Multimaterial coatings with reduced thermal noise

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Multimaterial coatings with reduced thermal noise

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I. INTRODUCTION

Lasers and resonant optical cavities have become a ubiquitous tool in optical experiments which explore the bounds of physics through precise measurements of space and time [1–4]. The precision with which these measurements can be made is in large part determined by the fundamental thermal motion of the coatings used in optical resonators [5–7].

Interferometric gravitational wave detectors in particular set extremely stringent requirements on their coatings; they must simultaneously have good mechanical properties for low thermal noise, low optical absorption for high-power operation, and good surface figure to support multikilometer resonant cavities [8]. These requirements are, however, very hard to meet in a single material [9–20].

This paper provides the theoretical foundation for a new approach to the search for high quality coating materials. We compute the coating thermal noises and absorption which will result from coatings comprised of more than two materials. This is to be contrasted with previous works, which have assumed that optical coatings are made of one low-index material and one high-index material [21,22].

In the next section we present our model of coating Brownian and thermo-optic noises, generalized from previous works to allow for multimaterial coatings. We go on to develop a simple model of absorption as a function of depth in the coating, from which we are able to assess the impact of using relatively high absorption materials deep in the coating. These calculations are followed by a few examples of how this new approach can be used to produce coatings with low thermal noise.

All frequently used symbols are given in Table I, and the Appendix connects the notation used in this work to that of previous authors.

II. MODEL OF COATING THERMAL NOISE

In order to elucidate the potential benefits of multimaterial coatings we will first describe briefly the model of thermal noise used in our calculations. For Brownian thermal noise we start with [21], and [22] is our starting point for thermo-optic noise, though similar treatments can be found in [23–25].

Since Hong et al. [21] conclude that changes in the ratio of shear to bulk mechanical loss do not significantly change the optimal coating design, and that photoelastic effects are relatively unimportant, we can simplify their result significantly by assuming that shear and bulk mechanical losses are equal, \( \phi_M = \phi_{\text{bulk}} = \phi_{\text{shear}} \), and that the photoelastic effects can be ignored. (While not important for optimization, the ratio of shear and bulk losses impacts the level of Brownian thermal noise at the ±30% level [21].)

The resulting equation for Brownian thermal noise is

\[
S_{\text{Br}} = \frac{4 k_B T}{\pi r_G^2 \omega} \frac{1 - \sigma_s - 2 \sigma_s^2}{\pi Y_s} \sum_j b_j d_j \phi_{Mj}
\]  

(1)

where the unitless weighting factor \( b_j \) for each layer is

\[
b_j = \frac{1}{1 - \sigma_j} \left[ \left( 1 - n_j \frac{\partial \phi_s}{\partial \phi_j} \right)^2 \frac{Y_j}{Y_s} + \frac{(1 - \sigma_j - 2 \sigma_j^2)^2 Y_j}{(1 + \sigma_j)^2 (1 - 2 \sigma_j) Y_s} \right].
\]

Under the assumption that the substrate and coating elastic parameters are equal \( (Y_j \rightarrow Y_s \text{ and } \sigma_j \rightarrow \sigma_s) \), and ignoring field penetration into the coating \( (\partial \phi_s / \partial \phi_j \rightarrow 0) \), \( b_j \rightarrow 2 \) for all layers.

For thermo-optic noise we use

\[
S_{\text{TO}} = \frac{4 k_B T^2}{\pi r_G^2 \sqrt{2 k_s C_s} \omega} \left[ \frac{\bar{\alpha}_s d - \bar{\beta} \lambda_0 - \bar{\alpha}_s}{C_s} \sum_j d_j C_j \right]^2
\]

(2)

where

\[
\bar{\alpha}_s = 2 (1 + \sigma_s) \alpha_s
\]

(3)
### TABLE I. Frequently used symbols for physical constants, the environment, material parameters, etc., are given above. Material parameters that appear with a subscript refer to either the substrate material, subscript s, the coating, subscript c, or are indexed to a particular coating layer, typically with the variable j. Note that k is occasionally used as a local index for summation or recursion when j is already in use.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Unit or value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_B$</td>
<td>Boltzmann’s constant</td>
<td>$1.38 \times 10^{-23}$ J/K</td>
</tr>
<tr>
<td>$T$</td>
<td>Mean temperature</td>
<td>290 K</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Vacuum laser wavelength</td>
<td>1064 nm</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Laser wave number</td>
<td>$2\pi/\lambda_0$</td>
</tr>
<tr>
<td>$r_G$</td>
<td>Gaussian beam radius ($1/e^2$ power)</td>
<td>6 cm</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
<td>rad/s</td>
</tr>
<tr>
<td>$S_{Br}^T$</td>
<td>Brownian noise</td>
<td>m$^2$/Hz</td>
</tr>
<tr>
<td>$S_{TO}^T$</td>
<td>Thermo-optic noise</td>
<td>m$^2$/Hz</td>
</tr>
<tr>
<td>$n$</td>
<td>Refractive index</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal expansion</td>
<td>1/K</td>
</tr>
<tr>
<td>$\beta$</td>
<td>(\partial n/\partial T)</td>
<td>1/K</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Thermal conductivity</td>
<td>W/K m</td>
</tr>
<tr>
<td>$C$</td>
<td>Heat capacity per volume</td>
<td>J/Km$^3$</td>
</tr>
<tr>
<td>$Y$</td>
<td>Young’s Modulus</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Poisson ratio</td>
<td></td>
</tr>
<tr>
<td>$\phi_M$</td>
<td>Mechanical loss angle</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Optical absorption</td>
<td>1/m</td>
</tr>
<tr>
<td>$d$</td>
<td>Coating thickness</td>
<td>m</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depth in the coating (negative)</td>
<td>m</td>
</tr>
<tr>
<td>$P$</td>
<td>Laser power arriving at each layer</td>
<td>W</td>
</tr>
<tr>
<td>$E$</td>
<td>Complex electric field amplitude</td>
<td>N/C</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Power absorption ratio</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>Brownian weight coefficient</td>
<td></td>
</tr>
</tbody>
</table>

In this section we summarize the model for coating reflectivity presented in Appendix B of [22], as this calculation forms the basis for computing the coating phase sensitivity to mechanical and thermal fluctuations [e.g., $\delta \phi_x/\delta \phi_j$ in Eqs. (1) and (5)]. In the next section, we extend this computation to include distributed absorption in coating materials, which is an essential ingredient in the primary result of this paper.

As in [22], we express the reflectivity of the interface between two coating layers, seen by a field moving from layer $j$ to layer $j+1$ (see Fig. 1), as

$$r_j = \frac{n_j - n_{j+1}}{n_j + n_{j+1}}. \quad (6)$$

By recursively combining the interface reflectivities $r_j$, we can find the reflectivity of layer $j$ and all of the layers between it and the substrate

$$\tilde{r}_j = \frac{E_j}{E_j} = e^{\phi_j} \frac{r_j + \tilde{r}_{j+1}}{1 + r_j \tilde{r}_{j+1}}. \quad (7)$$

Note that while $\tilde{r}_j$ includes the round-trip propagation phase in layer $j$, it does not include the reflectivity of the interface between layer $j-1$ and $j$.

The expression for $\tilde{r}_j$ is recursive and the base case is the transition from the $N$th coating layer to the substrate,

$$\tilde{r}_N = e^{\phi_N} r_N \quad (8)$$

which can be evaluated with (6) using $n_{N+1} = n_s$. Total coating reflectivity $\tilde{r}_0$ is evaluated with the external vacuum acting as layer zero such that $n_0 = 1$.

The sensitivity of the coating reflection phase to a change in layer $j$

$$\tilde{\phi} = \sum_{j=1}^{N} \frac{d_j}{\lambda_0} \left( \frac{1 + \sigma_j}{1 - \sigma_j} \right) \frac{(1 - 2\sigma_j) Y_j}{Y_s} \alpha_j \quad (4)$$

$$\tilde{\phi} = \sum_{j=1}^{N} \left( \frac{d_j}{\lambda_0} \left[ 1 + \frac{1 + \sigma_j}{1 - \sigma_j} \alpha_j n_j \right] \frac{\partial \phi_c}{\partial \phi_j} \right). \quad (5)$$

Note that the expression for $\tilde{\phi}$ is slightly different from that of [22], thanks to the correction by K. Yamamoto in Chap. 8.2.5 of [26].

In this paper we make a number of simplifying assumptions ($\phi_M = \phi_{bulk} = \phi_{shear}$, no photoelastic effect), and we ignore several correction factors (thick coating correction [22], finite size test-mass corrections [27,28]). All of this is to keep the formalism simple enough that the results can be easily understood and evaluated, but it should not be taken to mean that these corrections cannot be applied to multi-material coatings. Indeed, their application is expected to be straightforward if somewhat messy process.

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**FIG. 1 (color online).** The numbering of coating layers, interfaces and fields is shown above. The $z$-coordinate is zero at the coating surface, and is positive moving away from the coating. Note that the E-fields are evaluated just inside each coating layer (e.g., $E_1$ is evaluated at $z = -\epsilon$ in the limit of $\epsilon \to 0$).
\[ \frac{\partial \phi_j}{\partial \phi_j} = \text{Im} \left[ \frac{1}{\bar{r}_0 \partial \phi_j} \right] = \text{Im} \left[ \frac{\partial \log \bar{r}_0}{\partial \phi_j} \right] \]  

(9)

is given by the recursion relation

\[ \frac{\partial \bar{r}_k}{\partial \phi_j} = \begin{cases} \frac{e^{i \phi_j}}{\bar{r}_k} - \frac{1 - \rho_j^2}{1 + r_j \bar{r}_j} \frac{\partial \bar{r}_{j+1}}{\partial \phi_j}, & k < j \\ \bar{r}_k, & k = j \\ 0, & k > j \end{cases} \]  

(10)

with the recursion starting at \( k = 0 \), progressing through increasing values of \( k \), and terminating at \( k = j \).

### B. Optical absorption

Maintaining extremely low optical absorption in high-reflection coatings severely limits the choice of coating materials [29]. The key idea behind this paper and its experimental counterpart [30], is that this stringent requirement need not be applied to all layers in the coating, but only to those near the surface which are dominantly responsible for the absorption of the coating. This is true for both the input coupler and end mirror of any high-finesse cavity, since the power inside the cavity is much higher than either the incident or transmitted power.

To compute the depth dependence of optical absorption in a coating, we start by evaluating the electric field present in each layer of the coating

\[ E_{j+1} = \sqrt{\frac{1 - r_j^2}{1 + r_j \bar{r}_{j+1}}} E_j \]  

(11)

(see Fig. 1). This expression can be used iteratively to compute the field entering each coating layer given that the field entering the coating from the vacuum is \( E_0 = \sqrt{2 P_0 / \pi r_0 c_0} \), where \( P_0 \) is the power of the incident laser beam, while \( c \) and \( c_0 \) are the speed of light and the permittivity of free space (see [21] or Appendix A of [31]).

The field at any point in a given layer will be the sum of the two counterpropagating fields

\[ E(z_j, t) = \text{Re}[E_j e^{i (\omega t + k_0 n_j z_j)} + E_j^* e^{i (\omega t - k_0 n_j z_j)}] \]  

(12)

where \( z_j = z - \sum_{k=1}^{j-1} d_k \) (13)

such that \( z_j = 0 \) at the top of layer \( j \), and \( z_j = -d_j \) at the bottom. Optical absorption per unit length in a layer is assumed to be proportional to the time averaged field amplitude squared integrated over that layer, normalized by the power entering the layer and the layer thickness

\[ \rho_j = \frac{2}{|E_j|^2} \int_{-d_j}^{0} E(z_j, t)^2 \, dz_j \int_t \]  

(14)

\[ = (1 + |\tilde{r}_j|^2) + 2 \frac{\sin(k_0 n_j d_j)}{k_0 n_j d_j} \text{Re}[\tilde{r}_j e^{i k_0 n_j d_j}] \]  

(15)

### III. EXAMPLE COATINGS

Given the coating model described in the previous section, and a pallet of possible coating materials, we can evaluate the impact of using more than two materials to make a coating. In this paper we allow ourselves two hypothetical coating materials, metal-oxide A and B (MOA and MOB) as a means of demonstrating the types of optimizations which can occur (see Table II).

The coating examples presented in this section are designed to show how multimaterial coatings can
be used to produce low-noise coatings. For a detailed application of this approach to three-material coatings involving amorphous silicon see [30].

As a baseline, we start by computing the thermal noise seen by 1064 nm light for a 20-layer coating, made of 10 \( \text{SiO}_2 - \text{Ta}_2 \text{O}_5 \) layer pairs or “doublets.” The top layer, known as the “cap” has an optical thickness equal to half of the laser wavelength, such that \( d_1 n_1 / \lambda_0 = \frac{1}{2} \). All of the deeper coating layers are quarter wave with \( d_j n_j / \lambda_0 = \frac{1}{4} \). The results of calculations for this coating are shown in Figs. 2, 3 and 4. This coating transmits 0.1% of the incident light power, and absorbs 0.5 ppm.

A more extreme example is a coating made of 4 \( \text{SiO}_2 - \text{Ta}_2 \text{O}_5 \) doublets, and 3 \( \text{SiO}_2 - \text{MOB} \) doublets. This coating has less than 70% of the Brownian noise of the baseline coating, and only 0.8 ppm absorption. The high refractive index of this material means that fewer and thinner layers are needed relative to \( \text{Ta}_2 \text{O}_5 \) to produce the same transmission. This, in combination with the good mechanical properties of this coating, more than compensates for its high absorption of 100 ppm/\( \mu \text{m} \).

**IV. CONCLUSIONS**

Precision optical measurements are increasingly limited by coating thermal noise, and much time and effort has been and continues to be spent in the search for better coating materials [5]. In this work we suggest that the search for coating materials should not focus on finding a
single material which satisfies all requirements, but rather a pallet of materials which together can be used to make coatings which satisfy all requirements.

While a single high-index, low absorption and low mechanical loss material would be ideal, the examples in this work show that a high-index material with low mechanical loss, but not necessary low optical absorption, will suffice to make lower noise coatings possible. Since the material properties of a given coating layer depend not only on its constituents (e.g., doping), but also on the manufacturing process (e.g., annealing) a wide range of material properties have already been measured or are potentially accessible.

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APPENDIX: RELATION TO OTHER WORKS

The expressions in this work are related to those of [21] by

\[ \tilde{\rho}_0 = \rho_{\text{tot}} \]  

(A1)

\[ 2 \frac{\partial \phi_c}{\partial \phi_j} = \text{Im} \left[ \frac{\partial \log \rho}{\partial \phi_j} \right] \quad \forall 1 \leq j \leq N \]  

(A2)

\[ = \text{Im}(\epsilon_j/n_j) \text{ for their } \beta_j = 0 \]  

(A3)

with our expression on the left of each equality and theirs on the right. The factor of 2 results form our definition of \( \phi_j \) as a round-trip phase in each layer, while theirs is a one-way phase.

We did, however, follow the convention of [21] for the direction of the z-axis: normal to the surface of the coating and pointing into the vacuum. This represents a sign reversal relative to [22], such that \( \partial \phi_c / \partial \phi_j \) is generally positive in this work as in [21].

An earlier treatment of Brownian thermal noise which included field penetration effects was performed in [24]. They based their computation on the coating reflection phase sensitivity to interface displacements (rather than layer thickness changes) and their notation is connected to ours by

\[ n_j \frac{\partial \phi_c}{\partial \phi_j} - n_{j+1} \frac{\partial \phi_c}{\partial \phi_{j+1}} = \epsilon_j \]  

(A4)

again with our expression on the left of the equality and theirs on the right.

[27] V. B. Braginsky and S. P. Vyatchanin, Phys. Lett. A 312, 244 (2003).