Determination of $\phi$ and $2\sigma$ from charmless two-body decays of beauty mesons

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Determination of $\gamma$ and $-2\beta_3$ from charmless two-body decays of beauty mesons

**LHCb Collaboration**

**ABSTRACT**

Using the latest LHCb measurements of time-dependent CP violation in the $B^0 \to K^+K^-$ decay, a U-spin relation between the decay amplitudes of $B^0 \to K^+K^-$ and $B^0 \to \pi^+\pi^-$ decay processes allows constraints to be placed on the angle $\gamma$ of the unitarity triangle and on the $B^0$ mixing phase $-2\beta_3$. Results from an extended approach, which uses additional inputs on $B^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ decays from other experiments and exploits isospin symmetry, are also presented. The dependence of the results on the maximum allowed amount of U-spin breaking is studied. At 68% probability, the value $\gamma = (63.5_{-6.7}^{+7.2})^\circ$ module 180$^\circ$ is determined. In an alternative analysis, the value $-2\beta_3 = -0.12_{-0.14}^{+0.16}$ rad is found. In both measurements, the uncertainties due to U-spin breaking effects up to 50% are included. 

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1. Introduction

The understanding of flavour dynamics is one of the most important aims of particle physics. Charge-parity ($CP$) violation and rare decay processes involving weak decays of $B$ mesons provide tests of the Cabibbo–Kobayashi–Maskawa (CKM) mechanism [1, 2] in the Standard Model (SM). The CKM matrix describes all flavour changing transitions of quarks in the SM. These include tree-level decays, which are expected to be largely unaffected by non-SM contributions, and flavour changing neutral current transitions characterized by the presence of loops in the relevant diagrams, which are sensitive to the presence of non-SM physics. Tests of the CKM matrix structure, commonly represented by the unitarity triangle (UT), are of fundamental importance.

Although significant hadronic uncertainties usually complicate the experimental determination of the CKM matrix elements $V_{ij}$, there are certain cases where the $V_{ij}$ can be derived with reduced or even negligible hadronic uncertainty. One of these cases involves the determination of the UT angle $\gamma$. The angle $\gamma$, defined as $\gamma = \arg \left(-\frac{V_{ub}V_{tb}^*}{V_{cb}V_{tb}^*}\right)$, can be measured using decays that involve tree diagrams only, with almost vanishing theoretical uncertainty [3]. However, $\gamma$ is experimentally the least known of the UT angles. World averages of the measurements performed by BaBar, Belle and LHCb [4–7], provided by the UTfit Collaboration and CKMfitter group, are $\gamma = (70.1 \pm 7.1)^\circ$ and $\gamma = (68.0_{-8.5}^{+8.0})^\circ$, respectively [8,9].

An alternative strategy to determine $\gamma$ using two-body charmless $B$ decays, namely $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+K^-$, has also been proposed [10–12]. Knowledge of the $B^0$ mixing phase $2\beta$, where $\beta = \arg \left(-\frac{V_{ub}V_{tb}^*}{V_{cb}V_{tb}^*}\right)$, is needed as an input. Due to the presence of penguin diagrams in the decay amplitudes, in addition to tree diagrams, the interpretation of the observables requires knowledge of hadronic factors that cannot at present be calculated accurately from quantum chromodynamics (QCD). However, the hadronic parameters entering the $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+K^-$ decays are related by the U-spin symmetry of strong interactions. This symmetry, related to the exchange of $d$ and $s$ quarks in the decay diagrams, can be exploited to determine the unknown hadronic factors. A more sophisticated analysis has also been proposed [13], where it is suggested to combine the U-spin analysis of $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+K^-$ decays with the isospin analysis of $B^0 \to \pi^0\pi^0$, $B^0 \to \pi^0\bar{\pi}^0$ and $B^+ \to \pi^+\pi^0$ decays [14], in order to achieve a more robust determination of $\gamma$ with respect to U-spin breaking effects. The $B^0$ mixing phase $-2\beta_3$, where $\beta_3 = \arg \left(-\frac{V_{ub}V_{tb}^*}{V_{cb}V_{tb}^*}\right)$, can also be determined with either analysis approach. 

An analysis based on Bayesian statistics, aimed at determining probability density functions (PDFs) for $\gamma$ and $-2\beta_3$, is presented in this Letter. This uses the latest LHCb measurements of time-dependent $CP$ violation in the $B^0 \to K^+K^-$ decay, exploiting U-spin symmetry with the $B^0 \to \pi^+\pi^-$ decay. An extended analysis, including measurements on $B^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ decays from other experiments, is also performed. The Letter is organized as follows. First, the theoretical formalism needed to describe $CP$ violation is introduced in Section 2, including the SM parameterization of the decay amplitudes of the various decays.

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1 The measurements of $\gamma$ are given modulo 180$^\circ$ throughout this Letter.
The experimental status is given in Section 3. In Section 4 we present the determination of $\gamma$ and $-2\beta_0$ using $B^0 \to \pi^+\pi^-$ and $B^0_d \to K^+K^-$ decays, and in Section 5 we also add information from $B^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ decays. The dependence of the measurements of $\gamma$ and $-2\beta_0$ on the amount of U-spin breaking is studied in detail in both cases. Finally, conclusions are drawn in Section 6.

2. Theoretical formalism

Assuming CPT invariance, the CP asymmetry as a function of decay time for a neutral $B^0$ or $\bar{B}^0$ meson decaying to a self-conjugate final state $f$, with $f = \pi^+\pi^-$, $\pi^0\pi^0$ or $K^+K^-$, is given by

$$\mathcal{A}(t) \equiv \frac{\Gamma^\text{B}_0 \to f(t) - \Gamma^\text{B}_0 \to \bar{f}(t)}{\Gamma^\text{B}_0 \to f(t) + \Gamma^\text{B}_0 \to \bar{f}(t)} = -\frac{C_f \cos(\Delta m_{t0} t) + S_f \sin(\Delta m_{t0} t)}{\cos(\frac{\Delta m_{t0} t}{2}) + A_f \sinh(\frac{\Delta m_{t0} t}{2})},$$

(1)

where $\Delta m_{t0} \equiv m_{t0} - m_{t0}$, and $\Delta m_{t0} \equiv \Delta m_{t0} = \Delta m_{t0} - \Delta m_{t0}$. The mass and width differences of the $B^0$ or $\bar{B}^0$ system mass eigenstates. The subscripts H and L denote the heavy and light eigenstates. With this convention, the value of $\Delta m_{t0}$ is positive by definition, and that of $\Delta m_{t0}$ is measured to be positive [15], $\Delta m = 0.106 \pm 0.011$ (stat) $\pm 0.007$ (syst) ps$^{-1}$ [16]. The value of $\Delta m_{t0}$ is also positive in the SM and is expected to be much smaller than that of $\Delta m_{t0}$, $\Delta m \sim 3 \times 10^{-3}$ (stat) $\pm 3 \times 10^{-3}$ ps$^{-1}$ [8]. The quantities $C_f$, $S_f$ and $A_f$ are

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2},$$

$$S_f = \frac{2|\lambda_f|^2}{1 + |\lambda_f|^2},$$

$$\lambda_f \equiv \frac{q}{p} \bar{A}_f,$$

(2)

where $\lambda_f$ is given by

$$|\lambda_f|^2 \equiv \frac{2 \text{Re}\lambda_f}{1 + |\lambda_f|^2}.$$

(3)

The two mass eigenstates of the effective Hamiltonian in the $B^0(\bar{B}^0)$ system are $pB^0(\bar{B}^0)$, where $\bar{q}|\bar{p}|$, and $p$ and $q$ are complex parameters satisfying the relation $|p|^2 + |q|^2 = 1$. The parameter $\lambda_f$ is thus related to $pB^0(\bar{B}^0)$ mixing (via $q/p$) and to the decay amplitudes of the $B^0(\bar{B}^0) \to f$ decay $A_f$ and of the $B^0(\bar{B}^0) \to \bar{f}$ decay $A_f$. Assuming negligible CP violation in mixing ($|q/p| = 1$), as expected in the SM and supported by current experimental determinations [17,18], the terms $C_f$ and $S_f$ parameterize CP violation in the decay and in the interference between mixing and decay, respectively. From the definitions given in Eq. (2), it follows that

$$(C_f)^2 + (S_f)^2 + (A_f) = 1.$$}

(4)

It is then possible to express the magnitude (but not the sign) of $A_{f}$ as a function of $C_f$ and $S_f$. There are therefore two independent parameters, which can be chosen, for example, to be $\text{Re}\lambda_f$ and $\text{Im}\lambda_f$, or $C_f$ and $S_f$. In the latter case, the sign of $A_{f}$ carries additional information.

The CP-averaged branching fraction is given by

$$B_f = \frac{1}{2}\left(B^0 \to f\right)\left(\bar{A}_f \right)^2 + |A_f|^2,$$

(5)

where

$$F\left(B^0 \to \pi^+\pi^-ight) = \frac{\sqrt{m_{B^0}^2 - 4m_\pi^2}}{m_{B^0}^2 - 4m_\pi^2},$$

$$F\left(B^0 \to \pi^0\pi^0\right) = \frac{\sqrt{m_{B^0}^2 - 4m_\pi^2}}{m_{B^0}^2 - 4m_\pi^2},$$

$$F\left(B^0 \to K^+K^-ight) = \frac{2m_{B^0}^2 - 4m_{K^+}^2}{2m_{B^0}^2 - 4m_{K^+}^2}.$$
The LHCb measurement of $C_{K^+ K^-}$ and $S_{K^+ K^-}$ in Ref. [25] was obtained using the constraint

$$A_{K^+ K^-}^{\Lambda} = -\sqrt{1 - (C_{K^+ K^-})^2 - (S_{K^+ K^-})^2}$$

in the maximum likelihood fit. In the same analysis, the sign of $A_{K^+ K^-}^{\Lambda}$ was verified to be negative, as expected in the SM. A measurement of $A_{K^+ K^-}^{\Lambda}$ has also been made by LHCb via an effective lifetime measurement of the $B^0 \rightarrow K^+K^-$ decay, using the same data sample as in Ref. [25], but with different event selection. The result is $A_{K^+ K^-}^{\Lambda} = -0.87 \pm 0.17 \text{(stat) } \pm 0.13 \text{(syst)}$ [29]. In the analysis presented in this Letter, $A_{K^+ K^-}^{\Lambda}$ is constrained to have a negative value.

4. Determination of $\gamma$ and $-2\beta_s$ from $B^0 \rightarrow \pi^+ \pi^-$ and $B_s^0 \rightarrow K^+ K^-$ decays

A method to determine $\gamma$ and $-2\beta_s$ using CP asymmetries and branching fractions of $B^0 \rightarrow \pi^+ \pi^-$ and $B_s^0 \rightarrow K^+ K^-$ decays, exploiting the approximate U-spin symmetry of strong interactions, was proposed in Refs. [10–12]. Typical U-spin breaking corrections are expected to be around the 30% level [30,31]. In the limit of strict U-spin symmetry, one has $d = d', \vartheta = \vartheta'$, and $|D| = |D'|$. As pointed out in Ref. [10], the equalities $d = d'$ and $\vartheta = \vartheta'$ do not receive U-spin breaking corrections within the factorization approximation, in contrast to the equality $|D| = |D'|$.

$$D' = \frac{f_K}{f_I} \left( \frac{m_K^2}{m_{B_s}^2} \right) \frac{m_{B_s}^2 - m_K^2}{\Gamma_K^2}$$

where $f_K$ and $f_I$ are the kaon and pion decay constants, and $m_K$ and $m_{B_s}$ parameterize hadronic matrix elements. These quantities have been determined using QCD sum rules [32], yielding

$$D' = 1.41^{+0.20}_{-0.11}.$$ 

To take into account non-factorizable U-spin breaking corrections, we parameterize the effect of the breaking as

$$|D'| \rightarrow |D| \left( 1 + r_D e^{i(\delta_0 + \vartheta)} \right),$$

and

$$|D'| \rightarrow |D| \left( 1 + r_C e^{i(\delta_0 + \vartheta + \vartheta_0)} \right),$$

where $r_D$ and $r_C$ are relative corrections, and $\delta_0$ and $\vartheta_0$ are phase shifts caused by the breaking. In the absence of non-factorizable U-spin breaking, one has $r_D = 0$ and $r_C = 0$.

We perform two distinct analyses, to determine either $\gamma$ or $-2\beta_s$. They are referred to as analyses A and B, respectively. To improve the precision on the determination of $\gamma$, in analysis A the value of $-2\beta_s$ is constrained as

$$-2\beta_s = -2\lambda^2 \eta \left[ 1 + \lambda^2 \left( 1 - \rho \right) \right]$$

which is valid in the SM up to terms of order $\lambda^4$. The parameters $\rho$ and $\eta$ determine the apex of the UT, and are defined as $\rho = i\eta \equiv -i(V_{ud} V_{ub}^*)/(V_{ud} V_{ub}^*)^*$. Since $\rho$ and $\eta$ can be written as functions of $\beta$ and $\gamma$ as

$$\rho = \frac{\sin \beta \cos \gamma}{\sin(\beta + \gamma)} \quad \eta = \frac{\sin \beta \sin \gamma}{\sin(\beta + \gamma)},$$

we can express $-2\beta_s$ in terms of $\beta$ and $\gamma$. To determine $-2\beta_s$, in analysis B, the world average value of $\gamma$ from tree-level decays,
\(\gamma = (70.1 \pm 7.1)\)° [8], is used as an input, and \(-2\beta_i\) is left as a free parameter.

The inputs to the analyses are the measured values of \(C_{\pi^+\pi^-}, S_{\pi^+\pi^-}, C_{K^-\pi^+}, S_{K^-\pi^+}, B_{\pi^+\pi^-}^{B_{S}}\) and \(B_{K^-\pi^+}^{B_{S}}\). The corresponding constraints are given in Eqs. (12), (13), (16), (17), (18) and (21). In addition, the value of \(\Delta\theta_i^{max}\) is fixed to be negative. A summary of the experimental inputs is given in Table 2.

In both analyses, flat prior probability distributions, hereinafter referred to as priors, on \(d, \theta, t_{\theta}, t_{\rho}, \sigma_{t_{\theta}}, \sigma_{t_{\rho}}\), and, where appropriate, on \(\gamma\) and \(-2\beta_i\) are used. In particular, we allow the U-spin breaking phases \(\theta_{t_{\theta}}\) and \(\sigma_{t_{\rho}}\) to be completely undetermined, using flat priors between \(-180°\) and \(180°\). Concerning the parameters \(t_{\theta}\) and \(t_{\rho}\), we adopt uniform priors between 0 and \(\kappa\), where \(\kappa\) represents the maximum magnitude of non-factorizable U-spin breaking allowed. The ranges of the flat priors are summarized in Table 3. We study the sensitivity on \(\gamma\) and \(-2\beta_i\) as a function of \(\kappa\), ranging from 0 to 1, meaning from 0% up to 100% non-factorizable U-spin breaking. For all experimental inputs we use Gaussian PDFs. The values of \(|D'|, d'\) and \(\sigma'\) are determined using Eqs. (29) and (30).

The dependences on \(\kappa\) of the 68% and 95% posterior probability intervals for \(\gamma\) and \(-2\beta_i\) are shown in Fig. 1. When the allowed amount of U-spin breaking becomes large enough, the PDF for \(\gamma\) is poorly constrained. In particular, it can be noted that for values of \(\kappa\) exceeding 0.6 the sensitivity on \(\gamma\) reduces significantly as a function of increasing \(\kappa\). This fast transition is related to the non-linearity of the constraint equations. For \(-2\beta_i\), the dependence of the sensitivity on \(\kappa\) is mild, but for values of \(\kappa\) exceeding 0.6 a slight shift of the distribution towards more negative values is observed.

In Fig. 2 we show the PDFs for \(\gamma\) obtained from analysis A and for \(-2\beta_i\) obtained from analysis B, corresponding to \(\kappa = 0.5\). The numerical results from both analyses are reported in Table 4. The 68% probability interval for \(\gamma\) is [56°, 70°], and that for \(-2\beta_i\) is [−0.28, 0.02] rad.

### 5. Inclusion of physics observables from \(B^0 \to \pi^0\pi^0\) and \(B^+ \to \pi^+\pi^0\) decays

A method to determine the angle \(\alpha\) of the UT using CP asymmetries and branching fractions of \(B^0 \to \pi^+\pi^-\), \(B^0 \to \pi^0\pi^0\) and \(B^+ \to \pi^+\pi^0\) decays was proposed in Ref. [14]. This method relies on the isospin symmetry of strong interactions and on the assumption of negligible contributions from electroweak penguin amplitudes. Isospin breaking and electroweak Penguin contributions are known to be small, and their impact on the determination of the weak phase is at the level of 1° [36–39]. In Ref. [13] it was suggested to combine the isospin-based technique of Ref. [14] with that of Ref. [10] based on U-spin. Here we extend the study presented in Section 4 by including the experimental information on \(B^0 \to \pi^0\pi^0\) and \(B^+ \to \pi^+\pi^0\) decays, i.e., using also the observables \(C_{\pi^+\pi^0}, B_{\pi^+\pi^0}^{B_{S}}\) and \(B_{K^-\pi^+}^{B_{S}}\). The corresponding constraints are given in Eqs. (14), (19) and (20).

In complete analogy with the study presented in Section 4, we perform two distinct analyses, to determine either \(\gamma\) or \(-2\beta_i\). They are referred to as analyses C and D, respectively. In analysis C, the value of \(-2\beta_i\) is constrained as a function of \(\beta\) and \(\gamma\), and \(\gamma\) is determined, whereas in analysis D, the world average
value of $\gamma$ from tree-level decays is used as an input and $-2\beta_s$ is determined. A summary of the experimental inputs is given in Table 5.

In both analyses, flat priors on $d$, $\vartheta$, $q$, $\vartheta_2$, $r_D$, $\delta_{1\tau}$, $r_C$, $\delta_{1c}$ and, where appropriate, on $\gamma$ and $-2\beta_s$ are used. The ranges of the flat priors are summarized in Table 6. For all experimental inputs we use Gaussian PDFs. The values of $|D'|$, $d'$ and $\vartheta'$ are again determined using Eqs. (29) and (30).

The dependences on $\kappa$ of the 68% and 95% probability intervals for $\gamma$ and $-2\beta_s$ are shown in Fig. 3. Again, for $-2\beta_s$, the dependence of the sensitivity on $\kappa$ is very weak. In Fig. 4 we show the PDFs for $\gamma$ obtained from analysis C and for $-2\beta_s$ obtained from analysis D, corresponding to $\kappa = 0.5$. The numerical results from both analyses are reported in Table 7. The 68% probability interval for $\gamma$ is $[57^\circ, 71^\circ]$, and that for $-2\beta_s$ is $[-0.28, 0.02]$ rad.

It is worth emphasizing that, although this study is similar to that presented in Ref. [13], there are two relevant differences, in addition to the use of updated experimental inputs. First, the upper limits of the priors on $d$ and $q$ are chosen to be much larger, to include all nonzero likelihood regions and to remove any sizable dependence of the results on the choice of the priors. In particular, this leads to a bigger impact of U-spin breaking effects at very large $\kappa$ values. Second, the adopted parameterization of non-factorizable U-spin breaking is slightly different, in order to propagate equally the effects of the breaking on every topology contributing to the total decay amplitudes.

Table 5

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Analysis A</th>
<th>68% prob.</th>
<th>95% prob.</th>
<th>Analysis B</th>
<th>68% prob.</th>
<th>95% prob.</th>
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<td>$d$</td>
<td>0.32, 0.53</td>
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<td>0.36, 0.58</td>
<td>0.29, 0.75</td>
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<tr>
<td>$\vartheta$</td>
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<td>119, 165</td>
<td></td>
<td>141, 157</td>
<td>129, 163</td>
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<tr>
<td>$\vartheta'$</td>
<td>0.33, 0.50</td>
<td>0.28, 0.65</td>
<td></td>
<td>0.34, 0.52</td>
<td>0.28, 0.69</td>
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<tr>
<td>$\omega'$</td>
<td>132, 160</td>
<td>114, 176</td>
<td></td>
<td>132, 160</td>
<td>117, 175</td>
<td></td>
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<tr>
<td>$</td>
<td>D</td>
<td>$ [MeV ps$^{-1}$]</td>
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<td>0.094, 0.121</td>
<td></td>
<td>0.101, 0.112</td>
</tr>
<tr>
<td>$</td>
<td>D'</td>
<td>$ [MeV ps$^{-1}$]</td>
<td>0.130, 0.195</td>
<td>0.097, 0.231</td>
<td></td>
<td>0.122, 0.188</td>
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<tr>
<td>$\gamma$</td>
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<td>49, 82</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-2\beta_s$ [rad]</td>
<td>-</td>
<td>-</td>
<td></td>
<td>$-0.28, 0.02$</td>
<td>$-0.44, 0.17$</td>
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Fig. 1. Dependences of the 68% (hatched areas) and 95% (filled areas) probability intervals on the allowed amount of non-factorizable U-spin breaking, for (a) $\gamma$ from analysis A and (b) $-2\beta_s$ from analysis B.

Fig. 2. Distributions of (a) $\gamma$ from analysis A and (b) $-2\beta_s$ from analysis B, corresponding to $\kappa = 0.5$. The hatched areas correspond to 68% probability intervals, whereas the filled areas correspond to 95% probability intervals.

6. Results and conclusions

Using the latest LHCb measurements of time-dependent CP violation in the $B^0 \rightarrow K^+ K^-$ decay and the approaches outlined in Refs. [10,13], the angle $\gamma$ of the unitarity triangle and the $B^0$ mixing phase $-2\beta_s$ have been determined. The approach of Ref. [10] relies on the use of the U-spin symmetry of strong interactions relating $B^0 \rightarrow K^+ K^-$ with $B^0 \rightarrow \pi^+ \pi^-$ decay amplitudes, whereas that of Ref. [13] relies on both isospin and U-spin

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In the details provided, the content is structured as follows:

- **Fig. 1**: Dependent probabilities are shown, with hatched and filled areas indicating 68% and 95% probability intervals, respectively. The plots illustrate the dependence of $\gamma$ and $-2\beta_s$ on their respective parameters.

- **Fig. 2**: Distribution plots for $\gamma$ and $-2\beta_s$, showing hatched and filled areas corresponding to 68% and 95% probability intervals, respectively.

- **Table 5**: Lists the input parameters across different analyses, summarizing the 68% and 95% probability intervals for various quantities.

- **Table 6**: Provides a summary of the flat priors used in the analyses.

- **Table 7**: Reports the numerical results from both analyses, highlighting the probability intervals for $\gamma$ and $-2\beta_s$.

- **Section 6**: Outlines the conclusions reached by analyzing the data, focusing on the importance of the results and their implications for CP violation and U-spin breaking.
symmetries by combining the methods proposed in Refs. [10] and [14], i.e. considering also the information from $B^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ decays. To follow the latter approach, measure-
ments solely coming from other experiments have been included in the analysis.

We have studied the impact of large non-factorizable U-spin breaking corrections on the determination of $\gamma$ and $-2\beta$. The relevant results in terms of 68% and 95% probability intervals, which include uncertainties due to non-factorizable U-spin breaking effects up to 50%, are summarized in Fig. 5. Typical U-spin breaking effects, including factorizable contributions, are expected to be much smaller, around the 30% level [30,31].

With up to 50% non-factorizable U-spin breaking, the approach of Ref. [13] gives marginal improvements in precision with respect to that of Ref. [10]. The former approach gives considerably more robust results for larger U-spin breaking values. Following the approach of Ref. [13] and taking the most probable value as central value, at 68% probability we obtain

$$\gamma = (63.5^{+7.2}_{-6.7})^\circ.$$

\begin{table}
\caption{Experimental inputs used for the determination of $\gamma$ and $-2\beta$, from $B^0 \to \pi^0\pi^0$, $B^0 \to \pi^0\pi^+$, $B^+ \to \pi^+\pi^0$, and $B^+ \to K^+K^-$, using isospin and U-spin symmetries. The parameter $\rho(X,Y)$ is the statistical correlation between $X$ and $Y$. For $C_{\gamma\pi\gamma\pi}$ and $S_{\gamma\pi\gamma\pi}$ we perform our own weighted average of BaBar, Belle and LHCb results, accounting for correlations.}
\begin{tabular}{lll}
\hline
Quantity & Value & Source \\
\hline
$C_{\gamma\pi\gamma\pi}$ & $-0.30 \pm 0.05$ & This Letter \\
$S_{\gamma\pi\gamma\pi}$ & $-0.66 \pm 0.06$ & This Letter \\
$\rho(C_{\gamma\pi\pi\pi}, S_{\gamma\pi\pi\pi})$ & $-0.007$ & This Letter \\
$C_{B^0K^+K^-}$ & $-0.43 \pm 0.24$ & HFAG [17] \\
$C_{B^+\pi^+\pi^-}$ & $0.14 \pm 0.11$ & LHCb [25] \\
$S_{B^0K^+K^-}$ & $0.30 \pm 0.13$ & LHCb [25] \\
$\rho(C_{K^+K^-}, S_{K^+K^-})$ & $0.02$ & LHCb [25] \\
$B_{\gamma\pi\pi\pi} \times 10^6$ & $5.10 \pm 0.19$ & HFAG [17] \\
$B_{\gamma\pi\pi\pi} \times 10^6$ & $5.48 \pm 0.33$ & HFAG [17] \\
$B_{\gamma\pi\pi\pi} \times 10^6$ & $1.91 \pm 0.23$ & HFAG [17] \\
$B_{\gamma\pi\pi\pi} \times 10^6$ & $24.5 \pm 1.8$ & HFAG [17] \\
$\sin2\beta$ & $0.682 \pm 0.019$ & HFAG [17] \\
$\gamma$ (analysis D only) & $(70.1 \pm 7.1)^\circ$ & UTBH [8] \\
$\lambda$ & $0.2253 \pm 0.0007$ & PDG [33] \\
m_{B^0} [\text{MeV}/c^2]$ & $5279.55 \pm 0.26$ & PDG [33] \\
m_{B^+} [\text{MeV}/c^2]$ & $5279.25 \pm 0.26$ & PDG [33] \\
m_{B^0} [\text{MeV}/c^2]$ & $5366.7 \pm 0.4$ & PDG [33] \\
m_{B^+} [\text{MeV}/c^2]$ & $139.570 \pm 0.00035$ & PDG [33] \\
m_{B^0} [\text{MeV}/c^2]$ & $134.9766 \pm 0.0006$ & PDG [33] \\
m_{B^0} [\text{MeV}/c^2]$ & $493.677 \pm 0.011$ & PDG [33] \\
t_\gamma [\text{ps}] & $1.519 \pm 0.007$ & HFAG [17] \\
t_\rho [\text{ps}] & $1.641 \pm 0.008$ & HFAG [17] \\
t_\sigma [\text{ps}] & $1.516 \pm 0.011$ & HFAG [17] \\
$\Delta m_{\tau}/m_{\tau}$ & $0.160 \pm 0.020$ & LHCb [16] \\
$\tau(B^0 \to K^+K^-)$ & $1.452 \pm 0.042$ & LHCb [17,34,35] \\
\hline
\end{tabular}
\end{table}

\begin{table}
\caption{Ranges of flat priors used for the determination of $\gamma$ and $-2\beta$, from $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^0$, and $B^+ \to K^+K^-$, using isospin and U-spin symmetries.}
\begin{tabular}{ll}
\hline
Quantity & Prior range \\
\hline
$\theta$ & $[0, 20]$ \\
$q$ & $[-180^\circ, 180^\circ]$ \\
$\theta_\rho$ & $[0, 20]$ \\
$\theta_\sigma$ & $[-180^\circ, 180^\circ]$ \\
$\theta_\tau$ & $[0, 20]$ \\
$\theta_\kappa$ & $[-180^\circ, 180^\circ]$ \\
$\gamma$ (analysis C only) & $[-180^\circ, 180^\circ]$ \\
$-2\beta$ [rad] (analysis D only) & $[-\pi, \pi]$ \\
\hline
\end{tabular}
\end{table}

Fig. 3. Dependences of the 68% (hatched areas) and 95% (filled areas) probability intervals on the allowed amount of non-factorizable U-spin breaking, for (a) $\gamma$ from analysis C and (b) $-2\beta$, from analysis D.

Fig. 4. Distributions of (a) $\gamma$ from analysis C and (b) $-2\beta$, from analysis D, corresponding to $\kappa = 0.5$. The hatched areas correspond to 68% probability intervals, whereas the filled areas correspond to 95% probability intervals.
Table 7

Results obtained from analyses C and D with $\kappa = 0.5$. The results are given modulo $180^\circ$ for $\phi$, $\phi'$ and $\gamma$.

| Quantity | Analysis C | | Analysis D |
|----------|------------|------------|
|          | 68% prob. | 95% prob. | 68% prob. | 95% prob. |
| $d$      | [0.33, 0.57] | [0.28, 0.79] | [0.37, 0.59] | [0.31, 0.77] |
| $\phi$   | [139°, 157°] | [125°, 164°] | [142°, 157°] | [132°, 163°] |
| $d'$     | [0.34, 0.50] | [0.28, 0.65] | [0.34, 0.52] | [0.29, 0.70] |
| $\phi'$  | [112°, 160°] | [109°, 176°] | [113°, 160°] | [119°, 176°] |
| $q$      | [0.04, 1.21] | [0.04, 1.10] | [0.04, 1.11] | [0.05, 1.30] |
| $q_2$    | [82°, 85°] | [88°, 85°] | [88°, 85°] | [85°, 85°] |
| $|D|/|D'|$ | [0.101, 0.113] | [0.094, 0.118] | [0.100, 0.111] | [0.094, 0.116] |
| $|D'/D|/|D'|$ | [0.129, 0.193] | [0.097, 0.228] | [0.122, 0.187] | [0.089, 0.221] |
| $\gamma$ | [57°, 71°] | [52°, 82°] | [52°, 82°] | [52°, 82°] |
| $-2\beta_s$ [rad] | – | – | [–0.28, 0.02] | [–0.44, 0.17] |

Fig. 5. Results for (top) $\gamma$ and (bottom) $-2\beta_s$ with 50% ($\kappa = 0.5$) non-factorizable U-spin breaking. As a comparison, other reference values are also reported. The most likely values are indicated by the vertical lines inside the boxes. The boxes and the error bars delimit the 68% and 95% probability intervals, respectively.

and, in an alternative analysis,

$$-2\beta_s = -0.12_{-0.16}^{+0.14} \text{ rad}.$$  

These results have been verified to be robust with respect to the choice of the priors and of the parameterization of non-factorizable U-spin breaking contributions. The value of $\gamma$ shows no significant deviation from the averages of $\gamma$ from tree-level decays provided by the UTfit Collaboration and the CKMfitter group that quote $\gamma = (70.1 \pm 7.1)^\circ$ and $\gamma = (68.0 \pm 8.5)^\circ$, respectively [8,9]. Analogously, the value of $-2\beta_s$ is compatible with the LHCb result from $b \to c\bar{c}s$ transitions, $\phi_s = 0.01 \pm 0.07$ (stat) $\pm 0.01$ (syst) rad [16], obtained using a data sample of $pp$ collisions corresponding to an integrated luminosity of 1.0 fb$^{-1}$. 

In summary, the value of $\gamma$ from charmless two-body decays of beauty mesons is found to be compatible and competitive with that from tree-level decays. However, since the impact of U-spin breaking corrections is significant, further improvements in the measurement of $\gamma$ are primarily limited by theoretical understanding of U-spin breaking. By contrast, the impact of U-spin breaking effects on the value of $-2\beta_s$ is small, and significant improvements are anticipated with the advent of larger samples of data. It is worth emphasizing that the information on $-2\beta_s$ comes solely from the measurement of $CP$ violation in the $B^0 \to K^+ K^-$ decay [25], also based on a data sample of $pp$ collisions corresponding to an integrated luminosity of 1.0 fb$^{-1}$. At present, the overall uncertainty on $-2\beta_s$, which also includes theoretical uncertainties, is only two times larger than that obtained using $b \to c\bar{c}s$ transitions, as reported above.

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