospheres. Some of the degeneracies described may be broken by examining the colors of these atmospheres.

K.H. acknowledges financial and/or logistical support from the University of Bern, the Swiss-based MERAC Foundation, and the University of Zürich. We are grateful to Bruce Draine, Jaemin Lee, Michael Gillon, and Nikku Madhusudhan for useful conversations. K.H. benefited from discussions conducted at the Exeter–Oxford exoplanet workshop in 2013 April and at the PPVI conference in 2013 July. We thank the anonymous referee for constructive comments that improved the quality and clarity of the manuscript.

APPENDIX

MODELING SODIUM ABSORPTION AND RAYLEIGH SCATTERING

The quantum mechanical properties of the sodium D doublet are well known (e.g., Draine 2011). The absorption cross section is

$$\sigma_{Na} = \frac{\pi e^2}{m_e c} f_{\nu} \phi_{\nu},$$

where $e$ is the elementary unit of electric charge, $m_e$ is the mass of the electron, $c$ is the speed of light, $f_{\nu}$ is the oscillator strength,
and $\phi_v$ is the line profile function, described by a Lorentz profile,

$$\phi_v = \frac{4A_{ul}}{16\pi^2 (v - v_{ul})^2 + A_{ul}^2}, \quad (A2)$$

normalized such that $\int \phi_v dv = 1$ over all frequencies ($v$). The frequency of the line transition is given by $v_{ul}$, with “u” and “l” denoting the upper and lower atomic levels, respectively. The properties associated with the sodium doublet are

$$\lambda = 0.5891582 \mu m: \quad f_{ul} = 0.641,$$

$$g_u = 4, \quad g_l = 2, \quad A_{ul} \approx 6.159 \times 10^7 s^{-1},$$

$$\lambda = 0.5897558 \mu m: \quad f_{ul} = 0.320,$$

$$g_u = g_l = 2, \quad A_{ul} \approx 6.137 \times 10^7 s^{-1}. \quad (A3)$$

The Einstein $A$-coefficients are computed using

$$A_{ul} = \frac{8\pi^2 e^2 v_{ul}^3 g_l f_{ul}}{m_e c^3 g_u}, \quad (A4)$$

with $g_l$ and $g_u$ being the statistical weights of the lower and upper atomic levels, respectively. Essentially, there are no free parameters involved in specifying $\sigma_{\text{scat}}$. The assumption employed here is that Doppler broadening of the line may be neglected; otherwise, a Voigt profile has to be used in place of the Lorentz profile.

Rayleigh scattering by small particles is also a well-known phenomenon with a cross section given by

$$\sigma_{\text{scat}} = \frac{2\pi^5}{3} \left( \frac{n_r^2 - 1}{n_r^2 + 2} \right)^2 r_e^6 \lambda^{-4}, \quad (A5)$$

where $n_r$ is the real part of the index of refraction. For molecules, it is (Pierrehumbert 2010)

$$\sigma'_{\text{scat}} = \frac{32\pi^3}{3} \left( \frac{n_r' - 1}{n} \right)^2 \lambda^{-4}, \quad (A6)$$

where $n_r'$ is the real part of the index of refraction for the molecular gas and $n$ is its number density. For molecular gas, we have $(n_r' - 1) \ll 1$; for refractory condensates, we have $(n_r - 1) \sim 0.1$. Rayleigh scattering by molecules is weaker ($f_{\text{gas}} \sigma'_{\text{scat}} / \sigma_{\text{scat}} < 1$) as long as the particle radius exceeds a critical value,

$$r_e > \left[ \frac{4}{\pi} \frac{(n_r' - 1) (n_r^2 + 2)}{(n_r^2 - 1)} \right]^{1/3} f_{\text{gas}}^{1/6}, \quad (A7)$$

where $f_{\text{gas}}$ is the relative abundance of molecules to condensates by number. For $P = 0.1$ bar and $T = 1500$ K, we have $n \approx 5 \times 10^{17} \text{cm}^{-3}$. Using $n_r' = 1.0001$ and $n_r = 1.6$, we obtain $0.9 f_{\text{gas}}^{1/6}$ nm for the critical particle radius. For comparison, we note that the Bohr radius is about 0.05 nm. The relative weakness of Rayleigh scattering by hydrogen molecules may produce an albedo spectrum that is too low if sodium atoms are abundantly present. Measuring the abundance of sodium relative to hydrogen remains challenging, even for HD 189733b (Huitson et al. 2012).

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Received 2014 January 17; published 2014 March 27

Online-only material: color figures

We report on a technical error that affects some of the numbers reported in our study, but do not alter its general conclusions. A confusion between division and multiplication in a computer code used to post-process our three-dimensional (3D) simulations led to an under-estimation of the vertical velocity by several orders of magnitude. We note that this error does not affect previous, published work. Correcting this error implies that Figures 2, 4, and 5 of the published version of this article have to be amended. The previous claim that the vertical Mach number has an order-of-magnitude value of $10^{-6}$ is now revised to $10^{-3}$. These changes only affect the part of our study that deals with the lofting properties of condensates, but do not affect the parts dealing with the measured geometric albedos of hot Jupiters and interpreting the reflection spectrum of HD 189733b. For the latter, we correct a second error associated with the Rayleigh scattering cross section for particles, which amends Figure 3 and Equations (A5) and (A7) of the published article. All of the salient and major conclusions of our study, as stated in our original abstract, remain unchanged.

In 3D simulations of atmospheric circulation that solve the primitive equations of meteorology, it is common to specify the pressure ($P$) as the vertical coordinate (rather than the spatial coordinate $z$). A direct output of such 3D simulations, including ours, is the quantity

$$\omega \equiv \frac{DP}{Dt} \quad (1)$$

To compute the vertical velocity in more familiar physical units, we invoke the ideal gas law and use

$$v_z = \omega \left( \frac{\partial P}{\partial z} \right)^{-1} = -\frac{RT DP}{Pg \frac{DP}{Dt}}, \quad (2)$$

where $R$ denotes the specific gas constant, $T$ the temperature, and $g$ the surface gravity. Since $R$ and $g$ are input parameters, $P$ is obtained from the vertical grid and $T$ and $\omega$ are direct outputs of the simulation; $v_z$ may be straightforwardly computed.

Unfortunately, an error was made in the post-processing computer code used to evaluate $v_z$,

$$v_z = \omega \frac{\partial P}{\partial z} = -\frac{P_g DP}{RT \frac{DP}{Dt}}, \quad (3)$$

due to a confusion between division and multiplication. This error does not affect previous, published work as $\omega$, and not $v_z$, was the quantity being explicitly simulated/computed. It substantially revises Figure 2 in the published version of this article, which is corrected here. Again, rms values are presented. First, the order-of-magnitude values of $v_{z,\text{rms}}$ are now significantly larger, allowing us to achieve consistency with other published results of atmospheric circulation (e.g., Rauscher & Menou 2012). Second, both $v_{z,\text{rms}}$ and the vertical Mach number ($M_{z,\text{rms}}$) display greater sensitivity to the stellar irradiation flux and the absence or presence of a temperature inversion. Third, they are somewhat insensitive, across pressure, between $10^{-3}$ and $10^{-1}$ bar. Roughly speaking, one may adopt $M_{z,\text{rms}} \sim 10^{-3}$ within this pressure range. This is the value we now adopt, rather than the $M_{z,\text{rms}} \sim 10^{-6}$ value assumed in the published article.

The only equation affected by this error is Equation (11) of the published version of this article, which now reads

$$S \approx \frac{2.7 M_{z,y} P}{\rho_c \epsilon g} \sim 1 \left( \frac{M_z}{10^{-3}} \frac{\gamma}{7/5} \frac{P}{0.1 \text{ mbar}} \right) \left( \frac{\rho_c}{3 \text{ g cm}^{-3}} \frac{r_c}{1 \text{ gm}^{-3}} \frac{g}{10^3 \text{ cm s}^{-2}} \right)^{-1}. \quad (4)$$

Essentially, we expect micron-sized grains to be lofted up to $\sim 0.1$ mbar by atmospheric circulation.

In addition to Figure 2, Figures 4 and 5 of the published article are now revised and presented in this erratum. For the temperature–pressure profile presented in Section 3.2 and Figure 4 of the published article, we now adopt $\kappa_{IR} = 4 \text{ cm}^2 \text{ g}^{-1}$ and $\kappa_O = 3 \text{ cm}^2 \text{ g}^{-1}$ and place the bottom boundary of the model at 1 bar (instead of 100 bar). These changes allow us to examine the profile at higher altitudes (lower pressures). We see that the qualitative effect of changing the geometric albedo remains invariant. The temperature values are largely similar. The lofting parameter $S$ is then computed using these three temperature–pressure profiles.
Figure 2. The rms vertical velocity (left panel) and Mach number (right panel) as functions of pressure in the atmospheres of model hot Jupiters. (A color version of this figure is available in the online journal.)

Figure 3. Calculations of the geometric albedo. (A color version of this figure is available in the online journal.)
Figure 4. Examples of temperature–pressure profiles adopting some of the parameters of Kepler-7b. The dots indicate the locations of the photon deposition layers, where starlight is predominantly absorbed. The Kepler bandpass probes the photon deposition layer.

(A color version of this figure is available in the online journal.)

Figure 5. "Lofting parameter" $S$ as a function of $T_{\text{eq,0}}$ and the geometric albedo of the hot Jupiter for three different cloud particle radii ($r_c = 1, 10, \text{and } 100 \, \mu m$).

(A color version of this figure is available in the online journal.)

Figure 5 shows that the qualitative conclusion is unchanged: micron-sized particles are easily lofted, $r_c = 10 \, \mu m$ particles are partially lofted as a population, while we do not expect $r_c = 100 \, \mu m$ particles to be lofted by atmospheric circulation.

In Equation (A5) of the published article, we correct an error associated with the Rayleigh scattering cross section for particles,

$$\sigma_{\text{scat}} = \frac{2 \pi^5}{3} \left( \frac{n_r^2 - 1}{n^2 + 2} \right)^2 \frac{r_c^6 \lambda^{-4}}{\varphi},$$

where the extra factor of $2^6$ derives from the fact that it is the diameter and not the radius that should be used in the formula. Also, we correct a coding error that originates from the conversion between $\mu m$ and cm for the physical units of $\lambda$. These changes imply that the middle and bottom panels of Figure 3 of the published article have to be amended; a corrected version is shown in this erratum.

Furthermore, Equation (A7) is revised,

$$r_c > \left[ \frac{1}{2 \pi n} \left( \frac{n_r' - 1}{n_r^2 + 2} \right) \right]^{1/3} f_{\text{gas}}^{1/6}.$$  

Using $n = 5 \times 10^{17} \, \text{cm}^{-3}$, $n_r' = 1.0001$, and $n_r = 1.6$, we obtain $0.5 f_{\text{gas}}^{1/6} \, \text{nm}$ for the critical particle radius (instead of $0.9 f_{\text{gas}}^{1/6} \, \text{nm}$), an order of magnitude larger than the Bohr radius.

A technical error associated with the post-processing simulation output led to the vertical velocity of the atmospheric circulation to be underestimated, which we rectify in this erratum. We provide corrected versions of Figures 2, 4, and 5 and Equation (11). Some of the reported numbers associated with the part of the study dealing with the lofting of condensates have changed, but the overarching conclusions remain unchanged. The other parts of the study, which deal with the measured geometric albedos of hot Jupiters and the interpretation of the reflection spectrum of HD 189733b are unaffected. For the latter, we correct an error associated with the Rayleigh
scattering cross section, which amends Figure 3 and Equations (A5) and (A7) of the published version. The major conclusions of the study, as described in the published version of the abstract, remain unchanged.

We thank Vivien Parmentier and Tom Evans for bringing these issues to our attention.

REFERENCE